Stairway to Haven

Alejandro Rojas-Bernal*

Vancouver School of Economics, The University of British Columbia

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Abstract

This paper attempts to identify the main channels for the propagation of the macroeconomic effects from corporate profit shifting into tax havens. This question is answered by building a general equilibrium model that introduces firm profit shifting to tax havens in a multi-country environment with production networks. In this model, haven jurisdictions specialize and compete for shifted profits by trading concealment assets in a differentiated oligopolistic environment, and non-haven countries defend these profits by setting enforcement levels over capital flows. The central point of the model is that profit shifting introduces two classes of optimal distortions, first, rebated distortions that by modifying the terms of trade and the effective marginal tax rate alter the decision of firms, but also wasted distortions that optimally squander resources via enforcement policies and the corporate costs that firms have to incur in order to access and develop concealment strategies. I show that the main transmission channels for the propagation of these distortions occurs by increasing corporate dividends, the tax base, and wages in tax havens; while non-haven countries are affected by opposite effects in addition to the wasted distortions. We confirm these results in a three country one sector global economy that additionally provides evidence about the relevance of the structure of the production network and the consumption bundle in the magnitude of the effect from introducing profit shifting.

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1 Introduction

The complex entanglement of global finance with the concealment of wealth and assets provided by tax havens is an increasing source of instability for modern democracies. This challenges the possibility of a sustainable, fair fiscal and social state. By 2014, global household financial wealth added up to around $95.5 trillion, out of which, based on a conservative estimate, 8% or $7.6 trillion were in accounts located in haven jurisdictions, out of which 30% or around $2.3 trillion were held in Switzerland (Zucman, 2015).

The challenge that tax havens impose on modern democracies has been rising steadily since the Second World War and its effects are particularly concerning for emerging and developing countries with nascent fiscal systems. For instance, the wealth of Europeans located in tax havens had increased from around 2% in the post-war period to around 10% by 2013. By 2014, the share of financial wealth held offshore was 4% for the United States, 4% for Asia, 9% for Canada, 10% for Europe, 22% for Latin America, 30% for Africa, 52% for Russia and 57% for Gulf Countries. The literature on the effects from haven jurisdictions has been focused on tax revenue loss, which for 2014 was estimated globally at around $190 billions (bn), disaggregated locally in revenue losses of $0 bn for Gulf Countries, $1 bn for Russia, $6 bn for Canada, $14 bn for Africa, $21 bn for Latin America, $34 bn for Asia, $35 bn for the United States and $78 bn for Europe (Zucman, 2014, 2013, 2015).

Multinational firms are one of the main users of tax havens. The corporate sector globally raises nearly 8% of its equity and 10% of its bond financing via foreign subsidiaries located in tax havens. There are four main statistical consequences that have been unveiled by Coppola, Maggiori, Neiman, & Schreger (2020) from the use of tax havens by multinational corporations. First, the paradoxical small size of North-to-South capital flows (Lucas, 1990) is significantly larger once capital positions in third countries are taken into account, and this difference is primarily reflected by issuance of securities in tax havens.[1] Second, foreign currency denominated corporate bonds from emerging markets has a more significant role in capital flows than traditionally thought. As a consequence, contrary to traditional residency-based indicators, corporate bonds overshadow the importance of sovereign bonds, and there is a greater liability translation exposure for emerging economies.[2]. Finally, some investment positions are incorrectly considered foreign investment.[3]

[1]For instance, the holdings from the United States in Chinese equities in 2017 increases from $160 billion to $700 billion once positions located in third countries are taken into account, mainly those concerned with structures designed to avoid China's capital controls.

[2]For example, under traditional measures, U.S. investors hold three time more Brazilian government bonds than Brazilian corporate bonds, and the debt exposure to foreign currency is 20%, while once positions located in third countries are taken into account, corporate bonds double sovereign bonds and the debt exposure to foreign currency is 50%

[3]As an example, 12% of the foreign bond and 7% of the foreign equity holdings from the United States are actually domestic investment.

Corporations have a myriad of incentives to use tax havens (Coppola et al., 2020). First, by issuing securities via subsidiaries in tax havens such as Bermuda, the Cayman Islands, Guernsey, or Jersey, firms can eliminate completely the mandatory withholding on dividend and interest payments in such a way that the full payment is reflected on the funds' returns. Second, firms can reduce their effective corporate tax rate via strategies such as tax inversion (Seida & Wempe, 2004; Hwang, 2014; Marples & Gravelle, 2014; Capurso, 2016), transfer pricing (Swenson, 2001; Bartelsman & Beetsma, 2003; Clausing, 2003; Overesch, 2006; Neiman, 2010; Vicard, 2015; Cristea & Nguyen, 2016; Davies, Martin, Parenti, & Toubal, 2018), and intercorporate financing at higher rates via haven subsidiaries with the objective to locate after-interest profits on the haven country affiliate (Hines Jr & Rice, 1994). Third, to avoid capital controls that restrict foreign corporate ownership (Gillis & Lowry, 2014; Ziegler, 2016; Hopkins, Lang, & Zhao, 2016). Fourth, to hurdle specific corporate regulations, as for example the European Union Market Abuse Regulation that imposes disclosure of trades made by any manager, but does not apply to the Channel Islands, i.e. Guernsey and Jersey, which has generated an issuance shift of more informationally sensitive securities on these territories. Finally, emerging market firms have better access to capital flows originating in developed countries when securities are issued via a tax haven subsidiary.

The use of tax havens with the objective of minimizing tax obligations is influenced by intracorporate linkages and internal supply chains. Tax inversion, transfer pricing, and intercorporate financing strategies take advantage of the global structure of the multinational corporation and the location of its affiliates. In particular, Davies et al. (2018) present evidence that supply chains are relevant to explain corporate profit shifting via transfer pricing, as the export prices from French multinationals drops with the statutory corporate tax from the destination for intrafirm transactions that systematically involve tax havens as their destination.

Motivated by this, I address the gap in the literature about the channels through which corporate profit shifting into tax havens generates macroeconomic effects. With this objective, I assemble a multi-country general equilibrium model with production networks, and distortions at the sector level, in which multinational corporations have access to costly fiscal optimization technologies that reduce their tax expenditure by acquiring concealment financial assets from haven governments that allow them to shift profits into affiliated subsidiaries located in these jurisdictions. Simultaneously, tax havens operate in a differentiated oligopolistic environment in which they sell their heterogeneous concealment financial assets and optimally set their prices, while non haven governments influence the attractiveness of specific haven jurisdictions by setting optimal enforcement over specific cross-country capital flows. The corporate tax and the optimal decision from multinational corporations to allocate profits across its affiliate subsidiaries located in tax havens not only generate rebated distortions by modifying the terms of trade and the effective marginal tax rate, but also introduce wasted distortions that optimally squander resources.

The main question addressed in this paper is which are the channels for the propagation of the profit shifting distortions into macroeconomic effects. Adjacent questions that are also dealt with are how the intensity of these channels is influenced by: the production network; consumer preferences; the global capital allocation and supply of capital; tax differentials; the share of firms that have access to tax havens; the competitive environment from tax havens; and the role of global oversight on capital flows.

The static, multi-country, multi-sector, general equilibrium model of intersectoral trade with distortions at the sectoral level from this paper is based on Devereux, Gente, & Yu (2019) and Bigio & La'O (2020) representation of the Long & Plosser (1983) economy. In this framework, a heterogeneous set of domestic and multinational intermediate good firms at the country-sector level are connected via an intersectoral-trade market described by the input-output network, and face sectoral tax rates, markups and industry-level global capital markets.

The government of each country has access to a multidimensional and discontinuous space of competition in which it optimally sets the level of enforcement and the price of concealment assets that maximize domestic welfare. The solution to the policy problem assumes a bounded foresight of the government in which terms of trade are taken as given, which allows governments to focus on amplifying the household wealth effects. The international tax environment that I use is based on Slemrod & Wilson (2009) and Johannesen (2010). Even though my framework takes taxes and the decision to become haven jurisdictions as exogenous, it allows for specialization from tax havens on the bilateral linkages at the country-sector level. The policy variables and the amount of shifted profits are influenced by the solution to the global competitive equilibrium and the intersectoral supply chain. Under this scenario haven jurisdictions compete offensively by reducing the price of concealment financial assets with the objective to attract profits, while non-haven jurisdiction compete defensively by increasing enforcement over flows and curtailing their leakage of profits.

Under this framework, I find that corporate profit shifting parasitically relocates resources from households in non-haven countries to households in haven jurisdictions via four channels. First, by increasing corporate dividends from multinational subsidiaries located in tax havens. Second, by expanding the tax base in havens with low levels of taxation, and as a consequence increasing governmental transfers to households. Third, by increasing wages in tax havens, and as a consequence modifying the terms of trade. Finally, by creating opportunities for non-haven countries to optimally waste resources in enforcement policies, and in the corporate costs that firms have to incur in order to access and develop concealment fiscal optimization strategies.

I consider a simple three country one sector economy which I have nicknamed *The Bermuda Triangle*, because the introduction of profit shifting allows for wasted resources that vanish into thin air. In this economy there is a high, an intermediate, and a low tax country. I allow for profit shifting from the intermediate tax economy to the low tax economy, and from the high tax economy to both the low and intermediate tax economies. Introducing corporate profit shifting into this simplified scenario increases the nominal wage, the consumer price index, consumption and GDP, and creates a trade balance deficit in the low tax economy, while it has the opposite effect on the high tax economy. Additionally, there is an increase in the demand of capital from the multinational subsidiaries in the high tax jurisdiction and an increase in the global interest rate. Furthermore, the magnitude of the macroeconomic effects from introducing profit shifting into this model varies with the structure of the production network and the international tax environment, the consumption bundle, the tax differentials, the global allocation of capital, and the share of the multinational corporations across countries.

This paper connects three literature branches. First, based on the multi-sector environment from Long & Plosser (1983), there is a growing literature that studies the propagation of firm or sector specific distortions in economies with intermediate good trade (Basu, 1995; Ciccone, 2002; Yi, 2003; Jones, 2011, 2013; Asker et al., 2014; Devereux et al., 2019; Liu, 2019; Baqaee & Farhi, 2020; Bigio & La'O, 2020). This literature is based in the extensively covered question about the propagation of firm or sector specific productivity shocks through the production network (Horvath, 1998; Dupor, 1999; Horvath, 2000; Gabaix, 2011; Acemoglu et al., 2012; Carvalho, 2014). In particular, my model uses Cobb-Douglas production and utility functions just as in Jones (2013), Devereux et al. (2019) and Bigio & La'O (2020), which differentiates it from Baqaee & Farhi (2020), where general homogeneous of degree one production functions are used. Moreover, the model from this paper allows for an endogenous country specific elastic supply of labour as in Devereux et al. (2019), and Bigio & La'O (2020), which differentiates it from Jones (2013) and Baqaee & Farhi (2020), where there is an exogenous inelastic supply of labour, and for the market of capital, a more conservative inelastic global industry specific supply is imposed.

As in Bigio & La'O (2020) there is an exogenous distortion that is modeled as a tax. Additionally, there is a second distortion that comes from the decision of multinational corporations to allocate profits across its affiliated subsidiaries in tax havens. As a consequence of this second distortions: i) corporations reallocate resources in the budget of other governments by acquiring concealment assets; and ii) corporations waste resources to develop their fiscal optimization strategies. The revenue collected by the government both from taxes and from trading concealment assets is partially wasted in enforcement activities, and the leftovers are redistributed lump-sum to household. A clear difference in my model is the optimality from both the rebated distortions due to concealment asset acquisition and lump-sum transfers, and the wasted distortions due to corporate profit shifting costs and enforcement activities, while in Bigio & La'O (2020) these distortions are exogenous.

Second, this paper contributes to the literature on profit shifting to tax havens by introducing a differentiated oligopolistic environment based in Slemrod & Wilson (2009) into the imperfect competition model from Johannesen (2010). This is done by letting haven jurisdictions trade differentiated concealment financial asset. The effect that profit shifting has over the effective marginal tax rate allows multinational corporations to establish a *de facto* differentiated corporate tax rate that internalizes the *tax havens are good* argument presented by M. Desai, Foley, & Hines Jr (2006a), M. Desai, Foley, & Hines Jr (2006b), and Hong & Smart (2010). A structural contrast between our understanding of tax haven from the one in Slemrod & Wilson (2009), and Johannesen (2010), is that in these papers, haven jurisdictions are non-productive economies that have a zero statutory corporate tax rate. While, my notion of tax havens refers to economies with productive potential that receive shifted profits by selling concealment assets, and in which the statutory corporate tax rate does not have to be equal to zero. This allow me to take into account not only the small 35 countries with no corporate tax rate identified as *non-cooperating tax havens* in OECD (2000), but also countries such as Ireland and Switzerland that levy low corporate tax rates in order to attract real investment that leads to affiliated subsidiaries that can be used to shift profits from countries with higher tax rates as discussed in Hines Jr (2005).

Finally, this paper is part of the growing literature on the economic effects from corporate use of tax havens. This literature includes Hines Jr & Rice (1994), M. A. Desai, Foley, & Hines Jr (2004), Gravelle (2010), Zucman (2013, 2014, 2015), Guvenen, Mataloni Jr, Rassier, & Ruhl (2017), and Tørsløv, Wier, & Zucman (2018).

This paper is organized as follows. Section 2 lays out the model and solves the competitive equilibrium. Section 3 solves the optimal enforcement and concealment pricing policies and presents the analytical decomposition of its effects on government transfers and corporate dividends. Section 4 solves *The Bermuda Triangle* economy. Finally, section 5 concludes. The Appendix contains all proofs and supporting material.

2 The Environment

This static economy is built under a variation of the multi-country general equilibrium model of intersectoral-trade from Devereux, Gente, & Yu (2019), and the input-output model with sectoral distortions from Bigio & La'O (2020). This model contains R countries, where country r is populated by a representative household of size n_r. The total world population is normalized to unity, so that $\sum_{r=1}^{R} n_r = 1$. Additionally, the country r has N_r production sectors indexed by $i \in \{1, \ldots, N_r\}$, and in each sector there are three types of firms. Firstly, in each country-sector there is a unit mass of monopolistically-competitive firms that produce differentiated goods, indexed by $s \in [0, 1]$. This unit mass is partitioned in a share $\psi_{ri} \in [0, 1]$ of multinational subsidiaries linked to a continuum of multinational corporations for sector $i \in \left\{1, \ldots, \tilde{N}\right\}$

(where $\tilde{N} = Max_r\ N_r$). These corporations have access to a market of concealment financial assets supplied costlessly by sovereign governments that allow them to shift profits across subsidiaries. The complementary share $1-\psi_{ri}$ is composed by a set of domestic firms that have no access to the market of concealment assets. Secondly, for each country-sector there is a perfectly-competitive producer that aggregates and transforms the country-sector specific differentiated goods into a homogeneous uniform good that is traded as an input through global intersectoral-trade markets and as a final consumption good. Finally, the government of each country levies sector-specific revenue taxes from corporate gains, and from selling concealment assets to multinational corporations that allow them to transfer profits into the subsidiary located within the government jurisdiction. The government uses these resources to cover the wasteful expenses created by enforcement activities directed to curtail the size of shifted profits to other jurisdictions, and redistributes lump-sum the remainder to the domestic consumer.

2.1 Production

There are three kinds of input markets. First, the intersectoral-trade market for inputs. Second, country-specific labour markets with mobility across sectors within each country, but without migration of workers across countries. Finally, a global sector-specific market for capital, in which K stands for per capita world capital, each country-sector-specific continuum of intermediate firms is endowed with K_{ri} of these units, out of which a share ψ_{ri} belongs to the multinational subsidiaries and a fraction $1-\psi_{ri}$ to the domestic firms. Total supply on the sector-specific capital market is given by $K_i = \sum_{r=1}^{R} K_{ri}$, so that $K = \sum_{i=1}^{\tilde{N}} K_i$. Domestic firms can only use the endowed amount of capital, while multinational subsidiaries have access to this market in an unconstrained manner.

2.1.1 The Sectoral Aggregator Firm

For each country-sector there is a producer that aggregates the differentiated goods from intermediate multinational subsidiaries M and domestic firms D according to a constant elasticity of substitution production function with elasticity of substitution θ_{ri}

$$y_{ri} = \left[\int_0^{\psi_{ri}} x_{ri,Ms}^{\frac{\theta_{ri}-1}{\theta_{ri}}} ds + \int_{\psi_{ri}}^{1} x_{ri,Ds}^{\frac{\theta_{ri}-1}{\theta_{ri}}} ds\right]^{\frac{\theta_{ri}}{\theta_{ri}-1}} \tag{1}$$

where y_{ri} denotes gross output at the country-sector level, $x_{ri,Ms}$ stands for the demand of intermediate goods from multinational subsidiary s, and $x_{ri,Ds}$ is the demand of intermediate goods from domestic firm s.

The aggregator firm operates under a fully competitive environment, in equilibrium adds zero value to the global economy and has zero profits. Its purpose is to guarantee that there is a homogeneous good at the country-sector level despite the existence of heterogeneous production decisions between multinational subsidiaries and domestic firms. This type of firm demands goods from intermediate multinational subsidiaries and domestic firms to maximize its profit:

$$\underset{\left(X_{ri,Ms},X_{ri,Ds}\right)_{s\in[0,1]}}{Max} \quad \bar{\pi}_{ri} = (1-\tau_{ri})\left(P_{ri}y_{ri} - \int_0^{\psi_{ri}} P_{ri,Ms}x_{ri,Ms}ds - \int_{\psi_{ri}}^1 P_{ri,Ds}x_{ri,Ds}ds\right)$$

where P_{ri} stands for country-sector product price, $P_{ri,Ms}$ is the price for an intermediate good from multinational subsidiary s, and $P_{ri,Ds}$ is the price for an intermediate good from domestic firm s.

Demand of intermediate goods by the sectoral aggregator and price at the country-sector level satisfy

$$x_{ri,Ds} = y_{ri}\left(\frac{P_{ri}}{P_{ri,Ds}}\right)^{\theta_{ri}} \tag{2}$$

$$x_{ri,Ms} = y_{ri}\left(\frac{P_{ri}}{P_{ri,Ms}}\right)^{\theta_{ri}} \tag{3}$$

$$P_{ri} = \left(\int_0^{\psi_{ri}} P_{ri,Ms}^{1-\theta_{ri}}ds + \int_{\psi_{ri}}^1 P_{ri,Ds}^{1-\theta_{ri}}ds\right)^{\frac{1}{1-\theta_{ri}}}$$

2.1.2 Intermediate firms

Intermediate domestic firms and multinational subsidiaries within a country-sector share an identical technology that follows a constant returns to scale production function[4]

$$X_{ri,zs} = exp\left\{\alpha_{ri}\epsilon_{ri}\right\} l_{ri,zs}^{1-\alpha_{ri}-\alpha_{ri}^K} k_{ri,zs}^{\alpha_{ri}^K} M_{ri,zs}^{\alpha_{ri}} \text{ for } z \in \{M, D\} \tag{4}$$

where $X_{ri,zs}$ denotes gross output from intermediate firm s of type z, ϵ_{ri} is a common productivity term at the country-sector level, $l_{ri,zs}$ is labour demand, α_{ri}^K captures the capital cost share in production, $k_{ri,zs}$ is capital demand, α_{ri} captures the intermediate input cost share in production, and $M_{ri,zs}$ is the composite intermediate input demand

$$M_{ri,zs} = \prod_{m=1}^{R}\prod_{j=1}^{N_m} x_{rimj,zs}^{\omega_{rimj}}$$

with unitary elasticity of substitution across intermediate inputs,[5] $x_{rimj,zs}$ represents the use of country-sector mj product by firm s with type z in country-sector ri. The input-output matrix W has entries ω_{rimj} where $\sum_{m=1}^{R}\sum_{j=1}^{N_m}\omega_{rimj} = 1$, with $\omega_{rimj} \geq 0$.

[4] This follows the international trade literature Anderson & Van Wincoop (2003), Allen, Arkolakis, & Takahashi (2014) and, Caliendo, Parro, & Tsyvinski (2017).

[5] As in Acemoglu et al. (2016), Devereux et al. (2019), and Bigio & La'O (2020).

2.1.2.1 Domestic Firms

Domestic intermediate firm s in country-sector ri demands labour, capital, and intermediate inputs to solve

$$Max\ \pi_{ri,Ds} = (1-\tau_{ri})\left(P_{ri,Ds}X_{ri,Ds} - \tilde{w}_r l_{ri,Ds} - \sum_{m=1}^{R}\sum_{j=1}^{N_m} P_{mj}x_{rimj,Ds}\right) - \iota_i\left(k_{ri,Ds} - K_{ri}\right) \tag{5}$$

subject to (2) and (4). Where $X_{ri,Ds}$ is the domestic firm s output, $l_{ri,Ds}$ is labour demand by domestic firm s, $k_{ri,Ds}$ is capital demand by domestic firm s, $x_{rimj,Ds}$ is intermediate input demand by domestic firm s, τ_{ri} stands for the statutory tax over corporate gains with deductible labour and intermediate input costs for country-sector ri, $\tilde{w}_r$ is the nominal wage on country r, and ι_i is the nominal interest rate at the global sector-specific capital market.

Notwithstanding that some form of deductible capital interest costs is common across countries (OECD, 2015; Duff, 2019), the model assumes that capital interest costs are non-deductible from the tax base on corporate gains. The reason is that $\pi_{ri,zs}$ is defined as the after tax dividends, i.e. the disposable transferred resources from firms to shareholders, and by assuming a dividend tax credit that coincides with the capital interest costs, the deductible interest costs for the corporation is canceled out by the equivalent credit to the shareholder. The main consequence of this assumption in the model will be an equalization of after tax capital marginal productivity for domestic firms and multinational subsidiaries conditional on non-binding constraints for the tax base.

Let aggregate sales, domestic sales, and multinational sales in country sector ri be $S_{ri} = P_{ri}y_{ri}$, $S_{ri,D} = P_{ri}X_{ri,D}$, and $S_{ri,M} = P_{ri}X_{ri,M}$. The optimal demands from the domestic firms in country-sector ri for inputs from country-sector mj, capital, and labour are

$$P_{mj}\ x_{rimj,D} = \alpha_{ri}\ \phi_{ri}\ \omega_{rimj}\ S_{ri,D}^{\phi_{ri}}\ S_{ri}^{\frac{1}{\theta_{ri}}} \tag{6}$$

$$\iota_i\ k_{ri,D} = (1-\tau_{ri})\ \alpha_{ri}^{K}\ \phi_{ri}\ S_{ri,D}^{\phi_{ri}}\ S_{ri}^{\frac{1}{\theta_{ri}}} \tag{7}$$

$$\tilde{w}_r\ l_{ri,D} = \left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)\ \phi_{ri}\ S_{ri,D}^{\phi_{ri}}\ S_{ri}^{\frac{1}{\theta_{ri}}} \tag{8}$$

where $\phi_{ri} = \frac{\theta_{ri}-1}{\theta_{ri}}$ stands for a distortionary wedge that comes from the monopolistic markup.

2.1.2.2 Multinational Subsidiaries

The multinational corporation s in sector i demands labour, capital and intermediate inputs that are used by its subsidiary in country r, and demands concealment financial assets that allow shifting profits across its subsidiaries to solve

$$
\begin{aligned}
Max\ \pi_{i,Ms} = \sum_{r=1}^{R} \pi_{ri,Ms} = \sum_{r=1}^{R} \Bigg\{ (1-\tau_{ri}) \Bigg[P_{ri,Ms} X_{ri,Ms} - \tilde{w}_r l_{ri,Ms} - \sum_{m=1}^{R} \sum_{j=1}^{N_m} P_{mj} x_{rimj,Ms} \\
+ \sum_{m=1}^{R} (q_{mir,s} - q_{rim,s}) \Bigg] - \iota_i (k_{ri,Ms} - K_{ri}) - C_{ri,s} \Bigg\}
\end{aligned}
\tag{9}
$$

subject to (3), (4),

$$
q_{rim,s} = \frac{c_{rim,s}}{\gamma_i + b_{rim}} \tag{10}
$$

$$
C_{ri,s} = \frac{\left(\sum_{m=1}^{R} q_{rim,s}\right)^2}{2\alpha} + \frac{\sum_{m=1}^{R} q_{rim,s}^2}{2\beta} + \sum_{m=1}^{R} Q_{rim} c_{rim,s} + \Upsilon \tag{11}
$$

$$
q_{rim,s} \geq 0 \ \ \forall r, m \ \text{ and} \tag{12}
$$

$$
\Gamma_{ri,Ms} = \left[P_{ri,Ms} X_{ri,Ms} - \tilde{w}_r l_{ri,Ms} - \sum_{m=1}^{R} \sum_{j=1}^{N_m} P_{mj} x_{rimj,Ms} + \sum_{m=1}^{R} (q_{mir,s} - q_{rim,s}) \right] \geq 0 \ \ \forall r \tag{13}
$$

where $q_{rim,s}$ stands for the nominal level of shifted profits from country r to country m by multinational s in industry i, $C_{ri,s}$ is the cost function for the multinational corporations s in industry i of shifting profits out of country r, $c_{rim,s}$ is the quantity of concealment financial assets acquired from government m by the multinational corporation s in industry i that allow shifting profits from the subsidiary in country r to the subsidiary in country m, Q_{rim} is the unitary price charged by government m for these assets, b_{rim} is the level of enforcement by government of country r to capital outflows directed towards country m by firms in sector i, and γ_i is an industry common global parameter. Additionally the amount of shifted profits and tax base on corporate gains $\Gamma_{ri,Ms}$ are subject to non-negativity constraints.

The function $q_{rim,s}(c_{rim,s}, b_{rim})$ is based on Slemrod & Wilson (2009) representation of the amount of profits that can be shielded from tax authorities by acquiring concealment assets from haven jurisdictions, and satisfies $q(0,b) = 0$, $\partial q/\partial c > 0$, and $\partial^2 q/\partial c \partial b < 0$ for all $c, b \geq 0$, implying that: there are no shifted profits without concealment assets; increasing the demanded quantity of these assets increases the amount of shifted profits; and reductions in governmental enforcement augment the marginal productivity of concealment assets in the profit shifting technology. Furthermore, $q(c,0) < \infty$ due to the industry global parameter γ_i that represents the costless effect of global regulation on the oversight over capital flows from industry i. For instance, the effect of anti-money laundering, anti-terrorism financing regulations, or the moral suasion at the corporate level of principles such as source-based taxation or arm's length pricing.[6]

The function $C_{ri,s}$ describes an international tax environment in which the non-deductible costs

[6] A negative γ_i means that global regulation or corporate moral suasion abrade the effects of domestic enforcement on capital flow oversight.

of shifting profits comes not only from the cost of acquiring concealment assets from foreign governments, but also from the monetary risk of detection and the monetary effort costs from hiding tax evasion activities. The first element is the convex costs of total shifted profits, which is based on the two-country models from Haufler & Schjelderup (2000) and Stöwhase (2005), and represents the emphasis of tax or accounting auditors on inquiring into transfers with large irregularities. The second element is based on Johannesen (2010) and reflects a costs advantage from diversification in a multi-country model in a way such that shifting large amounts of profits exclusively to another jurisdiction is more noticeable and therefore carries a higher risk of detection than diversifying the portfolio of countries towards profits are shifted. The strength of this argument increases when we consider that the main corporate mechanisms to shift profits are price deviations from the arm's-length principle (transfer pricing) and intra-corporate loans with interest rates that do not match the market level. These operations are more difficult to unmask when diversified in individual operations across a portfolio of countries. The third element is the cost of buying concealment assets from independent countries and reflects the process of commercialization of state sovereignty described by Palan (2002). Under the absence of reputation costs, sovereign governments can costlessly manufacture regulatory environments in which legal vehicles such as shell companies are allowed to flourish and be used for financial maneuvers that enable shifting profits across jurisdictions. We assume that the concealment assets sold by governments grant access to these vehicles. Finally, only multinational firms that are big enough will be able to access profit shifting technologies by paying an exogenous fixed cost Υ.[7] The first two elements and the fixed cost represent the corporate resources that are wasted to develop profit shifting strategies, and the cost of acquiring concealment assets are the resources reallocated in the budget of tax haven governments.

The assumption that only multinational firms that are big enough will shift profits follows evidence from the literature on transfer-pricing as a mechanism of shifting profits. Where Davies, Martin, Parenti, & Toubal (2018) have found that multinational export prices drop with the destination corporate tax rate for intrafirm transactions directed to countries with very low tax rates that systematically involve tax havens, and this effect is concentrated in a small number of firms. Only 3.8% of the firms make intrafirm exports to the ten countries that are classified as tax havens following the definition in Hines Jr & Rice (1994), which was recently used by Dharmapala & Hines Jr (2009). With a scant 450 firms or 0.7% of the firms accounting for 90% of intrafirm exports to tax havens, and 25 firms accounting for almost 50% of intrafirm exports to these countries.[8]

As in Johannesen (2010), the parameters α and β shape the competitive environment of

[7] In our model Υ is small enough to keep the optimal marginal decision of shifting profits unaltered, but big enough to deter multinationals from demanding concealment assets when the total amount of shifted profits from country r is $\epsilon \to 0$.

[8] Davies et al. (2018) use a cross section of 64,633 French firms in 1999 that represent 98.8% of French exports and 95.2% of intrafirm exports.

countries as juridical entrepreneurs that compete for attracting the shifted profits from other jurisdictions. When $\beta \to \infty$ there is perfect competition in the sense that the role of portfolio diversification in our multi-country model becomes irrelevant and all of the profits from industry i in country r are shifted exclusively towards the jurisdiction with the highest marginal net gain from shifting profits. When $\alpha \to \infty$ there is monopolistic competition and the amount of profits shifted from country r towards country m depends exclusively on the marginal net gain from this operation.

The demands from the multinational corporations in country-sector ri of intermediate goods from country-sector mj, capital, and labour are

$$P_{mj}\; x_{rimj,M} = \alpha_{ri}\; \phi_{ri}\; \omega_{rimj}\; S_{ri,M}^{\phi_{ri}}\; S_{ri}^{\frac{1}{\theta_{ri}}}, \tag{14}$$

$$\iota_i\; k_{ri,M} = (1 - \tau_{ri} + \Omega_{ri})\;\; \alpha_{ri}^{K}\; \phi_{ri}\; S_{ri,M}^{\phi_{ri}}\; S_{ri}^{\frac{1}{\theta_{ri}}}, \tag{15}$$

$$\tilde{w}_r\; l_{ri,M} = \left(1 - \alpha_{ri} - \alpha_{ri}^{K}\right)\; \phi_{ri}\; S_{ri,M}^{\phi_{ri}}\; S_{ri}^{\frac{1}{\theta_{ri}}}. \tag{16}$$

where Ω_{ri} is the Lagrange multiplier from the non-negativity constraint over the corporate gains tax base.

Theorem 2.1. ***Symmetry - Asymmetry between domestic firms and multinational subsidiaries:***

1. *In an economy without capital, domestic firms and multinational subsidiaries are symmetric, i.e.* $X_{ri,D} = X_{ri,M}$*,* $l_{ri,D} = l_{ri,M}$*, and* $x_{rimj,D} = x_{rimj,M}$*.*

2. *In an economy with capital, domestic firms and multinational subsidiaries are asymmetric when the non-negativity constraint over the corporate gains tax base is binding and*

$$\frac{X_{ri,M}}{X_{ri,D}} = \left(\frac{1 - \tau_{ri} + \Omega_{ri}}{1 - \tau_{ri}}\right)^{\alpha_{ri}^{K}\theta_{ri}} \geq 1. \tag{17}$$

The optimality conditions in inputs between domestic firms and multinational subsidiaries differ only in the role that the Lagrange multiplier plays in the determination of capital for the latter. This multiplier satisfies $0 \leq \Omega_{ri} < \tau_{ri}$, as will be prove in the next theorem. When the multinational subsidiary has full profit shifting, i.e. when the non-negativity constraint over the corporate gains tax base is binding, the effective corporate income tax rate is lower and the after-tax marginal productivity of capital is higher than in the domestic firms, thus creating incentives for an increase in the productive capital used multinational firms in country-sector ri. In this sense, governments from countries with a high statutory tax over corporate income might find reasonable to set low levels of enforcement that allow corporations to fully shift profits out of their economies, and into subsidiaries located in haven jurisdictions, when the welfare gains from an increase in productive capital are greater than the negative welfare effects from the erosion of the tax base and the reduction in dividends by multinational subsidiaries.

This result is in line with the positive investment effect from tax havens that has been argued by M. Desai, Foley, & Hines Jr (2006a), M. Desai, Foley, & Hines Jr (2006b), and Hong & Smart (2010).

The first order condition for $q_{rim,s}$ when $q_{rim,s} > 0$ is given by

$$\frac{1}{\alpha}\sum_{h=1}^{R} q_{rih,s} + \frac{1}{\beta} q_{rim,s} + Q_{rim}\left(\gamma_i + b_{rim}\right) \leq \left(\tau_{ri} - \tau_{mi}\right) \tag{18}$$

where the right-hand side is the tax savings from shifting profits from country r to country m, and the left-hand side is the cost of this operation. This cost is decomposed into the marginal monetary cost of detection and hiding the operation, represented by the first two components, and the marginal cost of buying the amount of concealment financial assets from the government of country m that are necessary to shift one more unit of profit out of country r, represented by the third component.

Each country-sector has a ranking of preferences over jurisdictions that can be used as tax havens. This ranking of preferences is given by the net transactional gains from shifting profits. For instance, for country-sector ri the net transactional gains of shifting profits to country m are given by $\tau_{ri} - \tau_{mi} - Q_{rim}\left(\gamma_i + b_{rim}\right)$. If $\beta \to \infty$, multinational corporations in each country-sector optimally choose to shift profits only to the top jurisdiction in this ranking. Otherwise, their optimal decision involves selecting how many of the top countries in this ranking are going to be used as tax havens, and afterwards acquiring from this discrete number of countries the portfolio of concealment assets that will allow them to shift profits optimally.

The level of a country on the ranking of a country-sector can be the differentiating factor that includes it or not in the set of jurisdictions that are used as tax havens. Countries become more attractive as tax havens as the net transactional gains of shifting profits towards them increases. Once a country belongs to this list of jurisdictions that are used as tax havens, raising in the ranks of preferences increases the amount of concealment assets that are sold, and as a consequence the amount of profits that are received. For example, country m raises its position in the ranking of country-sector ri by reducing the statutory tax rate τ_{mi} or the price of concealment assets Q_{rim}, or when the government of country r reduces b_{rim}. This battle between countries to increase their position in the ranking will be described in the model by the metaphor of governments trying to locate their countries in the highest step of the country-sector stairway order.

Definition 2.1. Stairway order:

1. Let $g_{ri} : \{1, \ldots, R\} \to \{1, \ldots, R\}$ be a bijective order such that

   $$\left(\tau_{ri} - \tau_{g_{ri}(1)i}\right) - Q_{rig_{ri}(1)}\left(\gamma_i + b_{rig_{ri}(1)}\right) \geq \cdots \geq \left(\tau_{ri} - \tau_{g_{ri}(R)i}\right) - Q_{rig_{ri}(R)}\left(\gamma_i + b_{rig_{ri}(R)}\right).$$

2. $\mathscr{O}_{ri}(m) = g_{ri}^{-1}(m)$ indicates the position of country m in the stairway order g_{ri}. The

position of country r is given by $\mathscr{O}_{ri}(r) = z_{ri}$.

3. The net transactional gain of country-sector ri of shifting profits to the country of order e in the stairway order g_{ri} is given by $\eta_{ri}(e) = \left(\tau_{ri} - \tau_{g_{ri}(e)i}\right) - Q_{rig_{ri}(e)}\left(\gamma_i + b_{rig_{ri}(e)}\right) = \tau_{ri} - \Delta_{ri}(e)$.

This stairway order recognizes the possibility that certain haven jurisdictions might specialize in attracting shifted profits from specific country-sectors. The specialization of haven jurisdictions at the country level is justified by the way in which the international tax system is built upon bilateral treaties. These treaties modify the net transactional gains of shifting profits across the party jurisdictions. For instance, Luxembourg funds that invest in stocks from the United States have to pay no tax on dividends to the American government, and in the Grand Duchy neither earned or distributed dividends from these funds are taxed; the same story applies for funds in the Cayman Islands or Ireland. On the contrary, dividends distributed by Swiss funds are subject to a tax of 35%. *What is the consequence of this tax, which is intended to discourage tax fraud? Swiss funds have migrated to the Grand Duchy, and from their accounts in Geneva, investors now essentially buy Luxembourg funds* (Zucman, 2015, pp. 27). Additionally, the nationality of the investor influences the net transactional gains from shifting profits. For example, as a consequence of the European saving tax directive which has been applied since 2005, the governments of Luxembourg and Austria are excluded from reporting interest earned by citizens of the European Union to their corresponding country of nationality, but must tax at 35% the interest earned, and 3/4 of this revenue has to be sent back to the country of nationality of the investor. This directive only applies to interest, not to dividends, and not surprisingly, the main effect of the savings tax directive has been to encourage Europeans to conceal their nationality status by transferring their wealth to shell corporations, trust, and foundations in other haven jurisdictions (Zucman, 2015).

The specialization at the industry or firm level reflects the role that tax rulings have in allowing governments to offer tailored tax deals to specific multinational corporations. The Luxembourg Leaks revealed how the use of hybrid entities (characteristic of both partnership and corporation) and hybrid securities (both debt and equity features), create hybrid regulatory mismatches in at least two countries, which allows for double non-taxation. The consequence of these hybrid legal characterizations is to enable repatriation of profits to tax havens that have no withholding taxes over dividends such as Bermuda or the Cayman Islands (Hardeck & Wittenstein, 2018).

This type of specialization justifies the competition of tax havens for country-sector specific shifted profits in an oligopolistic environment with homogeneous concealment assets. But haven jurisdictions specialize themselves also in the type of financial assets that they offer. For instance, the Cayman Islands is known for the concealment possibilities for hedge funds, Switzerland specializes in concealing financial assets such as equity and bonds, and Luxembourg in mutual funds (Zucman, 2015). The heterogeneous offer of concealment assets justifies a differentiated oligopolistic environment, and the optimal portfolio of concealment assets held by each multinational corporation is given by the following theorem.

Theorem 2.2. ***Optimal Portfolio of Shifted Profits:*** *The optimal portfolio of shifted profits from country r by multinational corporations in industry i satisfies:*

1. $q_{rig_{ri(m)}} = 0$ *for* $m \in \{z_{ri}, \ldots, R\}$.
2. *If* $\tau_{mi} \geq \tau_{ri}$ *or* $\eta_{ri}\left(\mathscr{O}_{ri}(m)\right) \leq 0$ *then* $q_{rim} = 0$.
3. *If* $q_{rig_{ri}(m)} = 0$ *then* $q_{rig_{ri}(s)} = 0\ \forall s \geq m$.
4. *Optimal profit shifting is given by*

$$\begin{aligned} q_{rig_{ri}(m)} &= 1\{m \leq L_{ri}\} \frac{\beta}{\alpha + \beta L_{ri}} \left[\alpha\left(\eta_{ri}(m) - \Omega_{ri}\right) + \beta \sum_{s=1}^{L_{ri}} \left(\eta_{ri}(m) - \eta_{ri}(s)\right)\right] \\ &= 1\{m \leq L_{ri}\} \frac{\beta}{\alpha + \beta L_{ri}} \left[\alpha\left(\left(\tau_{ri} - \tau_{g_{ri}(m)i}\right) - Q_{rig_{ri}(m)}\left(\gamma_i + b_{rig_{ri}(m)}\right) - \Omega_{ri}\right)\right. \\ &\left. + \beta \sum_{s=1}^{L_{ri}} \left[\left(\tau_{g_{ri}(s)i} - \tau_{g_{ri}(m)i}\right) + \gamma_i\left(Q_{rig_{ri}(s)} - Q_{rig_{ri}(m)}\right) + \left(Q_{rig_{ri}(s)} b_{rig_{ri}(s)} - Q_{rig_{ri}(m)} b_{rig_{ri}(m)}\right)\right]\right] \end{aligned} \tag{19}$$

   where $1\{.\}$ *is the indicator function and the degree of competition is given by the lowest value of* L_{ri} *such that* $G_{ri}(L_{ri}) > 0$ *where*

$$G_{ri}(T) = \Delta_{ri}(T+1) - \Omega_{g_{ri}(T+1)i} - \frac{1}{\alpha + \beta T}\left(\alpha\left(\tau_{ri} - \Omega_{ri}\right) + \beta \sum_{m=1}^{T} \Delta_{ri}(m)\right) \tag{20}$$

   and

$$\Omega_{ri} = Max\left\{0, \tau_{ri} - \frac{1}{L_{ri}} \sum_{m=1}^{L_{ri}} \Delta_{ri}(m) - \frac{\alpha + \beta L_{ri}}{\alpha \beta L_{ri}}\left(S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}\left(1 - \left(1 - \alpha_{ri}^{K}\right)\phi_{ri}\right) + \sum_{m=1}^{R} q_{mir}\right)\right\}. \tag{21}$$

5. *The production network influences the profit shifting portfolio only when profits are fully shifted, i.e. when the corporate profit tax base non-negativity constraint is binding.*
6. *Imperfect re-shifting: If* $\exists\, r$ *such that* $q_{rim} > 0$ *then* $\Omega_{mi} = 0$.
7. *i) decreasing* τ_{ri} *or increasing* Ω_{ri} *have a positive effect on* $G(L_{ri})$ *that diminishes as the degree of competition* L_{ri} *increases; ii)* $G(z_{ri} - 1) > 0$ *if* $\Gamma_{ri,M} = 0$*; and iii)* $G(s) > G(s-1)$ *if* $\Delta_{ri}(s+1)(\alpha + \beta s) \geq \Delta_{ri}(s)(\alpha + \beta(s+1))$ *and* $\Gamma_{si,M} = 0\ \forall s$.

The optimal allocation of profits across countries is influenced by the tax differentials, not only between the dispatching and the destination countries, but also between all of the competing haven jurisdictions. This result follows the literature on the impact of international tax differences on the allocation of firm profits, and the evidence it provides about the use of transfer pricing mechanisms. This literature has found that: there is a response of prices to taxes and tariffs (Swenson, 2001); the value-added across manufacturing sectors in OECD countries depends on the corporate tax rate (Bartelsman & Beetsma, 2003); U.S. export and import price indexes that separate between intrafirm and interfirm prices are strongly influenced by taxes in a way consistent with transfer pricing (Clausing, 2003); the value of intrafirm trade for German multinational firms responds to the tax differential between Germany and the country in which the affiliate is located (Overesch, 2006); U.S. intrafirm prices are more flexible and have a greater pass-through than arm's-length prices (Neiman, 2010); evidence of transfer pricing in French (Vicard, 2015) and Danish (Cristea & Nguyen, 2016) firms; and French multinational export prices drop with the destination corporate tax rate for intrafirm transactions that systematically involve tax havens as the destination country (Davies et al., 2018).

This means that intermediate input supply chains are relevant to explain corporate profit shifting strategies mainly because they capture the intrafirm linkages that are used to shift profits via transfer pricing. As Davies et al. (2018) sample of French firms that cover 98.8% of French exports in 1999 show, for those firms in which positive intrafirm trade is observed, the share of intrafirm trade in a firm's total trade is above 40% for three quarters of the observations. Additionally, in a modified dynamic framework, the production network would also allow to explain profit shifting strategies due to intrafirm loans with modified interest rates.

In any case, the assumption of this model that profit shifting is accessed via concealment financial assets is agnostic about the mechanism that is used to shift profits across countries (e.g. transfer pricing or intrafirm loans). What is relevant, is that given a vector of enforcement values and concealment prices, the model allows for an influence of the production network on the optimal allocation of corporate profits across countries only in the particular case in which the corporate profit tax base non-negativity constraint is binding.

2.2 Households

Country r households preferences have the form

$$u\left(d_r, \tilde{L}_r\right) = \frac{\left(d_r\left(1-\tilde{L}_r\right)^{\lambda_r}\right)^{1-\sigma}}{1-\sigma}$$

with a total available labour supply $\tilde{L}_r$ normalized to unity, d_r is a Cobb-Douglas consumption aggregator for the consumption basket,

$$d_r = \prod_{m=1}^{R}\prod_{j=1}^{N_m} d_{rmj}^{\beta_{rmj}}$$

where $\beta_{rmj} \geq 0$, $\sum_{m=1}^{R}\sum_{j=1}^{N_m}\beta_{rmj} = 1$.

The households have perfect home-bias in shareholdings and their budget constraint is described by

$$P_r D_r = \sum_{m=1}^{R} \sum_{j=1}^{N_m} P_{mj} D_{rmj} = \tilde{w}_r L_r + \sum_{i=1}^{N_r} \bar{\pi}_{ri} + \sum_{i=1}^{N_r} \left(\int_0^{\psi_{ri}} \pi_{ri,Ms} ds + \int_{\psi_{ri}}^{1} \pi_{ri,Ds} ds \right) + T_r \tag{22}$$

where $D_r = n_r d_r$, $D_{rmj} = n_r d_{rmj}$, $L_r = n_r \tilde{L}_r$, and T_r represents lump-sum transfers from the government.

Optimal consumption is described by

$$P_{mj} d_{rmj} = \beta_{rmj} P_r d_r \tag{23}$$

where P_r is defined as

$$P_r = \prod_{m=1}^{R} \prod_{j=1}^{N_m} \left(\frac{P_{mj}}{\beta_{rmj}} \right)^{\beta_{rmj}}.$$

Optimal labour supply is given by

$$\begin{aligned} \tilde{w}_r \left(1 - \tilde{L}_r \right) &= \lambda_r P_r d_r \\ w_r - \tilde{w}_r L_r &= \lambda_r P_r D_r \\ \frac{\beta_{rmj}}{\lambda_r} \left(w_r - \tilde{w}_r L_r \right) &= P_{mj} D_{rmj} \end{aligned} \tag{24}$$

where $w_r = n_r \tilde{w}_r$ is the country-size weighted wage rate in country r.

2.3 Government Policy

Government of country r raises revenue from the corporate sector and by selling concealment financial assets to multinational corporations. This revenue is used by the government to invest in enforcement activities with the objective of curtailing profit shifting out of their jurisdictions and the proceeds are distributed in a lump-sum manner to the domestic households. The government constraint is then

$$T_r + \sum_{i=1}^{N_r} \sum_{m=1}^{R} b_{rim}^2 = \sum_{i=1}^{N_r} \tau_{ri} \left\{ \int_0^{\psi_{ri}} \Gamma_{ri,Ms} ds + \int_{\psi_{ri}}^{1} \Gamma_{ri,Ds} ds \right\} + \sum_{m=1}^{R} \sum_{j=1}^{N_m} Q_{mjr} \int_0^{\psi_{mj}} c_{mjk,s} ds. \tag{25}$$

The objective of the government is to maximize domestic welfare measured by $u\left(d_r, \tilde{L}_r\right)$ by choosing b_{rim} and Q_{mjr} subject to the government budget constraint, and

$$\begin{aligned} b_{rim} &> -\gamma_i; \\ \Gamma_{ri,Ds} &= \frac{\pi_{ri,Ds} + \iota_i \left(k_{ri,Ds} - K_{ri} \right)}{(1 - \tau_{ri})}. \end{aligned}$$

In this environment the price of concealment assets and the level of enforcement can be negative. On one hand, a negative concealment price means that the government has an incentive to pay multinational corporations for shifting profits towards its jurisdiction. This happens if the welfare effects derived from these capital flows outweigh the effects from the additional revenue that could be obtained by charging a positive price for concealment assets. On the other hand, a negative enforcement means that the government is better off by eroding the costless global oversight over capital flows from industry i. This happens when the welfare increases in the country as the amount of shifted profits out of the government jurisdictions raises, as for example is the case when the negative welfare effects from an elevated cost of shifting profits can be reduced by decreasing the total enforcement, or when the welfare gains from an increase in productive capital outweigh the welfare costs of an erosion of the tax base and a reduction in the dividends from multinational subsidiaries.

Households, firms and the government interact in a subgame perfect equilibrium with two stages. In the second stage, household and firms choose optimally, while in the first stage, based on the competitive equilibrium conditions from the second stage, the government sets the level of enforcement and the price for concealment assets. As it will be explained with more detail in the next section, the policy problem is not a Ramsey equilibrium because we assume that the government is a price-taker.

2.4 Market Clearing Conditions

The market for intermediate goods of country-sector ri are cleared when demand from the aggregator firm of goods produced by the multinational subsidiaries and the domestic firms match

$$x_{ri,M} = \int_0^{\psi_{ri}} X_{ri,Ms} ds; \; x_{ri,D} = \int_{\psi_{ri}}^1 X_{ri,Ds} ds. \tag{26}$$

The labour market in country r is cleared when the supply from the households matches the demand from domestic firms and multinational subsidiaries

$$L_r = \sum_{i=1}^{N_r} \left(\int_0^{\psi_{ri}} l_{ri,Ms} ds + \int_{\psi_{ri}}^1 l_{ri,Ds} ds \right). \tag{27}$$

The international capital market for industry i is cleared when the global supply of capital matches the demand of domestic and multinational firms of sector i across the world, and the demand of domestic firms from country-sector ri matches their capital endowment

$$\sum_{r=1}^{R} K_{ri} = \sum_{r=1}^{R} \left(\int_0^{\psi_{ri}} k_{ri,Ms} ds + \int_{\psi_{ri}}^1 k_{ri,Ds} ds \right); \tag{28}$$

$$(1 - \psi_{ri}) K_{ri} = \int_{\psi_{ri}}^1 k_{ri,Ds} ds. \tag{29}$$

The market for goods produced by country-sector ri is cleared when the intermediate and final demand equals the supply of the aggregator firm

$$y_{ri} = \sum_{m=1}^{R} D_{mri} + \sum_{m=1}^{R} \sum_{j=1}^{N_m} \left\{ \int_0^{\psi_{mj}} x_{mjri,Ms} ds + \int_{\psi_{mj}}^{1} x_{mjri,Ds} ds \right\}. \tag{30}$$

2.5 Competitive Equilibrium

Theorem 2.3. ***Competitive Equilibrium:*** *The solution to the competitive equilibrium that identifies the second stage of the model is given by the following system of* $6N+\tilde{N}+R$ *equations with* $6N+\tilde{N}+R$ *unknowns* S_{ri}, $S_{ri,M}$, $S_{ri,D}$, P_{ri}, $P_{ri,M}$, $P_{ri,D}$, w_r, *and* ι_i.

The system is given by

$$S_{ri} = \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \phi_{mj} \left(1 - \alpha_{mj} - \alpha_{mj}^K\right) S_{mj} \right) + \sum_{m=1}^{R} \sum_{j=1}^{N_m} \phi_{mj} \alpha_{mj} \omega_{mjri} S_{mj} \tag{31}$$

$$\begin{aligned} w_r - \sum_{i=1}^{N_r} \phi_{ri} \left(1 - \alpha_{ri} - \alpha_{ri}^K\right) S_{ri} = \lambda_r \Bigg\{ & \sum_{m=1}^{R} \sum_{j=1}^{N_m} Q_{mjr} q_{mjr} \left(\gamma_i + b_{mjr}\right) + \sum_{i=1}^{N_r} \Bigg[\iota_i K_{ri} + \psi_{ri} \left(q_{ri} - C_{ri}\right) - \sum_{m=1}^{R} b_{rim}^2 \\ & + \left(1 - \phi_{ri} \left(\alpha_{ri} + \alpha_{ri}^K \left((1 - \tau_{ri}) + \frac{\psi_{ri} \Omega_{ri} \left(1 - \tau_{ri} + \Omega_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)}}{\psi_{ri} \left(1 - \tau_{ri} + \Omega_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)} + \left(1 - \psi_{ri}\right) \left(1 - \tau_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)}} \right) \right) \right) S_{ri} \Bigg] \Bigg\} \end{aligned} \tag{32}$$

$$\iota_i \sum_{r=1}^{R} \psi_{ri} K_{ri} = \sum_{r=1}^{R} \left(\frac{\psi_{ri} \alpha_{ri}^K \left(1 - \tau_{ri} + \Omega_{ri}\right)^{1 + \alpha_{ri}^K (\theta_{ri} - 1)}}{\psi_{ri} \left(1 - \tau_{ri} + \Omega_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)} + \left(1 - \psi_{ri}\right) \left(1 - \tau_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)}} \right) S_{ri} \tag{33}$$

$$\begin{aligned} P_{ri} = & \frac{1}{\phi_{ri}} exp\left\{-\alpha_{ri} \epsilon_{ri}\right\} \left(\psi_{ri} \left(1 - \tau_{ri} + \Omega_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)} + \left(1 - \psi_{ri}\right) \left(1 - \tau_{ri}\right)^{\alpha_{ri}^K (\theta_{ri} - 1)} \right)^{-\frac{1}{\theta_{ri} - 1}} \\ & \times \left(\frac{w_k}{n_k \left(1 - \alpha_{ri} - \alpha_{ri}^K\right)} \right)^{1 - \alpha_{ri} - \alpha_{ri}^K} \left(\frac{\iota_i}{\alpha_{ri}^K} \right)^{\alpha_{ri}^K} \left(\frac{\tilde{P}_{ri}}{\alpha_{ri}} \right)^{\alpha_{ri}} \end{aligned} \tag{34}$$

$$S_{ri} = S_{ri,M} \left(\psi_{ri} + \left(1 - \psi_{ri}\right) \left(\frac{1 - \tau_{ri}}{1 - \tau_{ri} + \Omega_{ri}} \right)^{\alpha_{ri}^K (\theta_{ri} - 1)} \right)^{\frac{1}{\phi_{ri}}} \tag{35}$$

$$S_{ri,D} = S_{ri,M} \left(\frac{1 - \tau_{ri}}{1 - \tau_{ri} + \Omega_{ri}} \right)^{\alpha_{ri}^K \theta_{ri}} \tag{36}$$

$$P_{ri} = P_{ri,M} \left(\psi_{ri} + \left(1 - \psi_{ri}\right) \left(\frac{1 - \tau_{ri}}{1 - \tau_{ri} + \Omega_{ri}} \right)^{\alpha_{ri}^K (1 - \theta_{ri})} \right)^{\frac{1}{1 - \theta_{ri}}} \tag{37}$$

$$P_{ri,D} = P_{ri,M} \left(\frac{1 - \tau_{ri}}{1 - \tau_{ri} + \Omega_{ri}} \right)^{\alpha_{ri}^K} \tag{38}$$

where $\tilde{P}_{ri} = \prod_{m=1}^{R} \prod_{j=1}^{N_m} \left(\frac{P_{mj}}{\omega_{rimj}} \right)^{\omega_{rimj}}$, *and* $q_{ri} = \sum_{m=1}^{R} \left(q_{mir} - q_{rim}\right)$ *stands for the unweighted net gain on the base of country-sector* ri *due to shifted profits from multinational corporations.*[9]

[9] The matrix form representation for the system of equations that characterize the second stage solution is presented in Appendix B.

One of the equations in the system defined by (31)-(33) is redundant by Walras Law.

From the solution we can see that corporate profits shifting directly influences wages, and via wages it modifies nominal production, the interest rate, and prices. First, from equation (32), wages are directly impacted by the amount of shifted profits, the concealment prices, the enforcement levels, and Lagrange multipliers when the non-negativity constraint for the corporate tax base is binding. From equation (31), nominal production for each country-sector is directly affected by wages. From equation (33), the interest rates are impacted by both nominal production and the Lagrange multipliers. Finally, from equation (34), prices are altered by wages, the interest rate, and Lagrange multipliers. The effect of profit shifting over the equilibrium values for the terms of trade and nominal production alters the optimal decision of firms, consumers, and governments.

Corollary 2.1. ***GDP and Consumption:*** *Real GDP and consumption for country r are given by*

$$GDP_r = \frac{1}{P_r^{GDP}} \sum_{i=1}^{N_r} \left(1 - \alpha_{ri}\phi_{ri}\right) S_{ri}; \tag{39}$$

$$Con_r = \frac{1}{P_r^{GDP}} \left(\sum_{i=1}^{N_r} \phi_{ri} \left(1 - \alpha_{ri} - \alpha_{ri}^K\right) S_{ri} + Div_r + T_r \right); \tag{40}$$

where Div_r stands for corporate dividends in country r, and the GDP deflator is given by P_r^{GDP}, which can be substituted by P_r if deflation is done by comparing purchasing power measured by CPI.

In this one period economy, GDP has to coincide with consumption and the trade balance has to be equal to 0 for those countries that have no net investment income from renting capital when there is no leakage of profits. In particular, for a global economy without capital, trade balance equals 0 when there are no profit shifting opportunities.

Net investment income for country r is given by $\sum_{i=1}^{N_r} \iota_i \psi_{ri} \left(K_{ri} - k_{ri,M}\right)$.

Figure 1 in page 29 captures the main interactions for the simple case of the three country, one sector *Bermuda Triangle* economy presented in section 4.

3 Sovereign Profit Shifting Warfare

In the context of an international tax environment as the one described in this model, governments have a multidimensional space of competition that allows them to influence the welfare effect of lump sum transfers and corporate dividends that are handed over to domestic households. This idea is developed by models in the literature such as Slemrod & Wilson (2009), and

Johannesen (2010). In Slemrod & Wilson (2009) countries endogenously choose to become tax havens by completely giving up taxation revenue and sustaining their expenses from earnings that are generated by the provision of concealment services to firms in other countries, while non-haven jurisdictions choose optimally their taxation and enforcement level. These optimal policy decisions are taken in an environment in which the price of these concealment services is unique across haven jurisdictions and inversely related with the number of tax havens.

In Johannesen (2010), countries optimally choose the tax level that maximizes tax revenue while a multinational corporation shifts profits across jurisdictions. He focuses on two types of equilibrium, a symmetric and an asymmetric one. In the symmetric equilibrium all countries have the same tax rate and there is no profit shifting, while in the asymmetric equilibrium, a set of countries become low tax jurisdictions that act as net recipients of shifted profits. Moreover, if there is an exogenous introduction of unproductive tax havens with zero taxation in which non-haven jurisdictions have the incentive to set a uniform tax rate, there is an unambiguous reduction of revenue when compared with the symmetric equilibrium without tax havens, but there is a potential increase in revenue when compared with the asymmetric equilibrium. In particular, high-tax countries under the asymmetric equilibrium without tax havens can increase their tax revenue under the uniform-tax equilibrium with tax havens through three channels: i) effective investment increases in non-haven jurisdictions because with unproductive tax havens there is no incentive to allocate productive capital in haven countries; ii) when the measure of the haven-jurisdictions is low, the increase in the amount of shifted profits due to the fact that tax havens set a lower tax rate than low-tax countries in the asymmetric equilibrium is outweighed by the reduction on total shifted profits due to the lower number of attracting jurisdictions; and iii) there is a reduction in the tax sensitivity of the high-tax countries which allows for a higher equilibrium tax rate.

These models characterize tax havens as economies that fully relinquish tax revenue and fully depend on the income created by the provision of concealment services to multinational corporations. This introduces a discrepancy between the assumptions of these models and the fact that many countries that are classified as haven jurisdictions have a strictly positive statutory tax rate over corporate gains (OECD, 1998; GAO, 2008). Additionally, once a country is considered a tax haven in these models, it acts as a haven for all firms or subsidiaries located in any other country. This does not allow for bilateral linkages at the industry level across countries, as the one promoted by bilateral tax treaties, in which one jurisdiction acts as a haven only for a subset of country-sector firms, while other country-sectors firms might even be interested in shifting profits out of this jurisdiction. Furthermore, the amount of shifted profits or the government policy variables are not influenced by supply chains and intersectoral-trade markets in any of these models.

In our model environment statutory tax rates are exogenous. The attention is going to be directed to the policy problem of selecting the optimal level of enforcement and prices for concealment assets. In particular, the government of country r can modify b_{rim} and Q_{mir} $\forall m, i$, and these values are going to be identified for the set of country-sector industries in which there is an effective demand for profit shifting, i.e. b_{rim} will be identified when $\mathscr{O}_{ri}(m) \leq L_{ri}$ and Q_{mir} will be identified when $\mathscr{O}_{mi}(r) \leq L_{mi}$. Reductions in b_{rim} or Q_{rim} increase the net transactional gains for country-sector ri of shifting profits towards country m, and under significant reductions, country m might raise in the rankings of the stairway order g_{ri}.

If we additionally assume that the government takes terms of trade as given (i.e. prices, wages and the interest rates), the optimal policy decision ignores its substitution effects and it only takes into account the partial equilibrium wealth effect that influences households. Under this assumption, taxation, enforcement, and concealment pricing are non distortionary for households. Taxation is distortionary for both domestic firms and multinational subsidiaries. Concealment pricing is not distortionary for both domestic firms and multinational subsidiaries. Enforcement is only distortionary for mutinational subsidiaries when the non-negativity constraint for the tax base is binding.[10]

Under this assumption of governments with bounded foresight of its policy effects, the optimal policy is to choose b_{rim} and Q_{rim} that maximize governmental and corporate transfers to households

$$\sum_{i=1}^{N_r} \left(\psi_{ri}\pi_{ri,M} + (1-\psi_{ri})\,\pi_{ri,D}\right) + T_r, \tag{41}$$

conditional on its own constraints, and the domestic firms and multinational subsidiaries optimality conditions. As it will be shown in the next section, the level of profits shifted, enforcement and concealment pricing have general equilibrium effects over the terms of trade of the global economy that are swayed by the structure of the production network.

Given that q_{rim} and the first derivative of Ω_{ri} are discontinuous functions on the enforcement levels and concealment prices, the following analysis holds under the local space under which none of these discontinuities are hit. More precisely, this means that the optimal values of b_{rim} and Q_{rim} that we will consider are conditioned on changes that: i) do not alter the degree of competition L_{ri}; ii) do not alter for each country-sector ri the set of countries that compete for shifted profits, i.e. the list of countries m such that $\mathscr{O}_{ri}(m) \leq L_{ri}$ (the stairway order among this set of countries is allowed to vary); and iii) do not change for each country-sector ri the binding status of the non-negativity constraint for the tax base.

[10]The optimal policy for Q_{mir} is not influenced by whether the non-negativity constraint for the tax base is binding because the pricing decision of Q_{mir} is only relevant, under discontinuity considerations, when $q_{mir} > 0$, which implies that $\Omega_{ri} = 0$ due to imperfect re-shifting.

3.1 Effect over lump sum transfers T_r

Conditional on firm optimality conditions lump sum transfers are given by

$$T_r = \sum_{i=1}^{N_r} \tau_{ri} \left(S_{ri} \left(1 - \left(1 - \alpha_{ri}^K \right) \phi_{ri} \right) + \psi_{ri} q_{ri} \right) + \sum_{m=1}^{R} \sum_{j=1}^{N_m} Q_{mjr} q_{mjr} \left(\gamma_i + b_{mjr} \right) - \sum_{i=1}^{N_r} \sum_{m=1}^{R} b_{rim}^2. \tag{42}$$

The government revenue has two sources, firstly, levied taxes on corporate gains, secondly revenue collected on traded concealment financial assets. From equation (42) we see that the corporate tax base is composed of profits due to the monopolistically competitive environment in which multinational subsidiaries operate and the tax base gains $\psi_{ri} q_{ri}$. These resources are used in enforcement activities aimed to curtail the erosion of the tax base and the proceeds are the lump sum transfers for households.

The effect of Q_{mir} on T_r can be decomposed into[11]

$$\begin{aligned} \frac{\partial T_r}{\partial Q_{mir}} &= 1 \left\{ \mathscr{O}_{mi}(r) \leq L_{mi} \right\} \left(\underbrace{\tau_{ri} \psi_{ri} \frac{\partial q_{ri}}{\partial Q_{mir}}}_{\substack{\text{Concealment} \\ \text{base effect}} <0} + \underbrace{Q_{mir} \left(\gamma_i + b_{mir} \right) \frac{\partial q_{mir}}{\partial Q_{mir}}}_{\substack{\text{Concealment} \\ \text{quantity effect}} <0} + \underbrace{q_{mir} \left(\gamma_i + b_{mir} \right)}_{\substack{\text{Concealment} \\ \text{price effect}} >0} \right) \\ &= \frac{\beta \left(\gamma_i + b_{mir} \right)}{\alpha + \beta L_{mi}} \left[\left(\alpha + \beta L_{mi} \right) \left(2 \eta_{mi}(r) - \left(\tau_{mi} - \tau_{ri} \right) - \tau_{ri} \psi_{ri} \right) - \alpha \Omega_{mi} - \beta \sum_{s=1}^{L_{mi}} \eta_{mi}(s) \right] \end{aligned} \tag{43}$$

where

$$\underbrace{\frac{\partial q_{ri}}{\partial Q_{mir}}}_{\substack{\text{Offensive} \\ \text{concealment} \\ \text{competition}}} = \frac{\partial q_{rim}}{\partial Q_{mir}} = - \left(\underbrace{\frac{\alpha \beta + \beta^2}{\alpha + \beta L_{mi}} \left(\gamma_i + b_{mir} \right)}_{\substack{\text{Concealment} \\ \text{creation effect}}} + \underbrace{\frac{\beta^2 \left(L_{mi} - 1 \right)}{\alpha + \beta L_{mi}} \left(\gamma_i + b_{mir} \right)}_{\substack{\text{Concealment} \\ \text{diversion effect}}} \right) = -\beta \left(\gamma_i + b_{mir} \right). \tag{44}$$

Decreasing Q_{mir} has an ambiguous effect on T_r which can be decomposed into the three separate shocks from equation (43). First, the net transactional gains for multinational subsidiaries in country-sector mi of shifting profits towards country r are increased and as a consequence there is a positive concealment effect over the tax base of country r due to the expansion of the leakage of profits created in jurisdiction m and directed to country r. Second, there is an expansion in governmental revenue due to the increase in the amount of concealment assets sold by government r and demanded by multinational corporations in sector i to shift profits out of country m. Third, there is a reduction in governmental income due to the negative price effect.

In equation (44) we label the reduction in Q_{mir} as offensive concealment competition in which

[11] Due to imperfect re-shifting $\Omega_{ri} = 0$ and under the assumption of a government with bounded foresight of its policy effects there is no effect over S_{ri}.

the leakage of profits out of country m directed to country r is increased, firstly, due to a creation effect that reflects an increase in profit shifting out country-sector mi, and secondly, due to a diversion effect that reflects redirection of profits which otherwise would have been directed to any of the other $L_{mi}-1$ jurisdictions that compete for the shifted profits from multinational subsidiaries in country-sector mi.

The effect of b_{rim} on T_r can be decomposed into

$$\begin{aligned}\frac{\partial T_r}{\partial b_{rim}} =& 1\left\{\mathscr{O}_{ri}(m) \leq L_{ri}\right\} \\ & \times\left(\underbrace{\tau_{ri}\psi_{ri}\frac{\partial q_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement} \\ \text{base effect}}>0} - \underbrace{2b_{rim}}_{\substack{\text{Marginal} \\ \text{enforcement cost}}>0} + \underbrace{\tau_{ri}\left(1-\phi_{ri}\left(1-\alpha_{ri}^{K}\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement effect} \\ \text{on base via investment}}<0}\right) \end{aligned} \tag{45}$$

where

$$\begin{aligned}\underbrace{\frac{\partial q_{ri}}{\partial b_{rim}}}_{\substack{\text{Defensive} \\ \text{enforcement} \\ \text{competition}}} &= \underbrace{\beta Q_{rim}}_{\substack{\text{Enforcement} \\ \text{destruction effect}}} \underbrace{-\frac{\beta^2\left(L_{ri}-1\right)}{\alpha+\beta L_{ri}}Q_{rim}}_{\substack{\text{Enforcement} \\ \text{diversion effect}}} = \frac{\alpha\beta+\beta^2}{\alpha+\beta L_{ri}}Q_{rim}, \\ \frac{\partial S_{ri}}{\partial b_{rim}} &= \frac{\partial S_{ri}}{\partial \Omega_{ri}}\frac{\partial \Omega_{ri}}{\partial b_{rim}}; \\ \frac{\partial S_{ri}}{\partial \Omega_{ri}} &= 1\left\{\Omega_{ri}>0\right\}\left(\frac{\psi_{ri}\alpha_{ri}^{K}\theta_{ri}A_{ri}^{\frac{1}{\theta_{ri}-1}}B_{ri}^{\theta_{ri}}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)-1}\left(1-\tau_{ri}\right)^{\alpha_{ri}^{K}}}{1-\left(1-\tau_{ri}\right)^{\alpha_{ri}^{K}\theta_{ri}}A_{ri}^{\frac{1}{\phi_{ri}}}B_{ri}^{\theta_{ri}}}\right)S_{ri}; \\ \frac{\partial \Omega_{ri}}{\partial b_{rim}} &= -\left[\frac{Q_{rim}+\alpha_{ri}^{K}\left(\theta_{ri}-1\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{-1}E_{ri}S_{ri}}{L_{ri}+E_{ri}\frac{\partial S_{ri}}{\partial \Omega_{ri}}}\right]; \\ A_{ri} &= \psi_{ri}\left(\frac{1-\tau_{ri}+\Omega_{ri}}{1-\tau_{ri}}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}+\left(1-\psi_{ri}\right); \\ B_{ri} &= P_{ri}exp\left\{\alpha_{ri}\epsilon_{ri}\right\}\phi_{ri}\left(\frac{1-\alpha_{ri}-\alpha_{ri}^{K}}{\tilde{w}_r}\right)^{1-\alpha_{ri}-\alpha_{ri}^{K}}\left(\frac{\left(1-\tau_{ri}\right)\alpha_{ri}^{K}}{\iota_i}\right)^{\alpha_{ri}^{K}}\left(\frac{\alpha_{ri}}{\tilde{P}_{ri}}\right)^{\alpha_{ri}}; \\ E_{ri} &= \frac{\alpha+\beta L_{ri}}{\alpha\beta}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}\theta_{ri}}B_{ri}^{\theta_{ri}}\left(\psi_{ri}+\left(1-\psi_{ri}\right)\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}\right)^{\frac{1}{\theta_{ri}-1}}.\end{aligned} \tag{46}$$

The sign of the effect on T_r of increasing b_{rim} is ambiguous, but it can be decomposed into the three separate shocks from equation (45). First, the net transactional gains for multinational subsidiaries in country-sector ri of shifting profits towards country m are reduced. As a consequence there is a positive enforcement effect that expands the tax base of country r due to the reduction of the leakage of profits created in jurisdiction r and directed to country m. Second, there is an increase in the wasted resources used by public authorities of country r in enforcement activities. Third, when the non-negativity constraint of the corporate tax base for country-sector ri is binding, an increase in b_{rim} raises the effective marginal tax rate payed by multinational subsidiaries and as a consequence reduces the level of capital investment and the production from these firms, which negatively affects the tax base and the amount of levied taxes by country r.

In equation (46) we label the increase in b_{rim} as a defensive enforcement competition that augments the tax base for the multinational subsidiaries of country-sector ri. This increase in the tax base is due to a destruction effect in which the leakage of profits created in country r and directed to country m is diminished, and this effect outweighs the diversion effect that increases shifted profits by country-sector ri to any other of the $L_{ri}-1$ jurisdictions that compete for these capital flows.

3.2 Effect over dividends

Conditional on firm optimality conditions corporate dividends Div_r are given by

$$\begin{aligned} Div_r = \sum_{i=1}^{N_r} \Big\{ & \left(1-\tau_{ri}\right) S_{ri} \left(1-\left(1-\alpha_{ri}^K\right)\phi_{ri}\right) + \psi_{ri}\left(\left(1-\tau_{ri}\right) q_{ri} - C_{ri}\right) \\ & + \psi_{ri}\left(\iota_i K_{ri} - \alpha_{ri}^K \phi_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{1+\alpha_{ri}^K(\theta_{ri}-1)} B_{ri}^{\theta_{ri}-1} S_{ri}\right)\Big\}. \end{aligned} \tag{47}$$

Corporate dividends are composed by the after-tax locally generated profits, the profit gains due to multinational shifts from other jurisdictions net of shifting costs, and the net income that comes from the endowed capital.

The effect of Q_{mir} on Div_r is given by

$$\frac{\partial Div_r}{\partial Q_{mir}} = 1\left\{\mathscr{O}_{mi}\left(r\right) \leq L_{mi}\right\} \underbrace{\left(1-\tau_{ri}\right)\psi_{ri}\frac{\partial q_{ri}}{\partial Q_{mir}}}_{\substack{\text{Concealment} \\ \text{effect on dividends}} <0}. \tag{48}$$

Decreasing Q_{mir} as part of an offensive concealment competition strategy has a positive effect on dividends because the increase in the net transactional gains will augment the leakage of profits created in jurisdiction m and shifted to country r.

The effect of b_{rim} on Div_r can be decomposed into

$$\begin{aligned} \frac{\partial Div_r}{\partial b_{rim}} = 1\left\{\mathscr{O}_{ri}\left(m\right) \leq L_{ri}\right\} \Bigg(& \underbrace{\left(1-\tau_{ri}\right)\psi_{ri}\frac{\partial q_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement} \\ \text{effect on dividends}} >0} + \underbrace{\left(1-\tau_{ri}\right)\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement effect on} \\ \text{dividends via investment}} <0} \\ & - \underbrace{\psi_{ri}\frac{\partial \iota_i k_{ri,M}}{\partial b_{rim}}}_{\substack{\text{Enforcement effect} \\ \text{on dividends} \\ \text{via capital costs}} <0} - \underbrace{\psi_{ri}\frac{\partial C_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement effect} \\ \text{on dividends} \\ \text{via shifting costs}}} \Bigg) \end{aligned} \tag{49}$$

where

$$\frac{\partial \iota_i k_{ri,M}}{\partial b_{rim}} = 1\left\{\Omega_{ri}>0\right\}\left(\underbrace{\alpha_{ri}^K \phi_{ri} B_{ri}^{\theta_{ri}-1}\left(1-\tau_{ri}+\Omega_{ri}\right)^{1+\alpha_{ri}^K(\theta_{ri}-1)}\frac{\partial S_{ri}}{\partial b_{rim}}}_{\substack{\text{Enforcement}\\ \text{quantity effect } <0\\ \text{on capital costs}}}\right. \tag{50}$$
$$\left.+\underbrace{\alpha_{ri}^K \phi_{ri} B_{ri}^{\theta_{ri}-1}\left(1-\alpha_{ri}^K+\alpha_{ri}^K\theta_{ri}\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}\frac{\partial \Omega_{ri}}{\partial b_{rim}} S_{ri}}_{\substack{\text{Enforcement effective}\\ \text{tax rate effect } <0\\ \text{on capital costs}}}\right);$$

$$\frac{\partial C_{ri}}{\partial b_{rim}} = 1\left\{\mathscr{O}_{ri}(m)\leq L_{ri}\right\}\left(\underbrace{\frac{\beta^2 Q_{rim}}{\alpha+\beta L_{ri}}\sum_{p=1,p\neq m}^{R}\left(\frac{1}{\alpha}\sum_{s=1}^{R} q_{ris}+\frac{1}{\beta}q_{rip}+Q_{rip}\left(\gamma_i+b_{rip}\right)\right)}_{\substack{\text{Cross effect of enforcement}\\ \text{on shifting costs}}}\right. \tag{51}$$
$$\left.\underbrace{-\beta Q_{rim}\left(\frac{1}{\alpha}\sum_{s=1}^{R} q_{ris}+\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)}_{\substack{\text{Direct effect of enforcement}\\ \text{on shifting costs}}}\right).$$

The sign of the effect on Div_r of increasing b_{rim} is ambiguous, but it can be decomposed into the four separate shocks presented in equation (49). First, the net transactional gains are reduced and as a consequence dividends are expanded due to a reduction of the leakage of profits created in country r and directed to jurisdiction m by country-sector ri. Second, when the non-negativity constraint of the corporate tax base for country-sector ri is binding, an increase in b_{rim} reduces capital investment, final production, and dividends created by multinational subsidiaries. Third, as shown in equation (50) when the non-negativity constraint for the tax base is binding there is a reduction in capital costs due to a reduction in the amount of capital investment as a consequence of both the reduction in the production of intermediate goods by multinational subsidiaries and the increase in the effective marginal tax rate. Finally, as shown in equation (51), when $Q_{rim}>0$, there is an ambiguous effect on the cost of shifting profits, because due to the enforcement destruction effect there is a reduction in the costs of shifting profits from country r to country m, while due to the enforcement diversion effect there is an increase in the cost of shifting profits to any other of the $L_{ri}-1$ jurisdictions that compete for these capital flows.

3.3 Optimal Policy

Theorem 3.1. ***Optimal Enforcement and Concealment Pricing:***

1. *Equation (19), $\frac{\partial T_r}{\partial b_{rim}}+\frac{\partial Div_r}{\partial b_{rim}}=0$ and $\frac{\partial T_m}{\partial Q_{rim}}+\frac{\partial Div_m}{\partial Q_{rim}}=0$ define a system of equations that for a certain stairway order g_{ri} and competition degree L_{ri} identifies b_{rim}, Q_{rim}, and q_{rim} whenever $\mathscr{O}_{ri}(m)\leq L_{ri}$. The system of equations that identifies the first stage of the*

model is given by[12]

$$q_{rim} = 1\left\{\mathscr{O}_{ri}(m) \leq L_{ri}\right\} \frac{\beta}{\alpha+\beta L_{ri}} \Big[\alpha\left((\tau_{ri}-\tau_{mi}) - Q_{rim}(\gamma_i+b_{rim}) - \Omega_{ri}\right)$$
$$+\beta\sum_{s=1}^{L_{ri}}\left[\left(\tau_{g_{ri}(s)i}-\tau_{mi}\right)+\gamma_i\left(Q_{rig_{ri}(s)}-Q_{rim}\right)+\left(Q_{rig_{ri}(s)}b_{rig_{ri}(s)}-Q_{rim}b_{rim}\right)\right]\Big]; \tag{52}$$

$$0 = 1\left\{\mathscr{O}_{ri}(m) \leq L_{ri}\right\}\left(q_{rim} - \beta\left(\psi_{mi}+Q_{rim}(\gamma_i+b_{rim})\right)\right); \tag{53}$$

$$0 = 1\left\{\mathscr{O}_{ri}(m) \leq L_{ri}\right\}\Bigg(\frac{\psi_{ri}\beta Q_{rim}}{\alpha+\beta L_{ri}}\Bigg(\alpha+\beta+(\alpha+\beta(L_{ri}+1))\left(\frac{1}{\beta}q_{rim}+Q_{rim}(\gamma_i+b_{rim})\right)$$
$$+\beta\sum_{p=1}^{R}\left(\frac{1}{\alpha}q_{rip}-Q_{rip}(\gamma_i+b_{rip})\right)\Bigg) - 2b_{rim}$$
$$-1\left\{\Omega_{ri}>0\right\}\left[\frac{Q_{rim}+\alpha_{ri}^K(\theta_{ri}-1)(1-\tau_{ri}+\Omega_{ri})^{-1}E_{ri}S_{ri}}{L_{ri}+E_{ri}\frac{\partial S_{ri}}{\partial\Omega_{ri}}}\right]J_{ri}\Bigg) \tag{54}$$

where

$$J_{ri} = \Bigg[\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}$$
$$+\alpha_{ri}^K\phi_{ri}B_{ri}^{\theta_{ri}-1}(1-\tau_{ri}+\Omega_{ri})^{\alpha_{ri}^K(\theta_{ri}-1)}\left((1-\tau_{ri}+\Omega_{ri})\frac{\partial S_{ri}}{\partial\Omega_{ri}}+\left(1+\alpha_{ri}^K(\theta_{ri}-1)\right)S_{ri}\right)\Bigg].$$

2. *The solution to this problem is a global maximum under the interior space in which discontinuities are not hit if*
   - *(a)* $\left(\frac{\beta^2\psi_{ri}}{\alpha+\beta L_{ri}}\right)^2 \approx 0$ *for* $L_{ri} \geq 2$*;*
   - *(b)* $\varrho_{g_{ri}(\mathscr{O}_{ri}(m))} = \beta\psi_{ri}Q_{rim}^2 - 2b_{rim}^2 < 0$ *for all* m *such that* $\mathscr{O}_{ri}(m) \leq L_{ri}$*;*
   - *(c)* $\left\{\varrho_{g_{ri}(s)}\right\}_{s=1}^{L_{ri}}$ *is composed of* L_{ri} *distinct values; and*
   - *(d) the non-symmetric submatrix of the Hessian* $Hb(L_{ri})$ *that comes from the second order conditions for enforcement is diagonalizable.*

3.4 Computational Algorithm

Definition 3.1. Algorithm: The algorithm to solve the model for a given set of L_{ri} countries that compete for each country-sector is composed of the following steps.

1. Solve the linear system of equations (31)-(38) that characterize the solution for the second stage competitive equilibrium without profit shifting, i.e. assuming $q_{rim} = b_{rim} = Q_{rim} = q_{ri} = C_{ri} = \Omega_{ri} = 0 \ \forall\ r$, i and m.

[12] The matrix form representation for the system of equations that characterize the first stage solution is presented in Appendix B.

2. Set an approximation accuracy ϵ_Ω for the Lagrange multipliers and ϵ_{fs} for the nonlinear system of equations that characterize the solution to the first stage.

3. Set an initial guess $\Omega_{ri,0} = 0 \; \forall$ r and i.

4. Solve the system of equations (52)-(54) that characterize the solution to the first stage using the trust-region method until the infinity norm of residuals L_∞-norm is less than ϵ_{fs} or until the max number of iterations for the trust-region methods is reached.

5. Using the solution to (52)-(54) from step 4 solve the linear system of equations (31)-(38).

6. With the results from steps 4 and 5 estimate Ω_{ri}, A_{ri}, B_{ri}, E_{ri}, $\frac{\partial S_{ri}}{\partial \Omega_{ri}}$ and J_{ri}.

7. Iterate in steps 4 to 6 at least two times until the L_∞-norm for iteration s given by
$$||\vec{\Omega}|| = max\,\{|\Omega_{ri,s} - \Omega_{ri,s-1}| : \forall r, i\}$$
is less than ϵ_Ω, or the maximum amount of iterations is reached.[13]

4 The Bermuda Triangle

4.1 Three Country, One Sector Economy

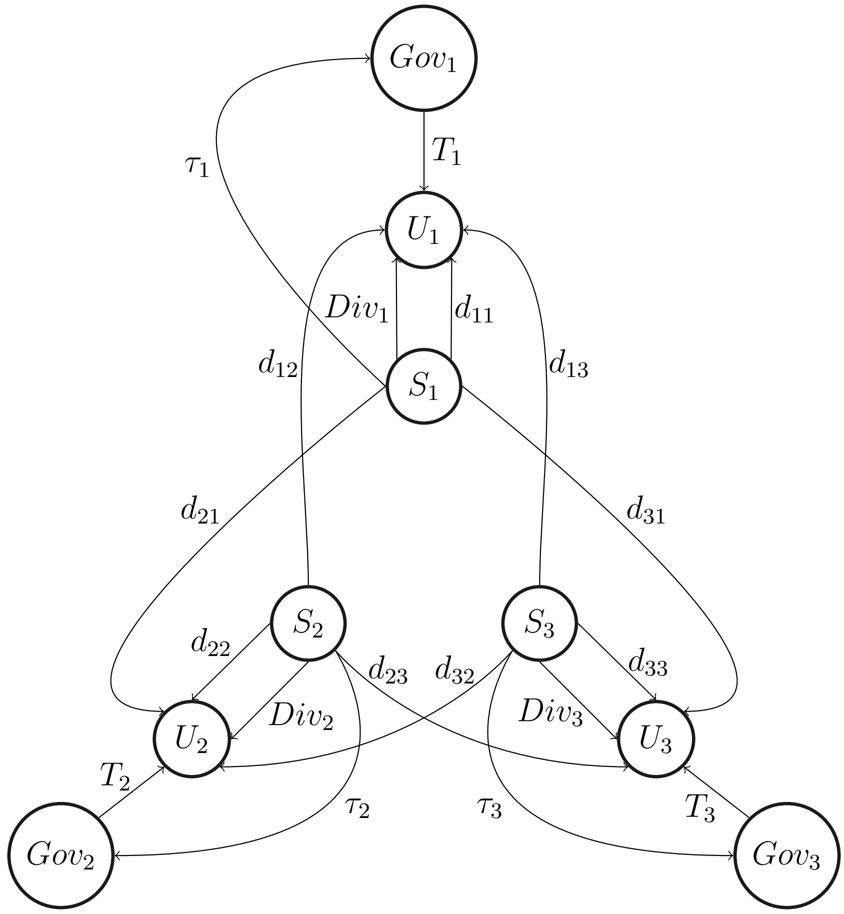
The most simple scenario to understand the intuition around this model is for a global economy with three countries and one sector in which the multinational subsidiaries have a size $\psi_r \in (0, 1]$ in countries $r \in \{1, 2, 3\}$. We assume $\tau_1 < \tau_2 < \tau_3$, and we allow for profit shifting from the intermediate tax economy to the low tax economy, and from the high tax economy to both the low and intermediate tax economies, i.e. $L_1 = 0$, $L_2 = 1$, and $L_3 = 2$. Figure 1 captures the flows of this economy between firms represented by S_r, households represented by U_r, and the governments represented by Gov_r. Subfigure (a) displays flows due to consumption, dividends, transfers, and taxes. Subfigure (b) present flows from the labour, capital, and intersectoral trade input markets. Finally, subfigure (c) exhibits flows due to corporate profit shifting, acquisition of concealment assets, capital flow enforcement, and profit shifting costs.

Profit shifting introduces sources of reallocation and waste of resources both by firms and governments. Profit shifting reallocates resources, first, from multinational subsidiaries to subsidiaries locates in jurisdictions with a lower tax rate, and second, it reallocates resources from multinational subsidiaries to government from countries with a lower tax rate via the acquisition of concealment financial assets. Profit shifting wastes resources, first, due to the enforcement costs that have to be covered by governments from leaking countries, and second, due to the non-reallocated costs from shifting profits that are covered by multinational subsidiaries. In

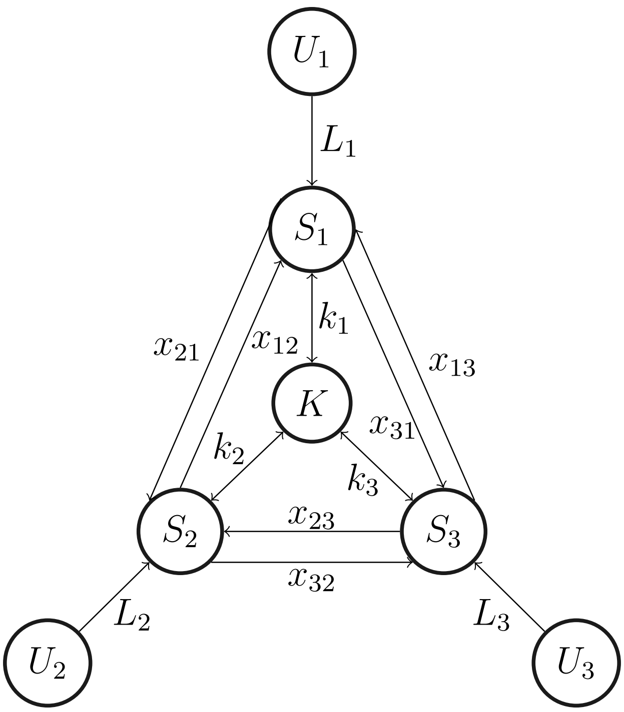
[13] The maximum number of iterations for steps 4 and 7 is set to 1000 in my estimations.

Figure 1: Flows in a Three Country One Sector Economy

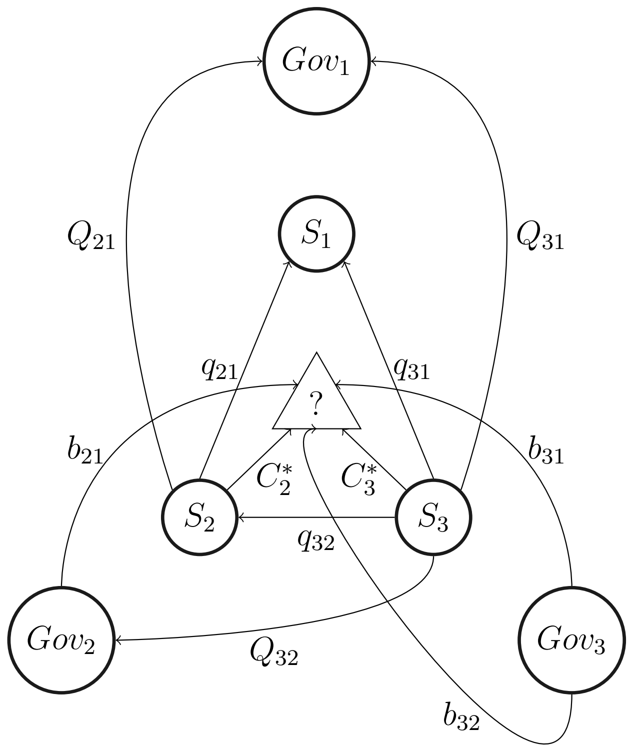
(a) Consumption, Dividends, Transfers and Taxes



(b) Input Market



(c) Profit Shifting



Note: Gov_r, U_r, and S_r stand for the government, household and firm of country r. T_r, Div_r, τ_r, L_r, and k_r stand for transfers, dividends, taxes, labour supply, and capital demand from country r. d_{rm} and x_{rm} stand for final and intermediate consumption from country r of country m good. q_{rm}, b_{rm}, and Q_{rm} stand for shifted profits, enforcement and price of the concealment assets for capital flows from country r to country m

subfigure (c), $C_2^* = C_2 - Q_{21}c_{21}$ and $C_3^* = C_3 - Q_{31}c_{31} - Q_{32}c_{32}$ stands for the non-reallocated costs, i.e. profit shifting costs net of payments for concealment assets. Additionally, the intersection between q's, b's and Q's signifies the stretch interconnection that exists between these variables. Finally, b_{21}, b_{31}, b_{32}, C_2^* and C_3^* are directed to the *Bermuda Triangle* in the center of subfigure (c) where they disappear, never to be seen again.

4.2 Policy Problem

Corollary 4.1. ***First stage for a three country one sector global economy:*** *From equations (52)-(54) the solution to the first stage of this model is characterized by a nonlinear system of nine equations that can be divided into two independent systems of equations.*

1. *The first system of three equations solves for the amount of shifted profits from the intermediate to the low tax economy q_{21}, and the policy variables b_{21} and Q_{21}:*

$$q_{21} = \frac{\alpha\beta}{2\alpha+\beta}\left(\tau_2 - \tau_1 + \psi_1\right);$$

$$b_{21}^2 + \gamma b_{21} - \frac{\beta\psi_2}{2\left(2\alpha+\beta\right)}\left(\alpha\left(\tau_2-\tau_1\right) - \psi_1\left(\alpha+\beta\right)\right)\left(1+\tau_2-\tau_1\right) = 0;$$

$$Q_{21} = \frac{\alpha\left(\tau_2-\tau_1\right) - \psi_1\left(\alpha+\beta\right)}{\left(2\alpha+\beta\right)\left(\gamma+b_{21}\right)}.$$

   The comparative statics of these variables on the model parameters follows

   (a) $\frac{\partial q_{21}}{\partial \alpha} > 0$, $\frac{\partial q_{21}}{\partial \beta} > 0$, $\frac{\partial^2 q_{21}}{\partial^2 \alpha} < 0$, $\frac{\partial^2 q_{21}}{\partial^2 \beta} < 0$, $\frac{\partial^2 q_{21}}{\partial \alpha \partial \beta} > 0$, $\frac{\partial b_{21}}{\partial \alpha} > 0$, $\frac{\partial b_{21}}{\partial \beta}$ *is ambiguous;* $\frac{\partial Q_{21}}{\partial \alpha} > 0$ *if* $\alpha > \beta$, *and* $\frac{\partial Q_{21}}{\partial \beta}$ *is ambiguous.*

   (b) $\frac{\partial q_{21}}{\partial(\tau_2-\tau_1)} > 0$, $\frac{\partial b_{21}}{\partial(\tau_2-\tau_1)} > 0$ *and* $\frac{\partial^2 b_{21}}{\partial b_{21}\partial(\tau_2-\tau_1)} < 0$ *if* $\alpha > \beta$ *and* $\tau_2 - \tau_1 > \psi_1$, *and* $\frac{\partial Q_{21}}{\partial(\tau_2-\tau_1)} > 0$ *if* $\alpha > \beta$ *and* $\tau_2 - \tau_1 > 2\psi_1$.

   (c) $\frac{\partial q_{21}}{\partial \psi_1} > 0$, $\frac{\partial b_{21}}{\partial \psi_1} < 0$, $\frac{\partial^2 b_{21}}{\partial b_{21}\partial \psi_1} > 0$, $\frac{\partial Q_{21}}{\partial \psi_1} < 0$ *if* $\alpha > \beta$ *and* $\tau_2 - \tau_1 > 2\psi_1$.

   (d) $\frac{\partial q_{21}}{\partial \psi_2} = 0$, $\frac{\partial b_{21}}{\partial \psi_2} > 0$ *and* $\frac{\partial Q_{21}}{\partial \psi_2} < 0$ *if* $\alpha > \beta$ *and* $\tau_2 - \tau_1 > 2\psi_1$.

   (e) $\frac{\partial q_{21}}{\partial \gamma} = 0$, $\frac{\partial b_{21}}{\partial \gamma} < 0$ *and* $\frac{\partial Q_{21}}{\partial \gamma} < 0$ *if* $\alpha > \beta$ *and* $\tau_2 - \tau_1 > 2\psi_1$.

2. *The second system of six equations solves for the amount of shifted profits from the high to the low tax economy q_{31} and for the amount of shifted profits from the high to the intermediate tax economy q_{32}, and its corresponding policy variables b_{31}, b_{32}, Q_{31} and Q_{32}*

$$\begin{aligned} q_{31} =& \frac{\alpha\beta}{2\left(\alpha+\beta\right)}\left(\tau_3 - \Omega_3\right) + \frac{\beta^2}{2\left(\alpha+2\beta\right)}\left(\tau_2-\tau_1+\psi_1-\psi_2\right) \\ &+ \frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\left(2\alpha+3\beta\right)\left(\psi_1-\tau_1\right)+\beta\left(\psi_2-\tau_2\right)\right); \\ q_{32} =& \frac{\alpha\beta}{2\left(\alpha+\beta\right)}\left(\tau_3 - \Omega_3\right) + \frac{\beta^2}{2\left(\alpha+2\beta\right)}\left(\tau_1-\tau_2+\psi_2-\psi_1\right) \\ &+ \frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\left(2\alpha+3\beta\right)\left(\psi_2-\tau_2\right)+\beta\left(\psi_1-\tau_1\right)\right); \end{aligned}$$

$$b_{31}^2+\gamma b_{31}=1\left\{\Omega_3>0\right\}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(\gamma+b_{31}\right)\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\frac{\Re_3}{\gamma+b_{31}}\right.$$
$$\left.+\frac{\psi_3\beta\left(q_{31}-\beta\psi_1\right)}{2\left(\alpha+2\beta\right)}\left(\alpha+\beta+\left(\alpha+3\beta\right)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}\left(q_{31}+q_{32}\right)+\psi_1+\psi_2\right)\right)\right)$$

$$b_{32}^2+\gamma b_{32}=1\left\{\Omega_3>0\right\}\left(\frac{q_{32}-\beta\psi_2}{2\beta\left(\gamma+b_{32}\right)\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\frac{\Re_3}{\gamma+b_{32}}\right.$$
$$\left.+\frac{\psi_3\beta\left(q_{32}-\beta\psi_2\right)}{2\left(\alpha+2\beta\right)}\left(\alpha+\beta+\left(\alpha+3\beta\right)\left(\frac{2}{\beta}q_{32}-\psi_2\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}\left(q_{31}+q_{32}\right)+\psi_1+\psi_2\right)\right)\right)$$

$$Q_{31}=\frac{q_{31}-\beta\psi_1}{\beta\left(\gamma+b_{31}\right)};$$

$$Q_{32}=\frac{q_{32}-\beta\psi_2}{\beta\left(\gamma+b_{32}\right)};$$

Where

$$\Omega_3=Max\left\{0,\tau_3-\frac{1}{2}\left(\tau_1+\tau_2+Q_{31}\left(\gamma+b_{31}\right)+Q_{32}\left(\gamma+b_{32}\right)\right)-\frac{\alpha+2\beta}{2\alpha\beta}\left(S_{3,M}^{\phi_3}S_3^{\frac{1}{\theta_3}}\left(1-\left(1-\alpha_3^K\right)\phi_3\right)\right)\right\};$$

$$\Re_3=\left[\frac{\alpha_3^K\beta\left(\theta_3-1\right)\left(\gamma+b_{31}\right)\left(1-\tau_3+\Omega_3\right)^{-1}E_3S_3}{2\beta\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}\right]J_3.$$

The comparative statics of these variables on the parameters follows

(a) *The sign of* $\frac{\partial q_{31}}{\partial\alpha}$, $\frac{\partial q_{32}}{\partial\alpha}$, $\frac{\partial q_{31}}{\partial\beta}$ *and* $\frac{\partial q_{32}}{\partial\beta}$ *is ambiguous.*

(b) $\frac{\partial q_{31}}{\partial\tau_1}=\frac{\partial q_{32}}{\partial\tau_2}<0$, $\frac{\partial q_{31}}{\partial\tau_2}=\frac{\partial q_{32}}{\partial\tau_1}>0$ *and* $\frac{\partial q_{31}}{\partial\tau_3}=\frac{\partial q_{32}}{\partial\tau_3}>0$. *If* $\alpha>0$ *and* $Q_{31}>0$ *then* $\frac{\partial b_{31}}{\partial\tau_1}<0$, $\frac{\partial b_{31}}{\partial\tau_2}>0$ *and* $\frac{\partial b_{31}}{\partial\tau_3}>0$. *If* $\alpha>0$ *and* $Q_{32}>0$ *then* $\frac{\partial b_{32}}{\partial\tau_1}>0$, $\frac{\partial b_{32}}{\partial\tau_2}<0$ *and* $\frac{\partial b_{32}}{\partial\tau_3}>0$. *The sign of* $\frac{\partial Q_{31}}{\partial\tau_1}$, $\frac{\partial Q_{31}}{\partial\tau_2}$, $\frac{\partial Q_{31}}{\partial\tau_3}$, $\frac{\partial Q_{32}}{\partial\tau_1}$, $\frac{\partial Q_{32}}{\partial\tau_2}$ *and* $\frac{\partial Q_{32}}{\partial\tau_3}$ *is ambiguous.*

(c) $\frac{\partial q_{31}}{\partial\psi_1}=\frac{\partial q_{32}}{\partial\psi_2}>0$, $\frac{\partial q_{31}}{\partial\psi_2}=\frac{\partial q_{32}}{\partial\psi_1}<0$ *and* $\frac{\partial q_{31}}{\partial\psi_3}=\frac{\partial q_{32}}{\partial\psi_3}=0$. *The sign of the effect of* ψ_1 *and* ψ_2 *on* b_{31}, Q_{31}, b_{32} *and* Q_{32} *is ambiguous. If* $Q_{31}>0$ *then* $\frac{\partial b_{31}}{\partial\psi_3}>0$ *and* $\frac{\partial Q_{31}}{\partial\psi_3}<0$. *If* $Q_{32}>0$ *then* $\frac{\partial b_{32}}{\partial\psi_3}>0$ *and* $\frac{\partial Q_{32}}{\partial\psi_3}<0$.

(d) $\frac{\partial q_{31}}{\partial\gamma}=\frac{\partial q_{32}}{\partial\gamma}=0$. *If* $Q_{31}>0$ *then* $\frac{\partial b_{31}}{\partial\gamma}<0$ *and* $\frac{\partial Q_{31}}{\partial\gamma}<0$. *If* $Q_{32}>0$ *then* $\frac{\partial b_{32}}{\partial\gamma}<0$ *and* $\frac{\partial Q_{32}}{\partial\gamma}<0$.

(e) $\frac{\partial q_{31}}{\partial\Omega_3}<0$. *The sign of the effect of* Ω_3 *on* b_{31}, Q_{31}, b_{32} *and* Q_{32} *is ambiguous.*

From the profit outflows originated in the intermediate tax economy we can learn about several points. Firstly, with respect to the international tax environment, as α increases and perfect competition becomes less relevant in the costs of shifting profits, or as β increases and monopolistic competition becomes less relevant in the costs of shifting profits, the amount of shifted profits is going to increase in a concave manner, and the cross effect of these parameters is positive. Furthermore, as perfect competition becomes less relevant the intermediate tax economy is going to increase its enforcement, while the low tax jurisdiction is going to raise its price for concealment assets. Secondly, as the tax gap $\tau_2 - \tau_1$ grows, the amount of shifted profits, enforcement from the intermediate economy, and pricing of concealment assets by the low tax economy rises. Thirdly, an increase in the share of the multinational corporations in the low tax economy ψ_1, increases profit shifting and reduces both enforcement and the price of concealment assets. The reduction in concealment pricing happens as a consequence of the increase in the magnitude of these flows for the low tax economy, this triggers an increase in shifted profits by subsidiaries in the intermediate tax economy, and as a consequence, due to the negative effect of the increase in shifting costs in dividends of the intermediate tax economy, its government decides to soothe this effect by reducing enforcement. Fourthly, as the share of the multinational corporations in the intermediate tax economy ψ_2 increases, the amount of shifted profits by each one of the subsidiaries does not change, but the enforcement level of the intermediate tax government rises in a defensive manner due to the increasing importance that multinationals play in both dividends and tax revenue, and as consequence the low tax economy mitigates the effects of the higher level of enforcement by offensively reducing concealment prices. Finally, as the effect of global regulation and oversight γ increases, the amount of shifted profits by each one of the subsidiaries does not change, and both the level of enforcement and the price of concealment are reduced.

From the profit outflows originated in the high tax economy we can conclude several additional points. Firstly, the direction of the effect of changes in α and β on shifted profits, enforcement, and concealment pricing is no longer clear, and as can be seen in the appendix, it depends on the relative size of α against β, τ_1 against ψ_1, and τ_2 against ψ_2. Secondly, just as before an increase in the tax gap augments the amount of profits shifted towards the haven jurisdiction for which the tax gap increases and the enforcement level of the country that leaks profits, but now the amount of shifted profits and the enforcement level is also positively affected by the tax rates of the other jurisdictions to which profits are directed (e.g. q_{31} and b_{31} increase if τ_3 or τ_2 rise, or if τ_1 falls). Thirdly, just as before, the share of the multinational corporations in the jurisdiction to which profits are leaked increases the amount of shifted profits by multinational subsidiaries, but now the share of the multinational corporations in the other jurisdiction to which profits are leaked has a negative effect on this amount (e.g. q_{31} increases if ψ_1 rises or if ψ_2 falls). Moreover, the direction of the effect of ψ_1 and ψ_2 on prices and enforcement is now ambiguous. Fourthly, just as before the share of the multinational corporation in the high tax economy ψ_3 has no effect on the amount of shifted profits, the enforcement increase defensively and the prices of concealment are reduced in an offensive manner. Fifthly, as before an increase in global regulation and oversight does not have an effect in the amount of shifted profits, and both the level of enforcement and concealment are reduced. Finally, an increase in Ω_3 reduces the amount of shifted profits while there is an ambiguous effect on enforcement and concealment pricing.

4.3 Competitive Equilibrium

To understand the effects of shifting profits to tax havens on the competitive equilibrium we are going to solve this simple model with and without capital under different production networks, consumption bundles, population sizes, capital allocations, supplies of capital, tax differentials, shares from multinational corporations in intermediate markets, and different competitive tax environments that change in α, β and γ.

Apart from the specific modifications on each table, all of the estimations from this section are solved under the assumption that $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$.[14]

Tables 1-22 present the solution for the competitive equilibrium under the aforementioned variations and from these results we can extract the following commonalities.

1. The amount of shifted profits by the multinational subsidiaries in the high tax jurisdiction are directed mainly to the low tax economy. The government from the high tax jurisdiction has an incentive to erode the costless global enforcement for the leakage of profits directed to the intermediate tax jurisdiction in almost all of the results. When there is capital in the model, the government from the high tax jurisdiction also has an incentive to erode the costless global oversight over capital flows directed to the low tax economy. Additionally, when there is no capital in the model, the government of the intermediate tax jurisdiction pays multinational subsidiaries in the high tax jurisdiction to shift profits into their economy.

   The introduction of capital in the model creates positive welfare effects in the high tax country that come from an increase of k_3 as the effective marginal tax rate is reduced when profit shifting increases. For this reason, the government of this country optimally decides to erode global oversight and allow profit shifting to occur. The added value that profit shifting has in reducing the effective marginal tax rate allows the intermediate tax government to charge firms a positive price for their concealment assets.

   In other words, the low tax economy will most certainly be better off as both dividends and transfers will increase as a consequence of the profits shifted by subsidiaries in the other two jurisdictions and the revenue collected by the government from the sale of concealment assets. The intermediate tax economy will have an ambiguous welfare shock as the positive effects from the high tax jurisdiction incoming profits will be counterbalanced by the negative effects from the leakage of profits to the low tax jurisdiction, the enforcement costs over these capital flows, and the compensation that is given to multinational

[14] Any deviation from these values is mentioned in the corresponding result tables.

subsidiaries from the high tax economy for the inflow of profits under a model without capital. Finally, the high tax jurisdiction will most certainly have a negative welfare shock as the negative effects from the leakage of profits directed to the other two jurisdictions and the enforcement over these activities is only counterbalanced by the positive welfare effects that come from an increase in capital as the effective marginal tax rate is reduced. But even under these negative effects, the impossibility from the government of the high tax economy to completely eliminate profits shifting, forces them to optimally choose a second best in which they allow profit shifting to increase by eroding costless global oversight.

2. In almost all of the results the corporate tax base non-negativity constraint for multinational subsidiaries in the high tax economy is binding.

3. The resource transfer from countries that leak profits to haven jurisdictions and the wasted resources by both multinational subsidiaries in shifting profits and governments in enforcement activities decreases the nominal wage and price of goods from the high tax country, while increasing the nominal wage and price of goods from the low tax economy. The direction of the effect on the nominal wage from the intermediate tax jurisdiction is negative most of the times, but on the price of goods the direction is ambiguous and depends on the parameterization of the model.

4. In almost all of the results the consumer price index increases in the low tax jurisdiction and decreases in the high tax country, while the direction of the effect on the consumer price index from the intermediary tax jurisdiction is ambiguous.[15]

5. Profit shifting introduces an upward pressure in the interest rate.[16]

6. Without profit shifting, the low tax economy demands more capital than the intermediate and the low tax jurisdictions, and the intermediate tax economy demands more capital than the low tax jurisdictions. This is explained by the increase in the after tax marginal productivity of capital as the statutory tax rate falls.

   Once profit shifting is introduced, the reduction in the effective marginal tax rate from the high tax jurisdictions increases the demand of capital from this country. This increase is moderated by the increase in the interest rate, which for the case of the intermediate tax economy leads to a decrease in their demand of capital. The effect over the demand of capital from the low tax economy is ambiguous.

7. In the model with capital, when the corporate tax base non-negativity constraint is binding, the nominal production and prices for intermediate goods from multinationals subsidiaries will be higher than the nominal production and prices from domestic firms.

[15] The exception to this result occurs when we use the same consumption bundle for all countries, as for example in the cases in which we use an equiweighted bundle.

[16] The only exception is under the circular consumption bundle from tables 4, 6, and 8.

8. In most of the results, real GDP increase in the low tax jurisdiction, while it falls in the high tax jurisdiction. The effect over this variable on the intermediary tax jurisdiction is ambiguous.

9. Consumption increases in the low tax jurisdiction, while it falls in both the intermediate and the high tax economy.[17] In most of the results there is a negative effect over real consumption from the intermediate tax country.

10. The transfer of resources from countries that leak profits to haven jurisdictions allows the low tax jurisdiction to have a trade balance deficit, while the waste of resources from both subsidiaries and the government forces the high tax country to sustain a trade balance surplus. In most of these results, the intermediate tax jurisdiction that has both a transfer and a waste of resources as a consequence of profit shifting has to sustain a trade balance surplus. In the model with capital and no profit shifting, the low tax jurisdiction uses an amount of capital that is greater than its endowment, which forces it to have a trade balance surplus that covers the negative net investment income, but this effect is to small to be noticeable on the graphs.

11. The magnitudes of the effects from shifting profits are amplified once capital is introduced.

4.3.1 Role of the production network

Assuming a consumption bundle with home bias such that $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$, we solve the equilibrium for three types of production networks. The three production networks that we use are: i) an equiweighted network in which $\omega_{rm} = 1/3$; ii) an autarkic network with $\omega_{rr} = 1$; and iii) a home bias network with $\omega_{rm} = 0.5$ when $r = m$ and $\omega_{rm} = 0.25$ when $r \neq m$.

Tables 1 and 2 contain the solution for the competitive equilibrium with and without capital under these three different network structures. The direction and the hierarchy from the effects of introducing profit shifting technologies is consistent across network structures both in the model with and without capital. The percentage differences from these effects vary across networks, and in particular, the size of the effects on terms of trade and nominal variables is greater under an autarkic network than under an equiweighted or a home bias network, while the size of the effect on real variables is greater on the latter. These differences are akin between the equiweighted and the home bias networks.

[17] The only exception for the high tax economy is under a circular bundle in which the low tax economy consumes exclusively the good from the high tax jurisdiction. In this case, there is an also an increase in the nominal and real consumption of the high tax jurisdiction.

4.3.2 Role of the consumption bundle

Using the same market shares for multinational subsidiaries and the same three types of networks as before we study the differentiated effects from profit shifting when we change the consumption bundle. The three types of consumption bundles that we consider are: i) an equiweighted bundle in which $\beta_{rm} = 1/3$; ii) and autarkic bundle in which $\beta_{rr} = 1$; and iii) a circular bundle in which $\beta_{13} = 1$, $\beta_{21} = 1$ and $\beta_{32} = 1$.

Tables 3-8 contain the solution for the competitive equilibrium under different consumption bundles and production networks in a model with and without capital. On one hand, under the same consumption bundle and different production networks the magnitude, but not the direction of these effects, changes slightly. On the other hand, under the same production network and different consumption bundles both the magnitude and the direction of some of these effects changes significantly.

Even under all of these changes there is a common denominator, the low tax economy has an increase in wages and consumption, and a decrease in their trade balance, while the intermediate and the high tax economy have a reduction in wages and consumption, and an increase in their trade balance.

The only exception to this rule is under a circular bundle in which the high tax economy has also an increase in real consumption, which is explained by the fact that the more wealthy households from the low tax jurisdiction increase their consumption demand for the goods produced by the high tax jurisdiction, and in this way increase the production and the wealth via governmental transfers and dividends of the households from the latter. Additionally, the reduction in purchasing power for the households in the intermediate tax economy reduces their demand for good produced by the low tax jurisdiction, which reduces the demand for capital from the firms in the latter, and as a consequence there is a negative effect on the interest rate.

This last exception shows how an increase in productive capital is not the only way in which a purely leaking country might be benefited by the introduction of profit shifting technologies. In particular, there are general equilibrium effects through which purchasing power is transmitted from wealthy household in tax havens to household in non-haven jurisdictions if the bundle of the former is heavily biased towards consuming goods from the latter.

Furthermore, tables 1-8 give some us evidence that the structure of the consumption bundle appears to be more relevant for the transmission of the effects from profit shifting than the structure of the production network.

4.3.3 Role of the population size

Tables 9 and 10 present the solution for the competitive equilibrium under three population scenarios with: i) small low tax jurisdiction and big high tax jurisdiction ($n_1 = 0.1$ and $n_3 = 0.7$); ii) low and high tax intermediate size jurisdictions ($n_1 = n_3 = 0.4$); and iii) big low tax jurisdiction and small high tax jurisdiction ($n_1 = 0.7$ and $n_3 = 0.1$). Apart from the differences in equilibrium values, once profit shifting is introduced, the percentage difference on these magnitudes is the same across the three scenarios.

4.3.4 Role of the capital allocation and the global supply of capital

Table 11 shows the solution for the competitive equilibrium under three capital allocation scenarios with: i) low capital endowment in the low tax jurisdiction and high capital endowment in the high tax jurisdiction; ii) intermediate endowment of capital in both low and high tax jurisdictions; and iii) high capital endowment in the low tax jurisdiction and low capital endowment in the high tax jurisdiction. As the supply of capital in the low tax jurisdiction increases, the interest rate falls, and both the magnitude and the percentage difference over all of the variables differ under the three scenarios.

Table 12 shows the solution for the competitive equilibrium under three levels of global capital supply, and as the supply of capital increases the interest rate falls. Equilibrium level differ, but just as in the case of the population size, the percentage difference on these values once profit shifting is introduced is the same under any of the three global capital supply scenarios.

4.3.5 Role of the tax differentials

Tables 13 and 14 present the solution for the competitive equilibrium under tax gap differentials of 5%, 10% and 20%. As the tax differential increases, profits shifted to the low tax jurisdiction from the other two countries increase, but this increase is moderated by a rising Ω_3. The higher tax level and the increasing leaking of profits encourages governments to raise enforcement and prices of concealment assets. In this way, as tax differentials increase, the transmission of resources across countries due to profit shifting operations has an accentuating effect over the percentage differentials in all variables.

4.3.6 Role of the share of multinational corporation subsidiaries

Tables 15 and 16 present the solution for the competitive equilibrium under different common shares of multinational corporations ψ_r. As this share increases, keeping and attracting profits from multinational corporations becomes more attractive for governments, and as a consequence enforcement levels rise, concealment prices fall, and shifted profits increase. As a consequence, the increase of ψ_r accentuates the effects over the percentage differentials of all the model outcomes.

4.3.7 Role of the international tax environment

Tables 17 and 18 contain the solution under differences in the importance of the perfect competition component on the profit shifting technology, while tables 19 and 20 show the same results under changes in the importance of the monopolistic competition component. As α falls and the perfect competition cost component rises, shifted profits, enforcement and concealment prices are reduced. As a consequence, there is a decrease in the effects on the percentage differentials over the equilibrium outcomes. As β falls and the monopolistic competition component cost rises, there is an increase in enforcement and concealment asset prices, in the intermediate tax economy there is an increase in shifted profits because Ω_2 is significantly reduced, while in the high tax economy there is a reallocation of shifted profits directed initially to the low tax economy towards the intermediate tax jurisdiction.[18]

4.3.8 Role of global oversight

Finally, tables 21 and 22 contain the solution under different levels of global oversight. As γ increases, optimal investment in enforcement and concealment prices fall, but the sum of domestic and global enforcement increases, creating an ambiguous effect over the amount of shifted profits. This changes slightly the effect over the percentage differentials of the model outcomes once profit shifting is taken into account.

5 Conclusions

This paper studies the channels for the propagation of the rebated and wasted distortions from profit shifting into macroeconomic effects. Both the model and the results from *The Bermuda Triangle* case provide the following main results:

1. Profit flows to tax havens reallocate resources from households in non-haven jurisdictions to households in haven jurisdictions by increasing corporate dividends and governmental transfers.

[18] A binding tax base non-negativity constraint in the intermediate tax economy does not contradict imperfect re-shifting as this result requires a fixed cost and a discrete decision of whether to use or not profit shifting technologies. Here we are leaving this discrete decision aside and assuming $\Upsilon = 0$.

2. Profit shifting increase wages and purchasing power from households in tax havens, while creating a negative effect in non-haven jurisdictions. The changing wealth of households due to corporate profit shifting creates general equilibrium spillover effects that are swayed by the structure of production networks and the international tax environment, the consumption bundles, the tax differentials, and the size of multinational subsidiaries in intermediate good markets.

3. Tax haven's households have access to a higher real consumption even after facing higher price indexes than households in non-haven countries.

4. Corporate profit shifting reduces the effective marginal tax rate for multinational subsidiaries in profit leaking countries, and as a consequence introduces investment incentives, but these incentives are partially canceled out by an increase in the interest rate.

There are obvious limitations to this analysis that point towards future research developments. Firstly, using only constant return to scale production functions and assuming perfect mobility of capital and labour in their corresponding markets. Secondly, assuming a bounded foresight from the government of its policy effects. The uniformity of rationality across the model environment will be benefit if a fully Ramsey equilibrium is solved. Thirdly, the assumption of perfect home-bias in shareholding could be improved by a portfolio of stocks that reflects the effective portfolio positions from each country household over the global economy. Finally, an empirical estimation of the model in which the input-output global production network and information about country-sector level bilateral linkages for corporate profits shifting are used to estimate the welfare effects from the existence of tax haven opacity. This would allow me to answer empirically, under the limitations of the model, which countries are the effective winners and losers out of the existence of tax havens.

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Appendices

A Proofs

A.1 The sectoral aggregator firm

FOC for the goods of the multinational subsidiary is given by $P_{ri}\left(\frac{\theta_{ri}}{\theta_{ri}-1}\right) y_{ri}^{\frac{1}{\theta_{ri}}} \left(\frac{\theta_{ri}-1}{\theta_{ri}}\right) X_{ri,Ms}^{-\frac{1}{\theta_{ri}}} ds = P_{ri,Ms} ds$ from where we get the demand for intermediate goods from the multinational subsidiaries $x_{ri,Ms} = y_{ri}\left(\frac{P_{ri}}{P_{ri,Ms}}\right)^{\theta_{ri}}$. Similarly we obtain the demand for intermediate domestic goods.

From zero profit $P_{ri}y_{ri} = \int_0^{\psi_{ri}} P_{ri,Ms}x_{ri,Ms}ds + \int_{\psi_{ri}}^1 P_{ri,Ds}x_{ri,Ds}ds$. Substituting intermediate good demand equations $P_{ri}y_{ri} = \int_0^{\psi_{ri}} P_{ri,Ms}^{1-\theta_{ri}} P_{ri}^{\theta_{ri}} y_{ri} ds + \int_{\psi_{ri}}^1 P_{ri,Ds}^{1-\theta_{ri}} P_{ri}^{\theta_{ri}} y_{ri} ds$. Canceling common terms $P_{ri} = \left(\int_0^{\psi_{ri}} P_{ri,Ms}^{1-\theta_{ri}} ds + \int_{\psi_{ri}}^1 P_{ri,Ds}^{1-\theta_{ri}} ds\right)^{\frac{1}{1-\theta_{ri}}}$.

A.2 The domestic intermediate firms

The Lagrangian for the domestic firm is given by

$$\mathscr{L}_{ri,D} = (1-\tau_{ri})\left(P_{ri,Ds}X_{ri,Ds} - \tilde{w}_r l_{ri,Ds} - \sum_{m=1}^{R}\sum_{j=1}^{N_m} P_{mj}x_{rimj,Ds}\right) - \iota_i\left(k_{ri,Ds} - K_{ri}\right).$$

With FOC for $x_{rimj,Ds}$, $k_{ri,Ds}$, and $l_{ri,Ds}$ respectively given by

$$(1-\tau_{ri})\left[\left(\frac{\partial P_{ri,Ds}}{\partial X_{ri,Ds}}X_{ri,Ds} + P_{ri,Ds}\right)\frac{\partial X_{ri,Ds}}{\partial x_{rimj,Ds}} - P_{mj}\right] = \alpha_{ri}\phi_{ri}\omega_{rimj}S_{ri,D}\left(\frac{S_{ri}}{S_{ri,Ds}}\right)^{\frac{1}{\theta_{ri}}} - P_{mj}x_{rimj,D} = 0$$

$$(1-\tau_{ri})\left(\frac{\partial P_{ri,Ds}}{\partial X_{ri,Ds}}X_{ri,Ds} + P_{ri,Ds}\right)\frac{\partial X_{ri,Ds}}{\partial k_{ri,Ds}} - \iota_i = (1-\tau_{ri})\,\alpha^K_{ri}\phi_{ri}S_{ri,D}\left(\frac{S_{ri}}{S_{ri,Ds}}\right)^{\frac{1}{\theta_{ri}}} - \iota_i k_{ri,D} = 0$$

$$(1-\tau_{ri})\left[\left(\frac{\partial P_{ri,Ds}}{\partial X_{ri,Ds}}X_{ri,Ds} + P_{ri,Ds}\right)\frac{\partial X_{ri,Ds}}{\partial l_{ri,Ds}} - \tilde{w}_r\right] = \left(1-\alpha_{ri}-\alpha^K_{ri}\right)\phi_{ri}S_{ri,D}\left(\frac{S_{ri}}{S_{ri,Ds}}\right)^{\frac{1}{\theta_{ri}}} - \tilde{w}_r l_{ri,D} = 0$$

Equality of FOC guarantees symmetry within domestic firms of country-sector ri. By symmetry within multinational subsidiaries we obtain by multiplying (1) by P_{ri} that

$$S_{ri} = \left(\psi_{ri}S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1-\psi_{ri})\,S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)^{\frac{\theta_{ri}}{\theta_{ri}-1}}.$$

Dividing by $S_{ri,D}$ we obtain $\frac{S_{ri}}{S_{ri,D}} = \left(\psi_{ri}\left(\frac{S_{ri,M}}{S_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1-\psi_{ri})\right)^{\frac{\theta_{ri}}{\theta_{ri}-1}}.$

Then

$$\begin{aligned}
S_{ri,D}\left(\frac{S_{ri}}{S_{ri,Ds}}\right)^{\frac{1}{\theta_{ri}}} &= \left(\psi_{ri}S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}}\,S_{ri,D}^{\frac{(\theta_{ri}-1)^2}{\theta_{ri}}} + (1-\psi_{ri})\,S_{ri,D}^{\theta_{ri}-1}\right)^{\frac{1}{\theta_{ri}-1}} \\
&= S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}}\left(\psi_{ri}S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1-\psi_{ri})\,S_{ri,D}^{\theta_{ri}-1-\frac{(\theta_{ri}-1)^2}{\theta_{ri}}}\right)^{\frac{1}{\theta_{ri}-1}} \\
&= S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}}\left(\psi_{ri}S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1-\psi_{ri})\,S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)^{\frac{1}{\theta_{ri}-1}} = S_{ri,D}^{\phi_{ri}}\,S_{ri}^{\frac{1}{\theta_{ri}}}.
\end{aligned}$$

A.3 The multinational subsidiary

A.3.1 Input demands

The Lagrangian for the multinational corporation is given by

$$\mathscr{L}_{i,M} = \sum_{r=1}^{R} \left\{ (1 - \tau_{ri} + \Omega_{ri,s}) \left[P_{ri,Ms} X_{ri,Ms} - \tilde{w}_r l_{ri,Ms} - \sum_{m=1}^{R} \sum_{j=1}^{N_m} P_{mj} x_{rimj,Ms} + \sum_{m=1}^{R} (q_{mir} - q_{rim}) \right] \right.$$
$$\left. - \iota_i (k_{ri,Ms} - K_{ri}) - \left[\frac{\left(\sum_{m=1}^{R} q_{rim,s} \right)^2}{2\alpha} + \frac{\sum_{m=1}^{R} q_{rim,s}^2}{2\beta} + \sum_{m=1}^{R} Q_{rim} c_{rim,s} + \Upsilon \right] + \sum_{m=1}^{R} \Lambda_{rim,s} q_{rim,s} \right\}$$

where $\Lambda_{rim,s}$ is the Lagrange multiplier over the non-negativity constraint of $q_{rim,s}$.

With FOC for $x_{rimj,Ms}$, $k_{ri,Ms}$, and $l_{ri,Ms}$ respectively given by

$$(1 - \tau_{ri} + \Omega_{ri,s}) \left[\left(\frac{\partial P_{ri,Ms}}{\partial X_{ri,Ms}} X_{ri,Ms} + P_{ri,Ms} \right) \frac{\partial X_{ri,Ms}}{\partial x_{rimj,Ms}} - P_{mj} \right]$$
$$= \alpha_{ri} \phi_{ri} \omega_{rimj} S_{ri,M} \left(\frac{S_{ri}}{S_{ri,Ms}} \right)^{\frac{1}{\theta_{ri}}} - P_{mj} x_{rimj,M} = 0$$

$$(1 - \tau_{ri} + \Omega_{ri,s}) \left(\frac{\partial P_{ri,Ms}}{\partial X_{ri,Ms}} X_{ri,Ms} + P_{ri,Ds} \right) \frac{\partial X_{ri,Ms}}{\partial k_{ri,Ms}} - \iota_i$$
$$= (1 - \tau_{ri} + \Omega_{ri,s}) \alpha_{ri}^K \phi_{ri} S_{ri,M} \left(\frac{S_{ri}}{S_{ri,Ms}} \right)^{\frac{1}{\theta_{ri}}} - \iota_i k_{ri,M} = 0$$

$$(1 - \tau_{ri} + \Omega_{ri,s}) \left[\left(\frac{\partial P_{ri,Ms}}{\partial X_{ri,Ms}} X_{ri,Ms} + P_{ri,Ms} \right) \frac{\partial X_{ri,Ms}}{\partial l_{ri,Ms}} - \tilde{w}_r \right]$$
$$= \left(1 - \alpha_{ri} - \alpha_{ri}^K\right) \phi_{ri} S_{ri,M} \left(\frac{S_{ri}}{S_{ri,Ms}} \right)^{\frac{1}{\theta_{ri}}} - \tilde{w}_r l_{ri,M} = 0$$

As before, dividing S_{ri} by $S_{ri,M}$ we obtain that $\frac{S_{ri}}{S_{ri,M}} = \left(\psi_{ri} + (1 - \psi_{ri}) \left(\frac{S_{ri,D}}{S_{ri,M}} \right)^{\frac{\theta_{ri}-1}{\theta_{ri}}} \right)^{\frac{\theta_{ri}}{\theta_{ri}-1}}$.

Then

$$S_{ri,M} \left(\frac{S_{ri}}{S_{ri,Ms}} \right)^{\frac{1}{\theta_{ri}}} = \left(\psi_{ri} S_{ri,M}^{\theta_{ri}-1} + (1 - \psi_{ri}) S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}} S_{ri,M}^{\frac{(\theta_{ri}-1)^2}{\theta_{ri}}} \right)^{\frac{1}{\theta_{ri}-1}}$$
$$= S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} \left(\psi_{ri} S_{ri,M}^{\theta_{ri}-1-\frac{(\theta_{ri}-1)^2}{\theta_{ri}}} + (1 - \psi_{ri}) S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}} \right)^{\frac{1}{\theta_{ri}-1}}$$
$$= S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} \left(\psi_{ri} S_{ri,M}^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1 - \psi_{ri}) S_{ri,D}^{\frac{\theta_{ri}-1}{\theta_{ri}}} \right)^{\frac{1}{\theta_{ri}-1}} = S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}.$$

A.3.2 Theorem 2.1

1. Symmetry between domestic and multinational firms in a model without capital: From the demand for intersectoral inputs for domestic firms and multinational subsidiaries we know that

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{x_{rimj,M}}{x_{rimj,D}}\right)^{\theta_{ri}-1}$$

and from the labour demands

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{l_{ri,M}}{l_{ri,D}}\right)^{\theta_{ri}-1}.$$

Then, from the last two equations

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)$$
$$\times\left(\frac{exp\left\{\alpha_{ri}\epsilon_{ri}\right\}l_{ri,M}^{1-\alpha_{ri}}}{exp\left\{\alpha_{ri}\epsilon_{ri}\right\}l_{ri,D}^{1-\alpha_{ri}}}\left(\prod_{m=1}^{R}\prod_{j=1}^{N_m}\left(\frac{x_{rimj,M}}{x_{rimj,D}}\right)^{\omega_{rimj}}\right)^{\alpha_{ri}}\right)^{\theta_{ri}-1}$$

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}$$

$$\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}=\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})$$
$$0=\psi_{ri}X^2+(1-2\psi_{ri})X-(1-\psi_{ri})$$

with $X=\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}$ is a quadratic equation with positive solution given by $X=1$, which implies that $X_{ri,M}=X_{ri,D}=X_{ri}$. From the previous two equations this implies that $l_{ri,M}=l_{ri,D}=l_{ri}$, and $x_{rimj,M}=x_{rimj,D}=x_{rimj}$. This proofs homogeneity across types of firms. Then, the FOC for both types of firms are given by

$$P_{mj}x_{rimj}=\alpha_{ri}\phi_{ri}\omega_{rimj}S_{ri}$$
$$\tilde{w}_r l_{ri}=(1-\alpha_{ri})\phi_{ri}S_{ri}.$$

2. Asymmetry between domestic and multinational firms in a model with capital: In this case, in addition to the relationship between the intermediate goods and labour demands we know that the capital demands for domestic firms and multinational subsidiaries are related by

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\theta_{ri}-1}$$
$$=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{k_{ri,M}}{k_{ri,D}}\right)^{\theta_{ri}-1}\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\theta_{ri}-1}.$$

Then, following the same steps as before including this last equation

$$\left(\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}\right)=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}$$
$$\psi_{ri}+(1-\psi_{ri})\left(\frac{X_{ri,D}}{X_{ri,M}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}=\left(\psi_{ri}\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}+(1-\psi_{ri})\right)\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}$$
$$0=\psi_{ri}\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}X^2+\left((1-\psi_{ri})\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}-\psi_{ri}\right)X-(1-\psi_{ri})$$

with $X=\left(\frac{X_{ri,M}}{X_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}}$ is a quadratic equation with positive solution given by $X=\left(\frac{1-\tau_{ri}+\Omega_{ri}}{1-\tau_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}$, which implies that $\frac{X_{ri,M}}{X_{ri,D}}=\left(\frac{1-\tau_{ri}+\Omega_{ri}}{1-\tau_{ri}}\right)^{\alpha_{ri}^K\theta_{ri}}$.

A.3.3 Theorem 2.2

The FOC for q_{kim} gives us a system of R equations such that

$$\frac{1}{\alpha}\sum_{h=1}^{R}q_{rih}+\frac{1}{\beta}q_{rim}=\eta_{ri}\left(\mathscr{O}_{ri}\left(m\right)\right)+\Lambda_{rim}+\Omega_{mi}-\Omega_{ri}\ \ \forall m\in\{1,\ldots,R\}\quad(A).$$

The solution for this system also needs to satisfy $[K1]$ $\Gamma_{ri,M}\geq 0$, $[K1]'$ $q_{rim}\geq 0$, $[K2]$ $\Omega_{ri}\geq 0$, $[K2]'$ $\Lambda_{rim}\geq 0$, $[K3]$ $\Omega_{ri}\Gamma_{ri,M}=0$, and $[K3]'$ $\Lambda_{rim}q_{rim}=0$.

1. Lets proof that $q_{rir}=0$. Let's assume that $q_{rir}>0$, then (A) becomes

$$\underbrace{\frac{1}{\alpha}\sum_{h=1}^{R}q_{rih}+\frac{1}{\beta}q_{rir}}_{>0}=\underbrace{-Q_{rir}\left(\gamma_i+b_{rir}\right)}_{\leq 0}+\Lambda_{rir}.$$

which implies that $\Lambda_{rir}>0$ and condition $[K3]'$ is not satisfied. Therefore $q_{rir}=0$ must hold given that $Q_{rir}\geq 0$. Otherwise, if $Q_{rir}<0$ the government from country r would be creating an incentive for multinationals to waste resources in profit shifting to its own jurisdiction and reduce dividends without having an increase in the tax base.

2. Lets proof that $\forall p\in\{z_{ri}+1,\ldots,R\}$ $q_{rig_{ri}(p)}=0$. Substracting (A) for country p from (A)

for country r and given $q_{rir} = 0$ we obtain that

$$\Lambda_{rig_{ri}(p)} + \Omega_{g_{ri}(p)} = \underbrace{\eta_{ri}(z_{ki}) - \eta_{ri}(p)}_{\geq 0} + \frac{1}{\beta} q_{rig_{ri}(p)} + \Lambda_{rir} + \Omega_{ri}.$$

Let us assume that $q_{rig_{ri}(p)} > 0$, then $\Lambda_{rig_{ri}(p)} = 0$ by $[K3]'$, and given that $\Gamma_{g_{ri}(p)} > 0$ due to positive profit shifting to country $g_{ri}(p)$ we obtain that $\Omega_{g_{ri}(p)} = 0$ by $[K3]$. But this would require that $\Lambda_{rir} + \Omega_{ri} < 0$ which would violate $[K2]$ or $[K2]'$. Therefore $q_{rig_{ri}(p)} = 0$ must hold.

3. Now lets proof that if $\eta_{ri}(m) \geq \eta_{ri}(p)$ and $q_{rig_{ri}(m)} = 0$, then $q_{rig_{ri}(s)} = 0$.

We already know that $q_{rig_{ri}(m)} = 0$ for $m \geq z_{ri}$. Now, for $p \in \{1, \ldots, z_{ri} - 1\}$ and $m \in \{1, \ldots, p-1\}$. To proof by contradiction for $q_{rig_{ri}(m)} = 0$, let us assume that $q_{rig_{ri}(p)} > 0$ which implies by $[K3]'$ that $\Lambda_{rig_{ri}(p)} = 0$ and by $[K3]$ $\Omega_{g_{ri}(p)i} = 0$, then substracting equation (A) for country $g_{ri}(p)$ from equation (A) for country $g_{ri}(m)$ we have that

$$0 = \underbrace{\eta_{ri}(m) - \eta_{ri}(p)}_{\geq 0} + \frac{1}{\beta} \underbrace{q_{rig_{ri}(p)}}_{>0} + \Lambda_{rig_{ri}(m)} + \Omega_{g_{ri}(m)i}$$

which would require $\Lambda_{rig_{ri}(m)} + \Omega_{g_{ri}(m)i} < 0$ and $[K2]$ or $[K2]'$ would be violated. Therefore $q_{rig_{ri}(p)} = 0$ must hold.

4. Now, lets proof that if $\tau_{mi} \geq \tau_{ri}$ then $q_{rim} = 0$. From (A)

$$\frac{1}{\alpha} \sum_{h=1}^{R} q_{rih} + \frac{1}{\beta} q_{rim} = \underbrace{\eta_{ri}(\mathscr{O}_{ri}(m))}_{\leq 0} + \Lambda_{rim} + \Omega_{mi} - \Omega_{ri}$$

Let us assume that $q_{rim} > 0$, from $[K3]$ and $[K3]'$ we know that $\Lambda_{rim} = \Omega_{mi} = 0$ and we would require $\Omega_{ri} < 0$ for the condition to hold, which would violate $[K2]$. Therefore $q_{rim} = 0$ must hold. Similarly, if $\eta_{ri}(\mathscr{O}_{ri}(m)) \leq 0$ we have that $q_{rim} = 0$.

5. Now, lets find when $q_{rig_{ri}(m)} > 0$ for $m \in \{1, \ldots, L_{ri}\}$ with $1 \leq L_{ri} \leq z_{ri} - 1$ and $q_{rig_{ri}(p)} = 0$ for $p \in \{L_{ri} + 1, \ldots, K\}$ is an optimal profit shifting strategy.

From $[K3]$ and $[K3]'$ as before $\Omega_{g_{ri}(m)i} = 0$ and $\Lambda_{rig_{ri}(m)} = 0$. Then the system of equations for $q_{rig_{ri}(m)}$ is given by

$$\frac{1}{\alpha} \sum_{h=1}^{R} q_{rih} + \frac{1}{\beta} q_{rig_{ri}(m)} = \eta_{ri}(m) - \Omega_{ri} = \delta_{ri}(m) \quad \forall m \in \{1, \ldots, L_{ri}\}$$

which can be represented in matrix form as:

$$\begin{pmatrix} a & f & \ldots & f \\ f & a & \ldots & f \\ \vdots & \vdots & \ddots & \vdots \\ f & f & \ldots & a \end{pmatrix} \begin{pmatrix} q_{rig_{ri}(1)} \\ q_{rig_{ri}(2)} \\ \ldots \\ q_{rig_{ri}(L_{ri})} \end{pmatrix} = \begin{pmatrix} \delta_{ri}(1) \\ \delta_{ri}(2) \\ \ldots \\ \delta_{ri}(L_{ri}) \end{pmatrix}$$

or $A_{ri} q_{ri} = \delta_{ri}$ with $a = \frac{\alpha+\beta}{\alpha\beta}$ and $f = \frac{1}{\alpha}$.

To use Cramer's rule let's start by finding

$$|A_{ri}| = \begin{vmatrix} a+f(L_{ri}-1) & f & \dots & f \\ a+f(L_{ri}-1) & a & \dots & f \\ \vdots & \vdots & \ddots & \vdots \\ a+f(L_{ri}-1) & f & \dots & a \end{vmatrix} = \begin{vmatrix} a+f(L_{ri}-1) & f & \dots & f \\ 0 & a-f & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a-f \end{vmatrix}$$
$$= (a+f(L_{ri}-1))(a-f)^{L_{ri}-1} = \frac{\alpha+\beta L_{ri}}{\alpha\beta^{L_{ri}}}$$

where the first equality comes from adding all the columns to the first one, and the second from substracting the first row from all others.

Now let us define $A_{ri,j}$ as the matrix in which δ_{ri} replaces the j-th column of A_{ri}. Then

$$|A_{ri,j}| = \begin{vmatrix} a & f & \dots & \delta_{ri}(1) & \dots & f \\ f & a & \dots & \delta_{ri}(2) & \dots & f \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ f & f & \dots & \delta_{ri}(j) & \dots & f \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ f & f & \dots & \delta_{ri}(L_{ri}) & \dots & a \end{vmatrix} = \begin{vmatrix} a+f(L_{ri}-2) & f & \dots & \delta_{ri}(1) & \dots & f \\ 0 & a-f & \dots & \delta_{ri}(2)-\delta_{ri}(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ f-a & 0 & \dots & \delta_{ri}(j)-\delta_{ri}(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_{ri}(L_{ri})-\delta_{ri}(1) & \dots & a-f \end{vmatrix}$$
$$= \begin{vmatrix} \delta_{ri}(j)-\delta_{ri}(1) & f-a & 0 & \dots & 0 & \dots & 0 \\ \delta_{ri}(1) & a+f(L_{ri}-2) & f & \dots & f & \dots & f \\ \delta_{ri}(3)-\delta_{ri}(1) & 0 & a-f & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \delta_{ri}(2)-\delta_{ri}(1) & 0 & 0 & \dots & a-f & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \delta_{ri}(L_{ri})-\delta_{ri}(1) & 0 & 0 & \dots & 0 & \dots & a-f \end{vmatrix} = \begin{vmatrix} H_{ri,j}(1) & H_{ri,j}(2) \\ H_{ri,j}(3) & H_{ri,j}(4) \end{vmatrix}$$

where the second equality comes from adding all columns different than j to the first column with a in the diagonal, and substracting the first row with $a+f(L_{ri}-2)$ in the diagonal from all other rows, and the third equality comes from the substitution of column 1 for column j, then column j for column 2, then row j for row 1, and finally row j for row 2. The number of substitutions is even, which keeps the sign of the determinant. $H_{ri,j}(1)$ and $H_{ri,j}(4)$ are square matrices of dimension 2 and $L_{ri}-2$, respectively.

Where $|H_{ri,j}(4)| = \beta^{-(L_{ri}-2)}$, $H_{ri,j}(4)^{-1} = \beta I$, and by Schur's complement

$$|A_{ri,j}| = |H_{ri,j}(4)| \times |H_{ri,j}(1) - H_{ri,j}(2) H_{ri,j}(4)^{-1} H_{ri,j}(3)|$$
$$= \frac{1}{\beta^{L_{ri}-2}} \begin{vmatrix} \delta_{ri}(j)-\delta_{ri}(1) & -\frac{1}{\beta} \\ \delta_{ri}(1) - \frac{\beta}{\alpha}\sum_{s=2;s\neq j}^{L_{ri}} (\delta_{ri}(s)-\delta_{ri}(1)) & \frac{1}{\beta}+\frac{1}{\alpha}(L_{ri}-1) \end{vmatrix}$$
$$= \frac{1}{\beta^{L_{ri}-2}} \left(\frac{1}{\beta}\delta_{ri}(j) - \frac{1}{\alpha}\sum_{s=1}^{L_{ri}} (\delta_{ri}(s)-\delta_{ri}(j)) \right)$$

and by Cramer's rule

$$
\begin{aligned}
q_{rig_{ri}(j)} &= \frac{|A_{ri,j}|}{|A_{ri}|} = \frac{\beta}{\alpha+\beta L_{ki}}\left\{\alpha\delta_{ri}(j)+\beta\sum_{s=1}^{L_{ri}}\left(\delta_{ri}(j)-\delta_{ri}(s)\right)\right\} \\
&= \frac{\beta}{\alpha+\beta L_{ki}}\left\{\alpha\delta_{ri}(j)+\beta\sum_{s=1}^{L_{ri}}\left(\Delta_{ri}(s)-\Delta_{ri}(j)\right)\right\} \\
&= \frac{\beta}{\alpha+\beta L_{ki}}\left\{\alpha\delta_{ri}(j)+\beta E_{rij}(L_{ri})\right\}
\end{aligned}
$$

where $\sum_{m=1}^{L_{ri}} E_{rim}(L_{ri}) = \sum_{m=1}^{L_{ri}}\sum_{s=1}^{L_{ri}}\left(\Delta_{ri}(s)-\Delta_{ri}(m)\right)=0$.

Now, in order to establish which is the marginal country L_{ri} we know from $[K3]'$ that if $\Lambda_{rig_{ri}(L_{ri}+1)}>0$, then $q_{rig_{ri}(L_{ri}+1)}=0$. From (A) we know that $\Lambda_{rig_{ri}(L_{ri}+1)}>0$ if and only if

$$
\begin{aligned}
\Lambda_{rig_{ri}(L_{ri}+1)} &= \frac{1}{\alpha}\sum_{m=1}^{L_{ri}} q_{rim}-\eta_{ri}(L_{ri}+1)+\Omega_{ri}-\Omega_{g_{ri}(L_{ri}+1)i} \\
&= \frac{1}{\alpha}\frac{\beta}{\alpha+\beta L_{ki}}\left(\alpha\sum_{m=1}^{L_{ri}}\delta_{ri}(m)+\beta\sum_{m=1}^{L_{ri}}E_{rim}(L_{ri})\right)-\eta_{ri}(L_{ri}+1)+\Omega_{ri}-\Omega_{g_{ri}(L_{ri}+1)i} \\
&= \frac{\beta}{\alpha+\beta L_{ki}}\sum_{m=1}^{L_{ri}}\delta_{ri}(m)-\eta_{ri}(L_{ri}+1)+\Omega_{ri}-\Omega_{g_{ri}(L_{ri}+1)i} \\
&= \Delta_{ri}(L_{ri}+1)-\Omega_{g_{ri}(L_{ri}+1)i}-\frac{1}{\alpha+\beta L_{ri}}\left(\alpha(\tau_{ri}-\Omega_{ri})+\beta\sum_{m=1}^{L_{ri}}\Delta_{ri}(m)\right) \\
&= G_{ri}(L_{ri})>0.
\end{aligned}
$$

This function $G_{ri}(s)$ defines the degree of competition L_{ri}.

6. Now we have enough elements to define Ω_{ri} using the demands of labour, capital, intersectoral inputs, and concealment assets from the multinational subsidiary and the demand of intermediate multinational goods from the sectoral aggregator

$$
\begin{aligned}
&\Gamma_{ri,M} = P_{ri,M}X_{ri,M}-\tilde{w_r}l_{ri,M}-\sum_{m=1}^{R}\sum_{j=1}^{N_m}P_{mj}x_{rimj,M}+\sum_{m=1}^{R}\left(q_{mir}-q_{rim}\right) \\
&= \frac{P_{ri,M}}{P_{ri}}S_{ri,M}-\left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)\phi_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}-\alpha_{ri}\phi_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\underbrace{\sum_{m=1}^{R}\sum_{j=1}^{N_m}\omega_{rimj}}_{=1}+\sum_{m=1}^{R}q_{mir}-\sum_{m=1}^{L_{ri}}q_{rim} \\
&= \left(\frac{S_{ri}}{S_{ri,M}}\right)^{\frac{1}{\theta_{ri}}}S_{ri,M}-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}+\sum_{m=1}^{R}q_{mir}-\frac{\beta}{\alpha+\beta L_{ri}}\left(\alpha\sum_{m=1}^{L_{ri}}\delta_{ri}(m)+\beta\sum_{m=1}^{L_{ri}}E_{rim}(L_{ri})\right) \\
&= S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right)+\sum_{m=1}^{R}q_{mir}-\frac{\alpha\beta}{\alpha+\beta L_{ri}}\left(L_{ri}\tau_{ri}-\sum_{m=1}^{L_{ri}}\Delta_{ri}(m)\right)+\frac{\alpha\beta L_{ri}}{\alpha+\beta L_{ri}}\Omega_{ri}.
\end{aligned}
$$

Then by $[K3]$

$$\Omega_{ri} = Max\left\{0, \tau_{ri} - \frac{1}{L_{ri}}\sum_{m=1}^{L_{ri}}\Delta_{ri}(m) - \frac{\alpha+\beta L_{ri}}{\alpha\beta L_{ri}}\left(S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right) + \sum_{m=1}^{R} q_{mir}\right)\right\}.$$

7. Now lets study the properties of $G_{ri}(L_{ri})$.

First $\partial G_{ri}(L_{ri})/\partial\tau_{ki} = -\alpha(\alpha+\beta L_{ri})^{-1} < 0$ and $\partial^2 G_{ri}(L_{ri})/\partial L_{ri}\partial\tau_{ki} = \alpha\beta(\alpha+\beta L_{ri})^{-2} > 0$.

Second when $\Gamma_{ri,M} \geq 0$ is not binding

$$\begin{aligned} G_{ri}(z_{ri}-1) &= \Delta_{ri}(z_{ri}) - \frac{\alpha}{\alpha+\beta(z_{ri}-1)}\tau_{ri} - \frac{\beta}{\alpha+\beta(z_{ri}-1)}\sum_{m=1}^{z_{ri}-1}\Delta_{ri}(m) \\ &\geq \Delta_{ri}(z_{ri})\left(1-\frac{\beta(z_{ri}-1)}{\alpha+\beta(z_{ri}-1)}\right) - \frac{\alpha}{\alpha+\beta(z_{ri}-1)}\tau_{ri} \\ &= Q_{rir}(\gamma_i + b_{rir})\left(1-\frac{\beta(z_{ri}-1)}{\alpha+\beta(z_{ri}-1)}\right) > 0 \end{aligned}$$

this means that the degree of competition $L_{ri} < z_{ri}$.

Finally, when the non-negativity constraint for $\Gamma_{si,M} \geq 0\ \forall s$ are not binding

$$G_{ri}(s) - G_{ri}(s-1)$$

$$= \left(\frac{1}{\alpha+\beta(s-1)} - \frac{1}{\alpha+\beta s}\right)\left(\alpha\tau_{ri} + \beta\sum_{m=1}^{s-1}\Delta_{ri}(m)\right) + \Delta_{ri}(s+1) - \frac{\alpha+\beta(s+1)}{\alpha+\beta s}\Delta_{ri}(s) > 0$$

if $\frac{\Delta_{ri}(s+1)}{\alpha+\beta(s+1)} \geq \frac{\Delta_{ri}(s)}{\alpha+\beta s}$.

8. In the proofs 2, 3 and 4 we used the assumption that $q_{rim} > 0$ implies that $\Omega_{mi} = 0$. In words this implies that shifted profits are not fully reshifted. When shifting an amount ϵ out of country r, a multinational can either shift directly to a country p, or if Q_{rip} or b_{rip} are too high shift ϵ to another economy s, and from s shift ϵ to another economy w. What we are going to proof is under which conditions it is not optimal for the firm to fully shift ϵ from s to w. The cost of shifting ϵ from r to p is given by

$$\frac{\left(\sum_{m=1}^{R} q_{rim}+\epsilon\right)^2}{2\alpha} + \frac{\sum_{m=1}^{R} q_{rim}^2 + 2\epsilon q_{rip} + \epsilon^2}{2\beta} + \sum_{m=1}^{R} Q_{rim}q_{rim}(\gamma_i + b_{rim}) + Q_{rip}\epsilon(\gamma_i + b_{rip}).$$

The cost of shifting ϵ from r to s and then from s to w is

$$\frac{\left(\sum_{m=1}^{R} q_{rim}+\epsilon\right)^2}{2\alpha} + \frac{\sum_{m=1}^{R} q_{rim}^2 + 2\epsilon q_{ris} + \epsilon^2}{2\beta} + \sum_{m=1}^{R} Q_{rim}q_{rim}(\gamma_i + b_{rim}) + Q_{ris}\epsilon(\gamma_i + b_{ris})$$

$$+\frac{\left(\sum_{m=1}^{R} q_{sim}+\epsilon\right)^2}{2\alpha} + \frac{\sum_{m=1}^{R} q_{sim}^2 + 2\epsilon q_{siw} + \epsilon^2}{2\beta} + \sum_{m=1}^{R} Q_{sim}q_{sim}(\gamma_i + b_{sim}) + Q_{siw}\epsilon(\gamma_i + b_{siw}).$$

The second cost is greater than the first if

$$Q_{rip}(\gamma_i + b_{rip}) < Q_{ris}(\gamma_i + b_{ris}) + Q_{siw}(\gamma_i + b_{siw}) + \frac{\left(\sum_{m=1}^{R} q_{sim}+\epsilon\right)^2}{2\alpha\epsilon} + \frac{\sum_{m=1}^{R} q_{sim}^2}{2\beta\epsilon} + \frac{q_{siw}}{\epsilon} + \frac{\epsilon}{2\beta} + \frac{1}{\epsilon}\sum_{m=1}^{R} Q_{sim}q_{sim}(\gamma_i + b_{sim}).$$

In particular this holds for any $\epsilon \to 0$ when $\sum_{m=1}^{R} q_{sim} > 0$, and also because the multinational subsidiary from country s wouldn't incur in fixed cost Υ just to transfer a small amount ϵ when $\sum_{m=1}^{R} q_{sim} = 0$.

A.4 Households

The FOC for $\tilde{L}_r$, d_r and d_{rmj} are respectively given by

$$\frac{\left(d_r\left(1-\tilde{L}_r\right)^{\lambda_r}\right)^{-\sigma} d_r\lambda_r\left(1-\tilde{L}_k\right)^{\lambda_k-1}}{n_r\tilde{w}_r} = \Xi_r$$

$$\frac{\left(d_r\left(1-\tilde{L}_r\right)^{\lambda_r}\right)^{-\sigma}\left(1-\tilde{L}_k\right)^{\lambda_k}}{n_rP_r} = \Xi_r$$

$$\frac{\left(d_r\left(1-\tilde{L}_r\right)^{\lambda_r}\right)^{-\sigma}\beta_{rmj}d_r\left(1-\tilde{L}_k\right)^{\lambda_k}}{n_rP_{mj}d_{rmj}} = \Xi_r$$

where Ξ_r stands for the Lagrange multiplier of the household budget constraint.

Thus from the FOC of $\tilde{L}_r$ and d_r.

$$\tilde{w}_r\left(1-\tilde{L}_r\right) = \lambda_rP_rd_r$$

multiplying by n_r

$$\tilde{w}_r\left(n_r-L_r\right) = w_r - \tilde{w}_rL_r = \lambda_rP_rD_r.$$

Also from the FOC of $\tilde{L}_r$ and d_{rmj} multiplied by n_r

$$\frac{\beta_{rmj}}{\lambda_r}\left(w_r-\tilde{w}_rL_r\right) = P_{mj}D_{rmj}.$$

From the FOC of d_{rmj} and d_{rts}

$$d_{rmj} = \frac{P_{ts}}{P_{mj}}\frac{\beta_{rmj}}{\beta_{rts}}d_{rts}.$$

Using the Cobb-Douglas consumption aggregator

$$d_r = \frac{P_{ts}}{\beta_{rts}}d_{rts}\prod_{m=1}^{R}\prod_{j}^{N_m}\left(\frac{\beta_{rmj}}{P_{mj}}\right)^{\beta_{rmj}}$$

then using $P_{ts}d_{rts} = \beta_{rts}P_rd_r$ that comes from the FOC of d_r and d_{rts}

$$P_r = \prod_{m=1}^{R}\prod_{j=1}^{N_m}\left(\frac{P_{mj}}{\beta_{rmj}}\right)^{\beta_{rmj}}.$$

A.5 Theorem 2.3

1. From the labour market equilibrium

$$L_r = \sum_{i=1}^{N_r} \left(\psi_{ri} l_{ri,M} + (1-\psi_{ri})\, l_{ri,D}\right).$$

From the market clearing condition for the goods produced by sector i of country k we have

$$S_{ri} = \sum_{m=1}^{R} P_{ri} D_{mri} + \sum_{m=1}^{R}\sum_{j=1}^{N_m} \left(\psi_{mj} P_{ri} x_{mjri,M} + (1-\psi_{mj})\, P_{ri} x_{mjri,D}\right).$$

First from households labour supply, the labour market equilibrium, and labour demand from domestic firms and multinational subsidiaries

$$\begin{aligned}
&\sum_{m=1}^{R} P_{ri} D_{mri} = \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \tilde{w}_m L_m\right) \\
&= \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \left(\psi_{mj} \tilde{w}_m l_{mj,M} + (1-\psi_{mj})\, \tilde{w}_m l_{mj,D}\right) \right) \\
&= \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \left(\psi_{mj} \left(1-\alpha_{mj}-\alpha_{mj}^K\right) \phi_{mj} S_{mj,M}^{\phi_{mj}} S_{mj}^{\frac{1}{\theta_{mj}}} + (1-\psi_{mj}) \left(1-\alpha_{mj}-\alpha_{mj}^K\right) \phi_{mj} S_{mj,D}^{\phi_{mj}} S_{mj}^{\frac{1}{\theta_{mj}}} \right) \right) \\
&= \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \phi_{mj} \left(1-\alpha_{mj}-\alpha_{mj}^K\right) S_{mj}^{\frac{1}{\theta_{mj}}} \left(\psi_{mj} S_{mj,M}^{\phi_{mj}} + (1-\psi_{mj})\, S_{mj,D}^{\phi_{mj}} \right) \right) \\
&= \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \phi_{mj} \left(1-\alpha_{mj}-\alpha_{mj}^K\right) S_{mj}^{\frac{1}{\theta_{mj}}+\phi_{mj}} \right) \\
&= \sum_{m=1}^{R} \frac{\beta_{mri}}{\lambda_m} \left(w_m - \sum_{j=1}^{N_m} \phi_{mj} \left(1-\alpha_{mj}-\alpha_{mj}^K\right) S_{mj} \right).
\end{aligned}$$

Second, from intermediate input demand from domestic firms and multinational subsidiaries

$$\begin{aligned}
&\sum_{m=1}^{R}\sum_{j=1}^{N_m} \left(\psi_{mj} P_{ri} x_{mjri,M} + (1-\psi_{mj})\, P_{ri} x_{mjri,D}\right) \\
&= \sum_{m=1}^{R}\sum_{j=1}^{N_m} \left(\psi_{mj} \alpha_{mj} \phi_{mj} \omega_{mjri} S_{mj,M}^{\phi_{mj}} S_{mj}^{\frac{1}{\theta_{mj}}} + (1-\psi_{mj})\, \alpha_{mj} \phi_{mj} \omega_{mjri} S_{mj,D}^{\phi_{mj}} S_{mj}^{\frac{1}{\theta_{mj}}} \right) \\
&= \sum_{m=1}^{R}\sum_{j=1}^{N_m} \phi_{mj} \alpha_{mj} \omega_{mjri} S_{mj}^{\frac{1}{\theta_{mj}}+\phi_{mj}} = \sum_{m=1}^{R}\sum_{j=1}^{N_m} \phi_{mj} \alpha_{mj} \omega_{mjri} S_{mj}.
\end{aligned}$$

2. From the household budget constraint, the household labour supply, dividends and lump sum taxes

$$\sum_{m=1}^{R}\sum_{j=1}^{N_m} P_{mj}D_{rmj} = \tilde{w}_r L_r + \sum_{i=1}^{N_r}\underbrace{\bar{\pi}_{ri}}_{=0} + \sum_{i=1}^{N_r}\left(\psi_{ri}\pi_{ri,M} + (1-\psi_{ri})\,\pi_{ri,D}\right) + T_r$$

$$\sum_{m=1}^{R}\sum_{j=1}^{N_m}\frac{\beta_{rmj}}{\lambda_r}\left(w_r - \tilde{w}_r L_r\right) = \tilde{w}_r L_r + Div_r + T_r$$

$$\frac{w_r - \tilde{w}_r L_r}{\lambda_r}\underbrace{\sum_{m=1}^{R}\sum_{j=1}^{N_m}\beta_{rmj}}_{=1} = \tilde{w}_r L_r$$

$$+\sum_{i=1}^{N_r}\left[(1-\tau_{ri})\left(\psi_{ri}\Gamma_{ri,M} + (1-\psi_{ri})\,\Gamma_{ri,D}\right) - \psi_{ri}\left(\iota_i\left(k_{ri,M} - K_{ri}\right) + C_{ri}\right) - (1-\psi_{ri})\,\iota_i\left(k_{ri,D} - K_{ri}\right)\right]$$

$$+\sum_{i=1}^{N_r}\left[\tau_{ri}\left(\psi_{ri}\Gamma_{ri,M} + (1-\psi_{ri})\,\Gamma_{ri,D}\right)\right] + \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) - \sum_{i=1}^{N_r}\sum_{m=1}^{R} b_{rim}^2$$

$$\frac{w_r - \tilde{w}_r L_r}{\lambda_r} = \tilde{w}_r L_r + \sum_{i=1}^{N_r}\left(\psi_{ri}\Gamma_{ri,M} + (1-\psi_{ri})\,\Gamma_{ri,D} + \iota_i K_{ri} - \iota_i\left(\psi_{ri}k_{ri,M} + (1-\psi_{ri})\,k_{ri,D}\right) - \psi_{ri}C_{ri} - \sum_{m=1}^{R} b_{rim}^2\right)$$

$$+\sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right)$$

Just as before $\tilde{w}_r L_r = \sum_{i=1}^{N_r}\phi_{ri}\left(1 - \alpha_{ri} - \alpha_{ri}^K\right)S_{ri}$. From firm demands of capital

$$\begin{aligned}
\psi_{ri}\iota_i k_{ri,M} + (1-\psi_{ri})\,\iota_i k_{ri,D} &= \psi_{ri}\left(1 - \tau_{ri} + \Omega_{ri}\right)\alpha_{ri}^K\phi_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}} + (1-\psi_{ri})\left(1-\tau_{ri}\right)\alpha_{ri}^K\phi_{ri}S_{ri,D}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\\
&= \phi_{ri}\alpha_{ri}^K\left(1-\tau_{ri}\right)S_{ri}^{\frac{1}{\theta_{ri}}}\left(\psi_{ri}S_{ri,M}^{\phi_{ri}} + (1-\psi_{ri})\,S_{ri,D}^{\phi_{ri}}\right) + \psi_{ri}\phi_{ri}\alpha_{ri}^K\Omega_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\\
&= \phi_{ri}\alpha_{ri}^K\left(\left(1-\tau_{ri}\right) + \frac{\psi_{ri}\Omega_{ri}}{\psi_{ri} + (1-\psi_{ri})\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}}\right)S_{ri}.
\end{aligned}$$

From previous definitions

$$\begin{aligned}
\Gamma_{ri,M} &= S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\left(1 - \left(1-\alpha_{ri}^K\right)\phi_{ri}\right) + \sum_{m=1}^{R} q_{mir} - \sum_{m=1}^{R} q_{rim}\\
\Gamma_{ri,D} &= S_{ri,D}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\left(1 - \left(1-\alpha_{ri}^K\right)\phi_{ri}\right)\\
\psi_{ri}\Gamma_{ri,M} + (1-\psi_{ri})\,\Gamma_{ri,D} &= \left(1 - \left(1-\alpha_{ri}^K\right)\phi_{ri}\right)S_{ri} + \psi_{ri}q_{ri}.
\end{aligned}$$

As a consequence

$$w_r - \sum_{i=1}^{N_r}\phi_{ri}\left(1-\alpha_{ri}-\alpha_{ri}^K\right)S_{ri} = \lambda_r\left[\sum_{i=1}^{N_r}\phi_{ri}\left(1-\alpha_{ri}-\alpha_{ri}^K\right)S_{ri} + \sum_{i=1}^{N_r}\left(\left(1 - \left(1-\alpha_{ri}^K\right)\phi_{ri}\right)S_{ri} + \psi_{ri}q_{ri} + \iota_i K_{ri}\right.\right.$$

$$\left.\left.-\phi_{ri}\alpha_{ri}^K\left(\left(1-\tau_{ri}\right) + \frac{\psi_{ri}\Omega_{ri}}{\psi_{ri} + (1-\psi_{ri})\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}}\right)S_{ri} - \psi_{ri}C_{ri} - \sum_{m=1}^{R} b_{rim}^2\right) + \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right)\right]$$

$$w_r - \sum_{i=1}^{N_r}\phi_{ri}\left(1-\alpha_{ri}-\alpha_{ri}^K\right)S_{ri} = \lambda_r\left\{\sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) + \sum_{i=1}^{N_r}\left[\iota_i K_{ri} + \psi_{ri}\left(q_{ri} - C_{ri}\right) - \sum_{m=1}^{R} b_{rim}^2\right.\right.$$

$$\left.\left.+S_{ri}\left(1 - \phi_{ri}\left(\alpha_{ri} + \alpha_{ri}^K\left(\left(1-\tau_{ri}\right) + \frac{\psi_{ri}\Omega_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}}{\psi_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)} + (1-\psi_{ri})\left(1-\tau_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}}\right)\right)\right)\right]\right\}.$$

3. For capital market of industry i, taking into account that $K_{ri} = k_{ri,D}$, and the demand of capital from multinational subsidiaries

$$
\begin{aligned}
\sum_{r=1}^{R} K_{ri} &= \sum_{r=1}^{R} \left(\psi_{ri} k_{ri,M} + (1-\psi_{ri}) k_{ri,D}\right) \\
\sum_{r=1}^{R} K_{ri} &= \sum_{r=1}^{R} \left(\psi_{ri} k_{ri,M} + (1-\psi_{ri}) K_{ri}\right) \\
\sum_{r=1}^{R} \psi_{ri} K_{ri} &= \sum_{r=1}^{R} \frac{\psi_{ri}}{\iota_i} \left(1-\tau_{ri}+\Omega_{ri}\right) \alpha_{ri}^{K} S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}} \\
\iota_i \sum_{r=1}^{R} \psi_{ri} K_{ri} &= \sum_{r=1}^{R} \left(\frac{\psi_{ri}\alpha_{ri}^{K} \left(1-\tau_{ri}+\Omega_{ri}\right)^{1+\alpha_{ri}^{K}(\theta_{ri}-1)}}{\psi_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)} + (1-\psi_{ri})\left(1-\tau_{ri}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}} \right) S_{ri}
\end{aligned}
$$

4. From the production function of the multinational subsidiaries and its input demand

$$
P_{ri} X_{ri,M} = P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\} l_{ri,M}^{1-\alpha_{ri}-\alpha_{ri}^{K}} k_{ri,M}^{\alpha_{ri}^{K}} \left(\prod_{m=1}^{R}\prod_{j}^{N_m} x_{rimj,M}^{\omega_{rimj}}\right)^{\alpha_{ri}}
$$

$$
\begin{aligned}
S_{ri,M} &= P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\} \left(\frac{\left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)\phi_{ri} S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}}{\tilde{w}_r} \right)^{1-\alpha_{ri}-\alpha_{ri}^{K}} \left(\frac{\left(1-\tau_{ri}+\Omega_{ri}\right)\alpha_{ri}^{K}\phi_{ri} S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}}{\iota_i} \right)^{\alpha_{ri}^{K}} \\
&\times \left(\prod_{m=1}^{R}\prod_{j=1}^{N_m} \left(\frac{\alpha_{ri}\phi_{ri}\omega_{rimj} S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}}}{P_{mj}} \right)^{\omega_{rimj}} \right)^{\alpha_{ri}}
\end{aligned}
$$

$$
S_{ri,M} = P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\} \phi_{ri} S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}} \left(\frac{\left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)}{\tilde{w}_r} \right)^{1-\alpha_{ri}-\alpha_{ri}^{K}} \left(\frac{\left(1-\tau_{ri}+\Omega_{ri}\right)\alpha_{ri}^{K}}{\iota_i} \right)^{\alpha_{ri}^{K}} \left(\alpha_{ri} \prod_{m=1}^{R}\prod_{j=1}^{N_m} \left(\frac{\omega_{rimj}}{P_{mj}}\right)^{\omega_{rimj}} \right)^{\alpha_{ri}}
$$

$$
\frac{S_{ri}}{\left(\psi_{ri} + (1-\psi_{ri})\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}\right)^{\frac{1}{\phi_{ri}}}} =
$$

$$
\frac{P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\}\phi_{ri} S_{ri}}{\psi_{ri} + (1-\psi_{ri})\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}} \left(\frac{\left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)}{\tilde{w}_r} \right)^{1-\alpha_{ri}-\alpha_{ri}^{K}} \left(\frac{\left(1-\tau_{ri}+\Omega_{ri}\right)\alpha_{ri}^{K}}{\iota_i} \right)^{\alpha_{ri}^{K}} \left(\alpha_{ri} \prod_{m=1}^{R}\prod_{j=1}^{N_m} \left(\frac{\omega_{rimj}}{P_{mj}}\right)^{\omega_{rimj}} \right)^{\alpha_{ri}}
$$

$$
\begin{aligned}
P_{ri} = &\frac{1}{\phi_{ri}} exp\left\{-\alpha_{ri}\epsilon_{ri}\right\} \left(\psi_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)} + (1-\psi_{ri})\left(1-\tau_{ri}\right)^{\alpha_{ri}^{K}(\theta_{ri}-1)}\right)^{-\frac{1}{\theta_{ri}-1}} \\
&\times \left(\frac{w_k}{n_k\left(1-\alpha_{ri}-\alpha_{ri}^{K}\right)} \right)^{1-\alpha_{ri}-\alpha_{ri}^{K}} \left(\frac{\iota_i}{\alpha_{ri}^{K}}\right)^{\alpha_{ri}^{K}} \left(\frac{\tilde{P}_{ri}}{\alpha_{ri}}\right)^{\alpha_{ri}}.
\end{aligned}
$$

5. The last four equations come directly from the production function of the sectoral aggregator, its demand for intermediate goods from the domestic firms and multinational subsidiaries, and the price composition derived from the zero profit condition and Theorem 2.1.

A.6 Corollary 2.1

From the goods market clearing condition we can define nominal domestic consumption and nominal exports

$$\sum_{i=1}^{N_r} S_{ri} = \underbrace{\sum_{i=1}^{N_r} P_{ri} D_{rri}}_{\text{Nominal Con}_r^{dom}} + \underbrace{\sum_{i=1}^{N_r} \sum_{\substack{m=1 \\ m \neq r}}^{R} \left(P_{ri} D_{mri} + \sum_{j=1}^{N_m} \alpha_{mj} \phi_{mj} \omega_{mjri} S_{mj} \right)}_{\text{Nominal Exp}_r} + \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \alpha_{ri} \phi_{ri} \omega_{rirj} S_{ri}$$

$$\sum_{i=1}^{N_r} S_{ri} \left(1 - \alpha_{ri} \phi_{ri} \sum_{j=1}^{N_r} \omega_{rirj} \right) = \text{Nominal Con}_r^{dom} + \text{Nominal Exp}_r.$$

Nominal Imports for country r are given by

$$\begin{aligned}
\text{Nominal Imp}_r &= \sum_{\substack{m=1 \\ m \neq r}}^{R} \sum_{j=1}^{N_m} P_{mj} D_{rmj} + \sum_{i=1}^{N_r} \sum_{\substack{m=1 \\ m \neq r}}^{R} \sum_{j=1}^{N_m} \alpha_{ri} \phi_{ri} \omega_{rimj} S_{ri} \\
&= \text{Nominal Con}_r^{for} + \sum_{i=1}^{N_r} \alpha_{ri} \phi_{ri} S_{ri} \sum_{\substack{m=1 \\ m \neq r}}^{R} \sum_{j=1}^{N_m} \omega_{rimj}.
\end{aligned}$$

Thus

$$\begin{aligned}
\text{Nominal GDP}_r &= \sum_{i=1}^{N_r} S_{ri} \left(1 - \alpha_{ri} \phi_{ri} \right) \\
&= \underbrace{\text{Nominal Con}_r^{dom} + \text{Nominal Con}_r^{for}}_{\text{Nominal Con}_r} + \text{Nominal Exp}_r - \text{Nominal Imp}_r.
\end{aligned}$$

A.7 Government Transfers T_r

We already know that

$$\begin{aligned}
\Gamma_{ri,M} &= S_{ri,M}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}} \left(1 - \left(1 - \alpha_{ri}^K \right) \phi_{ri} \right) + \sum_{m=1}^{R} q_{mir} - \sum_{m=1}^{R} q_{rim} \\
\Gamma_{ri,D} &= S_{ri,D}^{\phi_{ri}} S_{ri}^{\frac{1}{\theta_{ri}}} \left(1 - \left(1 - \alpha_{ri}^K \right) \phi_{ri} \right)
\end{aligned}$$

Then

$$
\begin{aligned}
T_r &= \sum_{i=1}^{N_r} \tau_{ri} \left(\psi_{ri}\Gamma_{ri,M} + (1-\psi_{ri})\,\Gamma_{ri,D}\right) + \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) - \sum_{i=1}^{N_r}\sum_{m=1}^{R} b_{rim}^2 \\
&= \sum_{i=1}^{N_r} \tau_{ri}\left[\left(\psi_{ri}S_{ri,M}^{\phi_{ri}} + (1-\psi_{ri})\,S_{ri,D}^{\phi_{ri}}\right) S_{ri}^{\frac{1}{\theta_{ri}}}\left(1-\left(1-\alpha_{ri}^K\right)\phi_{ri}\right) + \psi_{ri}q_{ri}\right] \\
&+ \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) - \sum_{i=1}^{N_r}\sum_{m=1}^{R} b_{rim}^2 \\
&= \sum_{i=1}^{N_r} \tau_{ri}\left[S_{ri}^{\frac{\theta_{ri}-1}{\theta_{ri}}} S_{ri}^{\frac{1}{\theta_{ri}}}\left(1-\left(1-\alpha_{ri}^K\right)\phi_{ri}\right) + \psi_{ri}q_{ri}\right] + \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) - \sum_{i=1}^{N_r}\sum_{m=1}^{R} b_{rim}^2 \\
&= \sum_{i=1}^{N_r} \tau_{ri}\left(S_{ri}\left(1-\left(1-\alpha_{ri}^K\right)\phi_{ri}\right) + \psi_{ri}q_{ri}\right) + \sum_{m=1}^{R}\sum_{j=1}^{N_m} Q_{mjr}q_{mjr}\left(\gamma_i + b_{mjr}\right) - \sum_{i=1}^{N_r}\sum_{m=1}^{R} b_{rim}^2.
\end{aligned}
$$

A.7.1 Effect of Q_{mir} on T_r

$$
\begin{aligned}
\frac{\partial T_r}{\partial Q_{mir}} = \tau_{ri}\psi_{ri}\frac{\partial q_{ri}}{\partial Q_{mir}} &+ Q_{mir}\left(\gamma_i + b_{mir}\right)\frac{\partial q_{mir}}{\partial Q_{mir}} + q_{mir}\left(\gamma_i + b_{mir}\right) \\
&+ \sum_{p=1,p\neq m}^{R}\sum_{i=1}^{N_p} Q_{pir}\left(\gamma_i + b_{pir}\right)\underbrace{\frac{\partial q_{pir}}{\partial Q_{mir}}}_{=0}.
\end{aligned}
$$

Where

$$
\begin{aligned}
\frac{\partial q_{mir}}{\partial Q_{mir}} &= -1\left\{\mathscr{O}_{mi}(r) \leq L_{mi}\right\}\frac{\beta}{\alpha+\beta L_{mi}}\left(\alpha\left(\gamma_i + b_{mir}\right) + \beta\sum_{s=1}^{L_{mi}}\left(\gamma_i + b_{mir}\right)\right) \\
&= -1\left\{\mathscr{O}_{mi}(r) \leq L_{mi}\right\}\frac{\beta\left(\gamma_i + b_{mir}\right)}{\alpha+\beta L_{mi}}\left(\alpha+\beta L_{mi}\right) = -1\left\{\mathscr{O}_{mi}(r) \leq L_{mi}\right\}\beta\left(\gamma_i + b_{mir}\right),
\end{aligned}
$$

and

$$
\frac{\partial q_{ri}}{\partial Q_{mir}} = \frac{\partial q_{mir}}{\partial Q_{mir}} - \underbrace{\frac{\partial q_{rim}}{\partial Q_{mir}}}_{=0} + \sum_{p=1,p\neq m}^{K}\left(\underbrace{\frac{\partial q_{pir}}{\partial Q_{mir}}}_{=0} - \underbrace{\frac{\partial q_{rip}}{\partial Q_{mir}}}_{=0}\right) = \frac{\partial q_{mir}}{\partial Q_{mir}}.
$$

Finally to proof the decomposition of the offensive concealment competition let us find the value for the concealment diversion effect:

$$
\begin{aligned}
\frac{\partial \sum_{p=1,p\neq r}^{R} q_{mip}}{\partial Q_{mip}} &= 1\left\{\mathscr{O}_{mi}(r) \leq L_{mi}\right\}\frac{\beta^2\left(\gamma_i + b_{mir}\right)}{\alpha+\beta L_{mi}}\sum_{p=1,p\neq r}^{L_{mi}} 1 \\
&= 1\left\{\mathscr{O}_{mi}(r) \leq L_{mi}\right\}\frac{\beta^2\left(L_{mi}-1\right)}{\alpha+\beta L_{mi}}\left(\gamma_i + b_{mir}\right).
\end{aligned}
$$

A.7.2 Effect of b_{rim} on T_r

We know from the proof for the input demand of domestic firms and Theorem 2.1 that

$$S_{ri} = S_{ri,D}\left(\psi_{ri}\left(\frac{S_{ri,M}}{S_{ri,D}}\right)^{\frac{\theta_{ri}-1}{\theta_{ri}}} + (1-\psi_{ri})\right)^{\frac{\theta_{ri}}{\theta_{ri}-1}}$$

$$= S_{ri,D}\left(\psi_{ri}\left(\frac{1-\tau_{ri}+\Omega_{ri}}{1-\tau_{ri}}\right)^{\alpha_{ri}^K(\theta_{ri}-1)} + (1-\psi_{ri})\right)^{\frac{\theta_{ri}}{\theta_{ri}-1}} = S_{ri,D}\, A_{ri}^{\frac{1}{\phi_{ri}}}.$$

Then

$$\frac{\partial S_{ri}}{\partial \Omega_{ri}} = 1\left\{\Omega_{ri} > 0\right\}\left(A_{ri}^{\frac{1}{\phi_{ri}}}\frac{\partial S_{ri,D}}{\partial \Omega_{ri}} + S_{ri,D}\psi_{ri}\alpha_{ri}^K\theta_{ri}A_{ri}^{\frac{1}{\theta_{ri}-1}}\frac{(1-\tau_{ri}+\Omega_{ri})^{\alpha_{ri}^K(\theta_{ri}-1)-1}}{(1-\tau_{ri})^{\alpha_{ri}^K(\theta_{ri}-1)}}\right).$$

Introducing the input demand functions of the domestic firm in the production function of sector ri we know that

$$S_{ri,D} = S_{ri}\left(P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\}\phi_{ri}\left(\frac{1-\alpha_{ri}-\alpha_{ri}^K}{\tilde{w}_r}\right)^{1-\alpha_{ri}-\alpha_{ri}^K}\left(\frac{(1-\tau_{ri})\,\alpha_{ri}^K}{\iota_i}\right)^{\alpha_{ri}^K}\left(\frac{\alpha_{ri}}{\tilde{P}_{ri}}\right)^{\alpha_{ri}}\right)^{\theta_{ri}}$$

$$= S_{ri}\,(1-\tau_{ri})^{\alpha_{ri}^K\theta_{ri}}\,B_{ri}^{\theta_{ri}}$$

where $\tilde{P}_{ri} = \prod_{m=1}^{R}\prod_{j=1}^{N_m}\left(\frac{P_{mj}}{\omega_{rimj}}\right)^{\omega_{rimj}}$. This means that $\frac{\partial S_{ri,D}}{\partial \Omega_{ri}} = \frac{\partial S_{ri}}{\partial \Omega_{ri}}(1-\tau_{ri})^{\alpha_{ri}^K\theta_{ri}}\,B_{ri}^{\theta_{ri}}$ and as a consequence

$$\frac{\partial S_{ri}}{\partial \Omega_{ri}} = 1\left\{\Omega_{ri} > 0\right\}\left(\frac{\psi_{ri}\alpha_{ri}^K\theta_{ri}A_{ri}^{\frac{1}{\theta_{ri}-1}}B_{ri}^{\theta_{ri}}(1-\tau_{ri}+\Omega_{ri})^{\alpha_{ri}^K(\theta_{ri}-1)-1}(1-\tau_{ri})^{\alpha_{ri}^K}}{1-(1-\tau_{ri})^{\alpha_{ri}^K\theta_{ri}}\,A_{ri}^{\frac{1}{\phi_{ri}}}B_{ri}^{\theta_{ri}}}\right)S_{ri}.$$

Now

$$\frac{\partial \Omega_{ri}}{\partial b_{rim}} = -1\left\{\mathscr{O}_{ri}(m) \le L_{ri}\right\}\frac{1}{L_{ri}}\left[Q_{rim} + \frac{\alpha+\beta L_{ri}}{\alpha\beta\theta_{ri}}\left(1-\left(1-\alpha_{ri}^K\right)\phi_{ri}\right)\left(\frac{S_{ri}}{S_{ri,M}}\right)^{\frac{1}{\theta_{ri}}}\right.$$
$$\left.\times\left(\frac{S_{ri,M}}{S_{ri}}\frac{\partial S_{ri}}{\partial b_{rim}} + (\theta_{ri}-1)\frac{\partial S_{ri,M}}{\partial b_{rim}}\right)\right].$$

Introducing the input demand of the multinational subsidiary in the production function of sector ri we know that

$$S_{ri,M} = S_{ri}\left(P_{ri} exp\left\{\alpha_{ri}\epsilon_{ri}\right\}\phi_{ri}\left(\frac{1-\alpha_{ri}-\alpha_{ri}^K}{\tilde{w}_r}\right)^{1-\alpha_{ri}-\alpha_{ri}^K}\left(\frac{(1-\tau_{ri}+\Omega_{ri})\,\alpha_{ri}^K}{\iota_i}\right)^{\alpha_{ri}^K}\left(\frac{\alpha_{ri}}{\tilde{P}_{ri}}\right)^{\alpha_{ri}}\right)^{\theta_{ri}}$$

$$= S_{ri}\,(1-\tau_{ri}+\Omega_{ri})^{\alpha_{ri}^K\theta_{ri}}\,B_{ri}^{\theta_{ri}}$$

which means that

$$\frac{\partial S_{ri,M}}{\partial b_{rim}} = (1-\tau_{ri}+\Omega_{ri})^{\alpha_{ri}^K\theta_{ri}}\,B_{ri}^{\theta_{ri}}\left[\frac{\partial S_{ri}}{\partial b_{rim}} + \frac{\alpha_{ri}^K\theta_{ri}}{1-\tau_{ri}+\Omega_{ri}}S_{ri}\right],$$

and as a consequence

$$\begin{aligned}\frac{\partial\Omega_{ri}}{\partial b_{rim}} =& -1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\frac{1}{L_{ri}}\left[Q_{rim}+\frac{\alpha+\beta L_{ri}}{\alpha\beta\theta_{ri}}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right)\left(\frac{S_{ri}}{S_{ri,M}}\right)^{\frac{1}{\theta_{ri}}}\right.\\ &\times\left(\left(\frac{S_{ri,M}}{S_{ri}}+\left(\theta_{ri}-1\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}\theta_{ri}}B_{ri}^{\theta_{ri}}\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}\frac{\partial\Omega_{ri}}{\partial b_{rim}}\right.\\ &\left.\left.+\alpha_{ri}^{K}\theta_{ri}\left(\theta_{ri}-1\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}\theta_{ri}-1}B_{ri}^{\theta_{ri}}S_{ri}\right)\right]\\ \frac{\partial\Omega_{ri}}{\partial b_{rim}} =& -1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left[\frac{Q_{rim}+\alpha_{ri}^{K}\left(\theta_{ri}-1\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{-1}E_{ri}S_{ri}}{L_{ri}+E_{ri}\frac{\partial S_{ri}}{\partial\Omega_{ri}}}\right].\end{aligned}$$

where

$$E_{ri}=\frac{\alpha+\beta L_{ri}}{\alpha\beta}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^{K}\theta_{ri}}B_{ri}^{\theta_{ri}}\left(\psi_{ri}+\left(1-\psi_{ri}\right)\left(\frac{1-\tau_{ri}}{1-\tau_{ri}+\Omega_{ri}}\right)^{\alpha_{ri}^{K}\left(\theta_{ri}-1\right)}\right)^{\frac{1}{\theta_{ri}-1}}.$$

A.8 Corporate Dividends Div_r

Following the same definitions that were used for T_r and using $k_{ri,D}=K_{ri}$

$$\begin{aligned}Div_r &= \sum_{i=1}^{N_r}\left(\psi_{ri}\pi_{ri,M}+\left(1-\psi_{ri}\right)\pi_{ri,D}\right)\\ &= \sum_{i=1}^{N_r}\left(1-\tau_{ri}\right)\left(\psi_{ri}\Gamma_{ri,M}+\left(1-\psi_{ri}\right)\Gamma_{ri,D}\right)+\psi_{ri}\left(\iota_i\left(K_{ri}-k_{ri,M}\right)-C_{ri}\right)+\left(1-\psi_{ri}\right)\iota_i\left(K_{ri}-k_{ri,D}\right)\\ &= \sum_{i=1}^{N_r}\left\{\left(1-\tau_{ri}\right)\left(S_{ri}\left(1-\left(1-\alpha_{ri}^{K}\right)\phi_{ri}\right)+\psi_{ri}q_{ri}\right)+\psi_{ri}\left(\iota_i\left(K_{ri}-k_{ri,M}\right)-C_{ri}\right)\right\}.\end{aligned}$$

where

$$\begin{aligned}\iota_i k_{ri,M} &= \left(1-\tau_{ri}+\Omega_{ri}\right)\alpha_{ri}^{K}\phi_{ri}S_{ri,M}^{\phi_{ri}}S_{ri}^{\frac{1}{\theta_{ri}}}\\ &= \alpha_{ri}^{K}\phi_{ri}\left(1-\tau_{ri}+\Omega_{ri}\right)^{1+\alpha_{ri}^{K}\left(\theta_{ri}-1\right)}B_{ri}^{\theta_{ri}-1}S_{ri}.\end{aligned}$$

A.8.1 Effect of Q_{mir} on Div_r

We have that

$$\frac{\partial Div_r}{\partial Q_{mir}}=\left(1-\tau_{ri}\right)\psi_{ri}\frac{\partial q_{ri}}{\partial Q_{mir}}-\psi_{ri}\frac{\partial C_{ri}}{\partial Q_{mir}}$$

where

$$\begin{aligned}\frac{\partial C_{ri}}{\partial Q_{mir}} =& \underbrace{\frac{\partial q_{rim}}{\partial Q_{mir}}}_{=0}\left(\frac{1}{\alpha}\sum_{s=1}^{R}q_{ris}+\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\\ &+\sum_{p=1,p\neq m}^{R}\underbrace{\frac{\partial q_{rip}}{\partial Q_{mir}}}_{=0}\left(\frac{1}{\alpha}\sum_{s=1}^{R}q_{ris}+\frac{1}{\beta}q_{rip}+Q_{rip}\left(\gamma_i+b_{rip}\right)\right).\end{aligned}$$

A.9 Theorem 3.1

A.9.1 First Part of Theorem 3.1

First lets find the FOC given by Q_{rim}

$$\begin{aligned}0=\frac{\partial T_m}{\partial Q_{rim}}+\frac{\partial Div_m}{\partial Q_{rim}} &= 1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\left(\left(\psi_{mi}\tau_{mi}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\frac{\partial q_{mi}}{\partial Q_{rim}}+q_{rim}\left(\gamma_i+b_{rim}\right)+\psi_{mi}\left(1-\tau_{mi}\right)\frac{\partial q_{mi}}{Q_{rim}}\right)\\ &= 1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\left(\left(\psi_{mi}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\frac{\partial q_{mi}}{\partial Q_{rim}}+q_{rim}\left(\gamma_i+b_{rim}\right)\right)\\ &= 1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\left(q_{rim}\left(\gamma_i+b_{rim}\right)-\beta\left(\gamma_i+b_{rim}\right)\left(\psi_{mi}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\\ &= 1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\left(q_{rim}-\beta\left(\psi_{mi}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\end{aligned}$$

For the FOC of b_{rim} lets start by simplifying $\frac{\partial C_{ri}}{\partial b_{rim}}$

$$\begin{aligned}\frac{\partial C_{ri}}{\partial b_{rim}} =&1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\left(\frac{\beta^2 Q_{rim}}{\alpha+\beta L_{ri}}\sum_{p=1,p\neq m}^{R}\left(\frac{1}{\alpha}\sum_{s=1}^{R}q_{ris}+\frac{1}{\beta}q_{rip}+Q_{rip}\left(\gamma_i+b_{rip}\right)\right)\right.\\ &\left.-\beta Q_{rim}\left(\frac{1}{\alpha}\sum_{s=1}^{R}q_{ris}+\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\\ =&1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\beta Q_{rim}\left(\left(\frac{\beta\left(L_{ri}-1\right)}{\alpha+\beta L_{ri}}-1\right)\frac{1}{\alpha}\sum_{s=1}^{R}q_{ris}+\frac{\beta}{\alpha+\beta L_{ri}}\sum_{p=1,p\neq m}^{R}\left(\frac{1}{\beta}q_{rip}+Q_{rip}\left(\gamma_i+b_{rip}\right)\right)\right.\\ &\left.-\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\\ =&1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\beta Q_{rim}\left(-\frac{\alpha+\beta}{\alpha\left(\alpha+\beta L_{ri}\right)}\sum_{s=1}^{R}q_{ris}+\frac{\beta}{\alpha+\beta L_{ri}}\sum_{p=1}^{R}\left(\frac{1}{\beta}q_{rip}+Q_{rip}\left(\gamma_i+b_{rip}\right)\right)\right.\\ &\left.-\left(\frac{\beta}{\alpha+\beta L_{ri}}+1\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\\ =&1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\frac{\beta}{\alpha+\beta L_{ri}}Q_{rim}\left(-\frac{\beta}{\alpha}\sum_{s=1}^{R}q_{ris}+\beta\sum_{p=1}^{R}\left(Q_{rip}\left(\gamma_i+b_{rip}\right)\right)-\left(\alpha+\beta\left(L_{ri}+1\right)\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)\\ =&1\left\{\mathscr{O}_{ri}(m)\le L_{ri}\right\}\frac{\beta}{\alpha+\beta L_{ri}}Q_{rim}\left(\beta\sum_{p=1}^{R}\left(Q_{rip}\left(\gamma_i+b_{rip}\right)-\frac{1}{\alpha}q_{rip}\right)-\left(\alpha+\beta\left(L_{ri}+1\right)\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right).\end{aligned}$$

Now, the FOC of b_{rim}

$$0=\frac{\partial T_r}{\partial b_{rim}}+\frac{\partial Div_r}{\partial b_{rim}}=1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left(\tau_{ri}\psi_{ri}\frac{\partial q_{ri}}{\partial b_{rim}}+\tau_{ri}\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}-2b_{rim}\right.$$
$$\left.+\left(1-\tau_{ri}\right)\psi_{ri}\frac{\partial q_{ri}}{\partial b_{rim}}+\left(1-\tau_{ri}\right)\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}-\psi_{ri}\frac{\partial \iota_i k_{ri,M}}{\partial b_{rim}}-\psi_{ri}\frac{\partial C_{ri}}{\partial b_{rim}}\right)$$
$$=1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left(\psi_{ri}\frac{\partial q_{ri}}{\partial b_{rim}}+\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}-2b_{rim}-\psi_{ri}\frac{\partial \iota_i k_{ri,M}}{\partial b_{rim}}-\psi_{ri}\frac{\partial C_{ri}}{\partial b_{rim}}\right)$$
$$=1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left\{\psi_{ri}\frac{\alpha\beta+\beta^2}{\alpha+\beta L_{ri}}Q_{rim}+\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial b_{rim}}-2b_{rim}\right.$$
$$-\psi_{ri}\frac{\beta}{\alpha+\beta L_{ri}}Q_{rim}\left(\beta\sum_{p=1}^{R}\left(Q_{rip}\left(\gamma_i+b_{rip}\right)-\frac{1}{\alpha}q_{rip}\right)-\left(\alpha+\beta\left(L_{ri}+1\right)\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right)+1\left\{\Omega_{ri}>0\right\}\frac{\partial\Omega_{ri}}{\partial b_{rim}}$$
$$\left.\times\left[\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}+\alpha_{ri}^K\phi_{ri}B_{ri}^{\theta_{ri}-1}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}\left(\left(1-\tau_{ri}+\Omega_{ri}\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}+\left(1+\alpha_{ri}^K\left(\theta_{ri}-1\right)\right)S_{ri}\right)\right]\right\}$$
$$=1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left(\frac{\psi_{ri}Q_{rim}}{\alpha+\beta L_{ri}}\left(\alpha\beta+\beta^2+\beta\left(\left(\alpha+\beta\left(L_{ri}+1\right)\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right.\right.\right.$$
$$\left.\left.+\beta\sum_{p=1}^{R}\left(\frac{1}{\alpha}q_{rip}-Q_{rip}\left(\gamma_i+b_{rip}\right)\right)\right)\right)-2b_{rim}+1\left\{\Omega_{ri}>0\right\}\frac{\partial\Omega_{ri}}{\partial b_{rim}}$$
$$\left.\times\left[\left(1-\phi_{ri}\left(1-\alpha_{ri}^K\right)\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}+\alpha_{ri}^K\phi_{ri}B_{ri}^{\theta_{ri}-1}\left(1-\tau_{ri}+\Omega_{ri}\right)^{\alpha_{ri}^K(\theta_{ri}-1)}\left(\left(1-\tau_{ri}+\Omega_{ri}\right)\frac{\partial S_{ri}}{\partial\Omega_{ri}}+\left(1+\alpha_{ri}^K\left(\theta_{ri}-1\right)\right)S_{ri}\right)\right]\right)$$
$$=1\left\{\mathscr{O}_{ri}\left(m\right)\leq L_{ri}\right\}\left(\frac{\psi_{ri}Q_{rim}}{\alpha+\beta L_{ri}}\left(\alpha\beta+\beta^2+\beta\left(\left(\alpha+\beta\left(L_{ri}+1\right)\right)\left(\frac{1}{\beta}q_{rim}+Q_{rim}\left(\gamma_i+b_{rim}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.+\beta\sum_{p=1}^{R}\left(\frac{1}{\alpha}q_{rip}-Q_{rip}\left(\gamma_i+b_{rip}\right)\right)\right)\right)-2b_{rim}-1\left\{\Omega_{ri}>0\right\}\left[\frac{Q_{rim}+\alpha_{ri}^K\left(\theta_{ri}-1\right)\left(1-\tau_{ri}+\Omega_{ri}\right)^{-1}E_{ri}S_{ri}}{L_{ri}+E_{ri}\frac{\partial S_{ri}}{\partial\Omega_{ri}}}\right]J_{ri}\right).$$

A.9.2 Second Part of Theorem 3.1

For country r the space of policy variables is given by b_{rim} for all m such that $\mathscr{O}_{ri}\left(m\right)\leq L_{ri}$ and Q_{mir} for all m such that $\mathscr{O}_{mi}\left(r\right)\leq L_{mi}$. The Hessian matrix for the policy problem of country r is given by

$$\begin{pmatrix} Hb\left(L_{ri}\right) & 0_{L_{ri}\times M_r} \\ 0_{M_r} & HQ\left(M_r\right)\end{pmatrix}$$
$$=\begin{pmatrix} \varrho_{g_{ri}(1)} & \dots & -\frac{\beta^2\psi_{ri}Q_{rig_{ri}(L_{ri})}b_{rig_{ri}(1)}}{\alpha+\beta L_{ri}} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\beta^2\psi_{ri}Q_{rig_{ri}(1)}b_{rig_{ri}(L_{ri})}}{\alpha+\beta L_{ri}} & \dots & \varrho_{g_{ri}(L_{ri})} & 0 & \dots & 0 \\ 0 & \dots & 0 & -\beta\left(\gamma_i+b_{m_1ir}\right) & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & -\beta\left(\gamma_i+b_{M_rir}\right)\end{pmatrix}$$

a square matrix of dimension $L_{ri}+M_r$ where M_r stands for the number of countries from which the multinational corporation in sector i shifts profits towards country r. To proof that the Hessian is negative definite we are going to show that all of its eigenvalues are strictly negative under conditions (a)-(d).

1. $Hb\left(L_{ri}\right)$ **is negative definite:** Recursively we can define $Hb\left(1\right)=\varrho_{g_{ri}(1)}$ and

$$Hb\left(s\right)=\begin{pmatrix}Hb\left(s-1\right) & B_s\\ C_s & \varrho_{g_{ri}(s)}\end{pmatrix}\text{ with}$$

$$B_s^T=-\frac{\beta^2\psi_{ri}Q_{rig_{ri}(s)}}{\alpha+\beta L_{ri}}\begin{pmatrix}b_{rig_{ri}(1)} & \dots & b_{rig_{ri}(s-1)}\end{pmatrix};$$

$$C_s=-\frac{\beta^2\psi_{ri}b_{rig_{ri}(s)}}{\alpha+\beta L_{ri}}\begin{pmatrix}Q_{rig_{ri}(1)} & \dots & Q_{rig_{ri}(s-1)}\end{pmatrix}.$$

In the following proof by induction we are going to follow the logic for the proof of the Haynsworth inertia additivity formula recognizing that $Hb\left(L_{ri}\right)$ is not a Hermite matrix as it is assumed in Haynsworth (1968). The inertia of a matrix $In\left(M\right)=\left(\pi_+\left(M\right),\pi_-\left(M\right),\pi_0\left(M\right)\right)$ is an ordered tripe whose components are the numbers of positive, negative and zero eigenvalues of M.

For the initial step of the proof by induction we know that $In\left(Hb\left(1\right)\right)=\left(0,1,0\right)$. For the inductive step we define an invertible matrix

$$M\left(s\right)=\begin{pmatrix}I_{s-1} & -Hb\left(s-1\right)^{-1}B_s\\ 0_{1\times(s-1)} & 1\end{pmatrix}$$

that we use to define

$$\begin{aligned}\hat{H}b\left(s\right)&=M\left(s\right)^THb\left(s\right)\hat{H}b\left(s\right)\\ &=\begin{pmatrix}Hb\left(s-1\right) & 0_{(s-1)\times 1}\\ C_s-B_s^T\left(Hb\left(s-1\right)^T\right)^{-1}Hb\left(s-1\right) & \varrho_{g_{ri}(s)}-C_sHb\left(s-1\right)^{-1}B_s\end{pmatrix}\end{aligned}$$

where $Hb\left(s\right)$ and $\hat{H}b\left(s\right)$ are congruent matrices.

Now given that $\left(\frac{\beta^2\psi_{ri}}{\alpha+\beta L_{ri}}\right)^2\approx 0$

$$\begin{aligned}&\varrho_{g_{ri}(s)}-C_sHb\left(s-1\right)^{-1}B_s\\ &=\varrho_{g_{ri}(s)}-\left(\frac{\beta^2\psi_{ri}}{\alpha+\beta L_{ri}}\right)^2b_{rig_{ri}(s)}Q_{rig_{ri}(s)}\begin{pmatrix}Q_{rig_{ri}(1)} & \dots & Q_{rig_{ri}(s-1)}\end{pmatrix}Hb\left(s-1\right)^{-1}\begin{pmatrix}b_{rig_{ri}(1)} & \dots & b_{rig_{ri}(s-1)}\end{pmatrix}^T\\ &\approx\varrho_{g_{ri}(s)}.\end{aligned}$$

Then by Schur complement the characteristic polynomial of $\hat{H}b\left(s\right)$ is given by

$$Det\left(\hat{H}b\left(s\right)-\lambda I_s\right)=Det\left(Hb\left(s-1\right)-\lambda I_{s-1}\right)Det\left(\varrho_{g_{ri}(s)}-\lambda\right)$$

which means that the eigenvalues λ of $\hat{H}b\left(s\right)$ are the $s-1$ eigenvalues of $Hb\left(s-1\right)$ and $\varrho_{g_{ri}(s)}$. This means that the eigenvalues of $\hat{H}b\left(s\right)$ are $\left\{\varrho_{g_{ri}(p)}\right\}_{p=1}^{s}$ and additionally

$$In\left(\hat{H}b\left(s\right)\right)=In\left(Hb\left(s-1\right)\right)+In\left(\varrho_{g_{ri}(s)}\right)=\left(0,s,0\right).$$

Now, given that $\left\{\varrho_{g_{ri}(p)}\right\}_{p=1}^{L_{ri}}$ is composed of L_{ri} distinct values, $\hat{H}b\left(L_{ri}\right)$ is diagonalizable. The rest of the proof follows steps similar to those for the Sylvester's Law of Inertia. Sylvester's Law of Inertia cannot be applied to solve this problem because $Hb\left(L_{ri}\right)$ is non-symmetric.

Let $\{\overrightarrow{v}_1, \ldots, \overrightarrow{v}_{L_{ri}}\}$ be the eigenbasis which diagonalizes $Hb\,(L_{ri})$, and a_i the eigenvalue that corresponds to $\overrightarrow{v}_i$. Likewise, let $\{\overrightarrow{w}_1, \ldots, \overrightarrow{w}_{L_{ri}}\}$ be the eigenbasis which diagonalizes $\hat{H}b\,(L_{ri})$, and b_i the eigenvalue that corresponds to $\overrightarrow{w}_i$. Given that $Hb\,(s)$ and $\hat{H}b\,(s)$ are congruent their rank is the same. To create a contradiction we will assume that their index is different. In particular let organize the eigenbasis of $Hb\,(L_{ri})$ in such a way that the first p elements represent the eigenvectors for positive eigenvalues, and for $\hat{H}b\,(L_{ri})$ the first q elements represent the eigenvectors for positive eigenvalues. Equivalent rank implies that in both eigenbasis the last $L_{ri} - s$ elements represent the eigenvectors for zero eigenvalues. Let us assume that $p \neq q$.

First, let us start by assuming $p > q$ and define the linear operator on $L : \mathbb{R}^{L_{ri}} \to \mathbb{R}^{p+(s-q)}$

$$L\,(\overrightarrow{x}) = \begin{pmatrix} \overrightarrow{v}_1^T Hb\,(L_{ri})\,\overrightarrow{x} & \ldots & \overrightarrow{v}_p^T Hb\,(L_{ri})\,\overrightarrow{x} & \overrightarrow{w}_{q+1}^T \hat{H}b\,(L_{ri})\,\overrightarrow{x} & \ldots & \overrightarrow{w}_s^T \hat{H}b\,(L_{ri})\,\overrightarrow{x} \end{pmatrix}.$$

From rank nullity $dim\,(ker\,(L)) = L_{ri} - rk\,(L) \geq L_{ri} - (p+s-q) > L_{ri} - s$. This means that $\exists \overrightarrow{v}_0 : \overrightarrow{v}_0 \in Ker\,(L)\,,\ \overrightarrow{v}_0 \notin span\,\{\overrightarrow{v}_{s+1}, \ldots, \overrightarrow{v}_{L_{ri}}\}\,,\ \overrightarrow{v}_0 \notin span\,\{\overrightarrow{w}_{s+1}, \ldots, \overrightarrow{w}_{L_{ri}}\}$. Additionally the vector $\overrightarrow{v}_0$ can be expressed in terms of the two basis $\overrightarrow{v}_0 = \sum_{i=1}^{L_{ri}} c_i \overrightarrow{v}_i = \sum_{i=1}^{L_{ri}} d_i \overrightarrow{w}_i$ where as we just showed at least one c_i and d_i must be non-zero for $i \leq s$.

Notice that $\overrightarrow{v}_k^T Hb\,(L_{ri})\,\overrightarrow{v}_0 = \sum_{i=1}^{L_{ri}} c_i \overrightarrow{v}_k^T Hb\,(L_{ri})\,\overrightarrow{v}_i = \sum_{i=1}^{L_{ri}} c_i a_i \overrightarrow{v}_k^T \overrightarrow{v}_i = c_k a_k$ due to orthonormality. With $1 \leq k \leq p$, since $L\,(\overrightarrow{v}_0) = \overrightarrow{0}$ we have that $c_k a_k = 0$. This means that $c_k = 0$ given that $a_k > 0$ and the $c_i \neq$ must be such that $p < i$. Hence $\overrightarrow{v}_0 = \sum_{i=p+1}^{L_{ri}} c_i \overrightarrow{v}_i$. Similarly $\overrightarrow{v}_0 = \sum_{1 \leq i \leq 1; s+1 \leq i \leq L_{ri}}^{L_{ri}} d_i \overrightarrow{w}_i$, and the $d_i \neq 0$ must satisfy $i \leq q$.

From here the value $\overrightarrow{v}_0^T Hb\,(L_{ri})\,\overrightarrow{v}_0 = \sum_{i=p+1}^{L_{ri}} \sum_{j=p+1}^{L_{ri}} c_i c_j \overrightarrow{v}_i^T Hb\,(L_{ri})\,\overrightarrow{v}_j^T = \sum_{i=p+1}^{L_{ri}} c_i^2 a_i < 0$ given that $\overrightarrow{v}_i^T Hb\,(L_{ri})\,\overrightarrow{v}_i^T = 0$ when $i \neq j$, a_i when $i = j$, $a_i \leq 0$, and at least one of the $c_i \neq 0$ for $p+1 \leq i \leq s$. Similarly $\overrightarrow{v}_0^T Hb\,(L_{ri})\,\overrightarrow{v}_0 = \sum_{1 \leq i \leq 1; s+1 \leq i \leq L_{ri}} d_i^2 b_i > 0$ for analogous reasons.

Thus creates a contradiction because $\overrightarrow{v}_0^T Hb\,(L_{ri})\,\overrightarrow{v}_0$ cannot be both strictly positive and negative, therefore $p \leq q$ must hold. Similarly we can proof that $q \leq p$ must hold which implies that $p = q$ and $Hb\,(L_{ri})$ is negative definite, i.e.

$$In\,(Hb\,(L_{ri})) = (0, L_{ri}, 0)\,.$$

2. Hessian is negative definite: $HQ\,(M_r)$ is a diagonal matrix with strictly negative entries, therefore all its eigenvalues are strictly negative and the matrix is negative definite. By Schur complement the characteristic polynomial for the Hessian is given by

$$Det\,(Hessian - \lambda I_{L_{ri}+M_r}) = Det\,(Hb\,(L_{ri}) - \lambda I_{L_{ri}})\,Det\,(HQ\,(M_r) - \lambda I_{M_r})\,.$$

This means that the L_{ri} strictly negative eigenvalues of $Hb\,(L_{ri})$ and the M_r strictly eigenvalues of $HQ\,(M_r)$ are the eigenvalues for the Hessian and the hessian is negative definite, i.e. $In\,(Hessian) = (0, L_{ri} + M_r, 0)$.

A.10 Corollary 4.1

A.10.1 First Part

From equations (52)-(54) we know that

$$q_{21} = \frac{\alpha\beta}{\alpha+\beta}\left(\tau_2 - \tau_1 - Q_{21}\left(\gamma + b_{21}\right)\right);$$

$$q_{21} = \beta\left(1 + Q_{21}\left(\gamma + b_{21}\right)\right);$$

$$2b_{21} = Q_{21}\frac{\psi_2\beta}{\alpha+\beta}\left(\alpha+\beta+(\alpha+2\beta)\left(\frac{1}{\beta}q_{21} + Q_{21}\left(\gamma+b_{21}\right)\right) + \beta\left(\frac{1}{\alpha}q_{21} - Q_{21}\left(\gamma+b_{21}\right)\right)\right).$$

From the first equation on the second we get that $Q_{21} = \frac{\alpha(\tau_2-\tau_1)-\psi_1(\alpha+\beta)}{(2\alpha+\beta)(\gamma+b_{21})}$ and then $q_{21} = \frac{\alpha\beta}{2\alpha+\beta}\left(\tau_2 - \tau_1 + \psi_1\right)$. The quadratic equation for b_{21} comes from introducing these two values into the third equation.

From these results

$$\frac{\partial q_{21}}{\partial\alpha} = \left(\frac{\beta}{\alpha+\beta}\right)^2\left(\tau_2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)\right) > 0$$

$$\frac{\partial q_{21}}{\partial\beta} = \left(\frac{\alpha}{\alpha+\beta}\right)^2\left(\tau_2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)\right) > 0$$

$$\frac{\partial^2 q_{21}}{\partial^2\alpha} = -\frac{\beta^2}{(\alpha+\beta)^3}\left(\tau_2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)\right) < 0$$

$$\frac{\partial^2 q_{21}}{\partial\beta} = -\frac{\alpha^2}{(\alpha+\beta)^3}\left(\tau_2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)\right) < 0$$

$$\frac{\partial^2 q_{21}}{\partial\alpha\partial\beta} = \frac{\partial^2 q_{21}}{\partial\beta\partial\alpha} = \frac{2\alpha\beta}{(\alpha+\beta)^3}\left(\tau_2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)\right) > 0$$

$$\frac{\partial b_{21}}{\partial\alpha} = \frac{\beta^2\psi_2\left(1+\tau_2-\tau_1\right)\left(\tau_2-\tau_1+\psi_1\right)}{2\left(2b_{21}+\gamma\right)\left(2\alpha+\beta\right)^2} > 0$$

$$\frac{\partial b_{21}}{\partial\beta} = \frac{\psi_2\left(1+\tau_2-\tau_1\right)\left(2\alpha\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-2\beta\psi_1\right)-\beta^2\psi_1\right)}{2\left(2b_{21}+\gamma\right)\left(2\alpha+\beta\right)^2}$$

$$\frac{\partial Q_{21}}{\partial\alpha} = \frac{\beta\left(\tau_2-\tau_1+\psi_1\right)\left(b_{21}+\gamma\right)-\left(2\alpha+\beta\right)\frac{\partial b_{21}}{\partial\alpha}\left(\beta\psi_1-\alpha\left(\tau_2-\tau_1+\psi_1\right)\right)}{\left(2\alpha+\beta\right)^2\left(b_{21}+\gamma\right)^2} > 0 \text{ if } \alpha > \beta$$

$$\frac{\partial Q_{21}}{\partial\beta} = -\frac{\psi_1\left(2\alpha+\beta\right)\left(b_{21}+\gamma\right)+\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1\right)\left(b_{21}+\gamma+\left(2\alpha+\beta\right)\frac{\partial b_{21}}{\partial\beta}\right)}{\left(2\alpha+\beta\right)^2\left(b_{21}+\gamma\right)^2}$$

$$\frac{\partial q_{21}}{\partial\left(\tau_2-\tau_1\right)} = \frac{\alpha\beta}{2\alpha+\beta} > 0$$

$$\frac{\partial b_{21}}{\partial\left(\tau_2-\tau_1\right)} = \frac{\beta\psi_2\left(\alpha+2\alpha\left(\tau_2-\tau_1\right)-\psi_1\left(\alpha+\beta\right)\right)}{2\left(2\alpha+\beta\right)\left(2b_{21}+\gamma\right)} > 0 \text{ if } \alpha > \beta \text{ and } \psi_1 < \tau_2-\tau_1$$

$$\frac{\partial^2 b_{21}}{\partial b_{21}\partial\left(\tau_2-\tau_1\right)} = -\frac{\beta\psi_2\left(\alpha+2\alpha\left(\tau_2-\tau_1\right)-\psi_1\left(\alpha+\beta\right)\right)}{\left(2\alpha+\beta\right)\left(2b_{21}+\gamma\right)^2} < 0 \text{ if } \alpha>\beta \text{ and } \psi_1<\tau_2-\tau_1$$

$$\frac{\partial Q_{21}}{\partial\left(\tau_2-\tau_1\right)} = \frac{\beta\psi_2\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1\right)\left(\alpha+\psi_1\left(\alpha+\beta\right)\right)+2\alpha\gamma\left(2\alpha+\beta\right)\left(b_{21}+\gamma\right)}{2\left(2b_{21}+\gamma\right)\left(2\alpha+\beta\right)^2\left(\gamma+b_{21}\right)^2} > 0$$

$$\text{if } \alpha>\beta \text{ and } 2\psi_1<\tau_2-\tau_1$$

$$\frac{\partial q_{21}}{\partial\psi_1} = \frac{\alpha\beta}{2\alpha+\beta} > 0$$

$$\frac{\partial b_{21}}{\partial\psi_1} = -\frac{\beta\psi_2\left(\alpha+\beta\right)\left(1+\tau_2-\tau_1\right)}{2\left(2\alpha+\beta\right)\left(2b_{21}+\gamma\right)} < 0$$

$$\frac{\partial^2 b_{21}}{\partial b_{21}\partial\psi_1} = \frac{\beta\psi_2\left(\alpha+\beta\right)\left(1+\tau_2-\tau_1\right)}{\left(2\alpha+\beta\right)\left(2b_{21}+\gamma\right)^2} > 0$$

$$\frac{\partial Q_{21}}{\partial\psi_1} = -\frac{\left(\alpha+\beta\right)\left(\beta\psi_2\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1\right)\left(1+\tau_2-\tau_1\right)+2\gamma\left(2\alpha+\beta\right)\left(b_{21}+\gamma\right)\right)}{2\left(2\alpha+\beta\right)^2\left(2b_{21}+\gamma\right)^3} < 0$$

$$\text{if } \alpha>\beta \text{ and } 2\psi_1<\tau_2-\tau_1$$

$$\frac{\partial q_{21}}{\partial\psi_2} = 0$$

$$\frac{\partial b_{21}}{\partial\psi_2} = \frac{\beta\psi_2\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1\right)\left(1+\tau_2-\tau_1\right)}{2\left(2\alpha+\beta\right)\left(2b_{21}+\gamma\right)} > 0 \text{ if } \alpha>\beta \text{ and } 2\psi_1<\tau_2-\tau_1$$

$$\frac{\partial Q_{21}}{\partial\psi_2} = -\frac{\left(\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1\right)}{\left(b_{21}+\gamma\right)^2}\frac{\partial b_{21}}{\partial\psi_2} < 0 \text{ if } \alpha>\beta \text{ and } 2\psi_1<\tau_2-\tau_1$$

$$\frac{\partial q_{21}}{\partial\gamma} = 0$$

$$\frac{\partial b_{21}}{\partial\gamma} = -\frac{b_{21}}{2b_{21}+\gamma} < 0$$

$$\frac{\partial Q_{21}}{\partial\gamma} = -\frac{\alpha\left(\tau_2-\tau_1-\psi_1\right)-\beta\psi_1}{\left(\gamma+b_{21}\right)\left(2b_{21}+\gamma\right)} < 0 \text{ if } \alpha>\beta \text{ and } 2\psi_1<\tau_2-\tau_1$$

A.10.2 Second Part

From equations (52) and (53) we know that

$$q_{31} = \frac{\beta}{\alpha+2\beta}\left[\alpha\left(\tau 3-\tau_1-Q_{31}\left(\gamma+b_{31}\right)-\Omega_3\right)+\beta\left(\tau_2-\tau_1+Q_{32}\left(\gamma+b_{32}\right)-Q_{31}\left(\gamma+b_{31}\right)\right)\right];$$

$$q_{32} = \frac{\beta}{\alpha+2\beta}\left[\alpha\left(\tau 2-\tau_1-Q_{21}\left(\gamma+b_{21}\right)-\Omega_3\right)+\beta\left(\tau_1-\tau_2+Q_{31}\left(\gamma+b_{31}\right)-Q_{32}\left(\gamma+b_{32}\right)\right)\right];$$

$$Q_{31} = \frac{q_{31}-\beta\psi_1}{\beta\left(\gamma+b_{31}\right)};$$

$$Q_{32} = \frac{q_{32}-\beta\psi_2}{\beta\left(\gamma+b_{32}\right)}.$$

Introducing the last two equations into the first two we obtain a linear system of two equations in two unknowns that by solving it we obtain

$$\begin{aligned}q_{31} =&\frac{\alpha\beta}{2\left(\alpha+\beta\right)}\left(\tau_3-\Omega_3\right)+\frac{\beta^2}{2\left(\alpha+2\beta\right)}\left(\tau_2-\tau_1+\psi_1-\psi_2\right)\\&+\frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\left(2\alpha+3\beta\right)\left(\psi_1-\tau_1\right)+\beta\left(\psi_2-\tau_2\right)\right);\\q_{32} =&\frac{\alpha\beta}{2\left(\alpha+\beta\right)}\left(\tau_3-\Omega_3\right)+\frac{\beta^2}{2\left(\alpha+2\beta\right)}\left(\tau_1-\tau_2+\psi_2-\psi_1\right)\\&+\frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\left(2\alpha+3\beta\right)\left(\psi_2-\tau_2\right)+\beta\left(\psi_1-\tau_1\right)\right).\end{aligned}$$

Finally from equation (54)

$$\begin{aligned}b_{31}^2+\gamma b_{31} = 1\left\{\Omega_3>0\right\}&\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(\gamma+b_{31}\right)\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\frac{\Re_3}{\gamma+b_{31}}\right)\\+\frac{\psi_3\beta\left(q_{31}-\beta\psi_1\right)}{2\left(\alpha+2\beta\right)}&\left(\alpha+\beta+\left(\alpha+3\beta\right)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}\left(q_{31}+q_{32}\right)+\psi_1+\psi_2\right)\right)\\b_{32}^2+\gamma b_{32} = 1\left\{\Omega_3>0\right\}&\left(\frac{q_{32}-\beta\psi_2}{2\beta\left(\gamma+b_{32}\right)\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\frac{\Re_3}{\gamma+b_{32}}\right)\\+\frac{\psi_3\beta\left(q_{32}-\beta\psi_2\right)}{2\left(\alpha+2\beta\right)}&\left(\alpha+\beta+\left(\alpha+3\beta\right)\left(\frac{2}{\beta}q_{32}-\psi_2\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}\left(q_{31}+q_{32}\right)+\psi_1+\psi_2\right)\right)\end{aligned}$$

From here we get that:

$$\begin{aligned}\frac{\partial q_{31}}{\partial\alpha} =&\frac{\beta^2}{2\left(\alpha+\beta\right)^2}\left(\tau_3-\Omega_3\right)-\frac{\beta^2}{2\left(\alpha+2\beta\right)^2}\left(\tau_2-\tau_1+\psi_1-\psi_2\right)\\&+\frac{\beta\left(2\beta^2-\alpha^2\right)}{4\left(\alpha+2\beta\right)^2\left(\alpha+\beta\right)^2}\left(\left(2\alpha+3\beta\right)\left(\psi_1-\tau_1\right)+\beta\left(\psi_2-\tau_2\right)\right)+\frac{\alpha\beta}{2\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\psi_1-\tau_1\right);\\\frac{\partial q_{32}}{\partial\alpha} =&\frac{\beta^2}{2\left(\alpha+\beta\right)^2}\left(\tau_3-\Omega_3\right)-\frac{\beta^2}{2\left(\alpha+2\beta\right)^2}\left(\tau_1-\tau_2+\psi_2-\psi_1\right)\\&+\frac{\beta\left(2\beta^2-\alpha^2\right)}{4\left(\alpha+2\beta\right)^2\left(\alpha+\beta\right)^2}\left(\left(2\alpha+3\beta\right)\left(\psi_2-\tau_2\right)+\beta\left(\psi_1-\tau_1\right)\right)+\frac{\alpha\beta}{2\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(\psi_2-\tau_2\right);\end{aligned}$$

$$\begin{aligned}\frac{\partial q_{31}}{\partial\beta} =&\frac{\alpha^2}{2\left(\alpha+\beta\right)^2}\left(\tau_3-\Omega_3\right)+\frac{\beta\left(\alpha+\beta\right)}{\left(\alpha+2\beta\right)^2}\left(\tau_2-\tau_1+\psi_1-\psi_2\right)+\frac{\alpha\left(4\alpha^2+9\alpha\beta+4\beta^2\right)}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\\&\times\left(\left(2\alpha+3\beta\right)\left(\psi_1-\tau_1\right)+\beta\left(\psi_2-\tau_2\right)\right)+\frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(3\left(\psi_1-\tau_1\right)+\psi_2-\tau_2\right);\\\frac{\partial q_{32}}{\partial\beta} =&\frac{\alpha^2}{2\left(\alpha+\beta\right)^2}\left(\tau_3-\Omega_3\right)+\frac{\beta\left(\alpha+\beta\right)}{\left(\alpha+2\beta\right)^2}\left(\tau_1-\tau_2+\psi_2-\psi_1\right)+\frac{\alpha\left(4\alpha^2+9\alpha\beta+4\beta^2\right)}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\\&\times\left(\left(2\alpha+3\beta\right)\left(\psi_2-\tau_2\right)+\beta\left(\psi_1-\tau_1\right)\right)+\frac{\alpha\beta}{4\left(\alpha+2\beta\right)\left(\alpha+\beta\right)}\left(3\left(\psi_2-\tau_2\right)+\psi_1-\tau_1\right);\end{aligned}$$

the last four equations have an ambiguous sign.

$$\frac{\partial q_{31}}{\partial\tau_1}=\frac{\partial q_{32}}{\partial\tau_2}=-\frac{\beta}{2\left(\alpha+2\beta\right)}\left[\beta+\frac{\alpha\left(2\alpha+3\beta\right)}{2\left(\alpha+\beta\right)}\right]<0;$$

$$\frac{\partial q_{31}}{\partial \tau_2}=\frac{\partial q_{32}}{\partial \tau_1}=\frac{\beta^2}{4(\alpha+\beta)}>0;$$

$$\frac{\partial q_{31}}{\partial \tau_3}=\frac{\partial q_{32}}{\partial \tau_3}=\frac{\alpha\beta}{2(\alpha+\beta)}>0;$$

If $\alpha>\beta$ and $Q_{31}\geq 0$

$$\frac{\partial b_{31}}{\partial \tau_1}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\Re_3\right)\right)$$
$$=\frac{1}{\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left[\frac{\psi_3\beta}{2(\alpha+2\beta)}\left((q_{31}-\beta\psi_1)\left(\frac{2\alpha^2+5\alpha\beta+\beta^2}{\alpha\beta}\frac{\partial q_{31}}{\partial \tau_1}+\frac{\beta-\alpha}{\alpha}\frac{\partial q_{32}}{\partial \tau_1}\right)\right.\right.$$
$$\left.\left.+\frac{\partial q_{31}}{\partial \tau_1}\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)\right)+\frac{1\{\Omega_3>0\}\frac{\partial q_{31}}{\partial \tau_1}}{2(\gamma+b_{32})\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}\right]<0;$$

$$\frac{\partial b_{31}}{\partial \tau_2}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\Re_3\right)\right)$$
$$=\frac{1}{\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left[\frac{\psi_3\beta}{2(\alpha+2\beta)}\left((q_{31}-\beta\psi_1)\left(\frac{2\alpha^2+5\alpha\beta+\beta^2}{\alpha\beta}\frac{\partial q_{31}}{\partial \tau_2}+\frac{\beta-\alpha}{\alpha}\frac{\partial q_{32}}{\partial \tau_2}\right)\right.\right.$$
$$\left.\left.+\frac{\partial q_{31}}{\partial \tau_2}\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)\right)+\frac{1\{\Omega_3>0\}\frac{\partial q_{31}}{\partial \tau_2}}{2(\gamma+b_{32})\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}\right]>0;$$

$$\frac{\partial b_{31}}{\partial \tau_3}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}+\Re_3\right)\right)$$
$$=\frac{1}{\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left[\frac{\psi_3\beta}{2(\alpha+2\beta)}\left((q_{31}-\beta\psi_1)\frac{\partial q_{31}}{\partial \tau_3}\left(\frac{2\alpha^2+7\alpha\beta-\beta^2}{\alpha\beta}\right)\right.\right.$$
$$\left.\left.+\frac{\partial q_{31}}{\partial \tau_3}\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)\right)+\frac{1\{\Omega_3>0\}\frac{\partial q_{31}}{\partial \tau_3}}{2(\gamma+b_{32})\left(2+E_3\frac{\partial S_3}{\partial\Omega_3}\right)}\right]>0;$$

Similarly $\frac{\partial b_{32}}{\partial \tau_1}>0$, $\frac{\partial b_{32}}{\partial \tau_2}<0$ and $\frac{\partial b_{32}}{\partial \tau_3}>0$ if $\alpha>\beta$ and $Q_{32}\geq 0$.

$$\frac{\partial Q_{31}}{\partial \tau_1}=\frac{\frac{\partial q_{31}}{\partial \tau_1}(\gamma+b_{31})-\frac{\partial b_{31}}{\partial \tau_1}(q_{31}-\beta\psi_1)}{\beta(\gamma+b_{31})^2}\text{ is ambiguous;}$$
$$\frac{\partial Q_{31}}{\partial \tau_2}=\frac{\frac{\partial q_{31}}{\partial \tau_2}(\gamma+b_{31})-\frac{\partial b_{31}}{\partial \tau_2}(q_{31}-\beta\psi_1)}{\beta(\gamma+b_{31})^2}\text{ is ambiguous;}$$

$$\frac{\partial Q_{31}}{\partial \tau_3}=\frac{\frac{\partial q_{31}}{\partial \tau_3}(\gamma+b_{31})-\frac{\partial b_{31}}{\partial \tau_3}(q_{31}-\beta\psi_1)}{\beta(\gamma+b_{31})^2}\text{ is ambiguous;}$$
$$\frac{\partial q_{31}}{\partial \psi_1}=\frac{\partial q_{32}}{\partial \psi_2}=\frac{\beta}{2(\alpha+2\beta)}\left(\beta+\frac{\alpha(2\alpha+3\beta)}{2(\alpha+\beta)}\right)>0;$$
$$\frac{\partial q_{31}}{\partial \psi_2}=\frac{\partial q_{32}}{\partial \psi_1}=-\frac{\beta^2}{4(\alpha+\beta)}<0;$$

$$\frac{\partial q_{31}}{\partial \psi_3}=\frac{\partial q_{32}}{\partial \psi_3}=0;$$

$$\frac{\partial b_{31}}{\partial \psi_1}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)=$$

$$\frac{1}{\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left[\frac{\psi_3\beta}{2(\alpha+2\beta)}\left((q_{31}-\beta\psi_1)\left(\frac{2\alpha^2+5\alpha\beta+\beta^2}{\alpha\beta}\frac{\partial q_{31}}{\partial \psi_1}+\frac{\beta-\alpha}{\alpha}\frac{\partial q_{32}}{\partial \psi_1}-(\alpha+2\beta)\right)\right.\right.$$

$$\left.\left.+\left(\frac{\partial q_{31}}{\partial \psi_1}-\beta\right)\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)\right)+\frac{1\{\Omega_3>0\}\left(\frac{\partial q_{31}}{\partial \tau_1}-\beta\right)}{2(\gamma+b_{32})\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}\right];$$

$$\frac{\partial b_{31}}{\partial \psi_2}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)=$$

$$\frac{1}{\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left[\frac{\psi_3\beta}{2(\alpha+2\beta)}\left((q_{31}-\beta\psi_1)\left(\frac{2\alpha^2+5\alpha\beta+\beta^2}{\alpha\beta}\frac{\partial q_{31}}{\partial \psi_2}+\frac{\beta-\alpha}{\alpha}\frac{\partial q_{32}}{\partial \psi_2}+\beta\right)\right.\right.$$

$$\left.\left.+\frac{\partial q_{31}}{\partial \psi_2}\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)\right)+\frac{1\{\Omega_3>0\}\frac{\partial q_{31}}{\partial \psi_2}}{2(\gamma+b_{32})\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}\right];$$

The last two equations have an ambiguous sign. Therefore the effect on prices of ψ_1 and ψ_2 is ambiguous.

$$\frac{\partial b_{31}}{\partial \psi_3}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)=$$

$$\frac{\beta(q_{31}-\beta\psi_1)\left(\alpha+\beta+(\alpha+3\beta)\left(\frac{2}{\beta}q_{31}-\psi_1\right)+\beta\left(\frac{\beta-\alpha}{\alpha\beta}(q_{31}+q_{32})+\psi_1+\psi_2\right)\right)}{2(\alpha+2\beta)\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}>0 \text{ if } Q_{31}>0;$$

$$\frac{\partial Q_{31}}{\partial \psi_3}=-\frac{q_{31}-\beta\psi_1}{\beta(\gamma+b_{31})^2}\frac{\partial b_{31}}{\partial \psi_3}<0 \text{ if } Q_{31}>0$$

Similarly $\frac{\partial b_{32}}{\partial \psi_3}>0$ and $\frac{\partial Q_{32}}{\partial \psi_3}<0$ if $Q_{32}>0$.

$$\frac{\partial q_{31}}{\partial \gamma}=\frac{\partial q_{32}}{\partial \gamma}=0;$$

$$\frac{\partial b_{31}}{\partial \gamma}\left(1+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2\left(2b_{31}+\gamma+\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta(\gamma+b_{31})^2\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)\right)$$

$$=-\frac{1\{\Omega_3>0\}}{(\gamma+b_{31})^2}\left(\frac{q_{31}-\beta\psi_1}{2\beta\left(2+E_3\frac{\partial S_3}{\partial \Omega_3}\right)}+\Re_3\right)<0 \text{ if } Q_{31}>0;$$

$$\frac{\partial Q_{31}}{\partial \gamma}=-\frac{q_{31}-\beta\psi_1}{\beta(\gamma+b_{31})^2}<0 \text{ if } Q_{31}>0$$

Similarly $\frac{\partial b_{32}}{\partial \gamma}<0$ and $\frac{\partial Q_{32}}{\partial \gamma}<0$ if $Q_{32}>0$.

$$\frac{\partial q_{31}}{\partial \Omega_3}=\frac{\partial q_{32}}{\partial \Omega_3}=-\frac{\alpha\beta}{2(\alpha+\beta)}<0;$$

$\frac{\partial b_{31}}{\partial \Omega_3}$ and $\frac{\partial Q_{31}}{\partial \Omega_3}$ have an ambiguous sign.

B Matrix Form Representation

B.1 Second Stage Matrix Form Representation

The system of equations that characterizes the second stage has a representation in matrix form given by

$$S = H\tilde{\beta}\,\tilde{\lambda}^{-1}w$$

$$w = T\,S + \tilde{\lambda}\left[\left(\tilde{\psi}\circ q\right)' e_N + IR\left(diag(K)\,IP\,\iota - b^2 e_R\right) - \Psi\left(qe_R + C\right) + \left(Q\circ q\circ(\gamma e_R + b)\right)' e_N\right]$$

$$diag\left(IK\left(\psi\circ K\right)\right)\iota = IK\;V\;S$$

$$Ln\;P = D\left[\left(e_N - \tilde{\alpha} - \alpha^K\right)\circ\hat{w}_N + \alpha^K\circ Ln\;\left(IP\,\iota\right) - \left(\tilde{\alpha}\circ\epsilon\right)\right] - \mathscr{L}$$

$$S_M = \left(\psi + (e_N - \psi)\circ\left(diag\left(e_N - \tau + \Omega\right)^{-1}\left(e_N - \tau\right)\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)}\right)^{-\diamond\left(diag(\phi)^{-1}e_N\right)}\circ S$$

$$S_D = \left(diag\left(e_N - \tau + \Omega\right)^{-1}\left(e_N - \tau\right)\right)^{\diamond\left(\alpha^K\circ\theta\right)}\circ S_M$$

$$Ln\,P_M = Ln\,P + diag\left(\theta - e_N\right)^{-1}Ln\;\left(\psi + (e_N - \psi)\left(diag\left(e_N - \tau + \Omega\right)^{-1}\left(e_N - \tau\right)\right)^{\diamond\left(\alpha^K\circ(e_N - \theta)\right)}\right)$$

$$Ln\;P_D = Ln\;P_M + \alpha^K\circ\left(Ln\;\left(e_N - \tau\right) - Ln\;\left(e_N - \tau + \Omega\right)\right)$$

where the effective Leontief matrix is given by

$$H = \left[I_N + \tilde{\beta}\,\tilde{\lambda}^{-1}\left(\Phi\circ\Gamma\right) - \left(\left(e_N\,\tilde{\alpha}'\right)\circ W'\circ\left(e_N\,\phi'\right)\right)\right]^{-1} \quad \text{and}$$

$$C = \frac{1}{2\alpha}\left(\left(q\,e_R\right)\circ\left(q\,e_R\right)\right) + \frac{1}{2\beta}\left(q\circ q\right)e_R + \left(Q\circ q\circ\left(\gamma\,e_N' + b\right)\right)e_R + \Upsilon\;e_N$$

$$T = \left(\Phi\circ\left(IR - \Gamma_K\right)\right) + \tilde{\lambda}\left[IR - \left(\Phi\circ\Gamma_M\right) - \left(\Phi\circ\Gamma_K\right)diag\left(e_N - \tau + diag\left(\psi\circ\left(e_N - \tau + \Omega\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)}\right.\right.\right.$$
$$\left.\left.\left. + \left(e_N - \psi\right)\circ\left(e_N - \tau\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)}\right)^{-1}\times\left(\psi\circ\Omega\circ\left(e_N - \tau + \Omega\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)}\right)\right)\right] - \left(\Phi\circ\Gamma_M\right)$$

$$V = diag\left(diag\left(\psi\circ\left(e_N - \tau + \Omega\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)} + \left(e_N - \psi\right)\circ\left(e_N - \tau\right)^{\diamond\left(\alpha^K\circ(\theta - e_N)\right)}\right)^{-1}\right.$$
$$\left.\times\left(\psi\circ\alpha^K\circ\left(e_N - \tau + \Omega\right)^{\diamond\left(e_N + \left(\alpha^K\circ(\theta - e_N)\right)\right)}\right)\right)$$

$$D = \left[I_N - \left(\left(\tilde{\alpha}\;e_N'\right)\circ W\right)\right]^{-1}$$

$$\begin{aligned}\tilde{\mathscr{L}} =& \tilde{\alpha}\circ Ln\ \tilde{\alpha}+\alpha^K\circ Ln\ \alpha^K+\left(e_N-\tilde{\alpha}-\alpha^K\right)\circ Ln\ \left(e_N-\tilde{\alpha}-\alpha^K\right)+Ln\ \phi+\tilde{\alpha}\circ\left(\left(W\circ Ln\ W\right)e_N\right)\\ &+diag\left(\theta-e_N\right)^{-1}Ln\ \left(\psi\circ\left(e_N-\tau+\Omega\right)^{\diamond\left(\alpha^K\circ\left(\theta-e_N\right)\right)}+\left(e_N-\psi\right)\circ\left(e_N-\tau\right)^{\diamond\left(\alpha^K\circ\left(\theta-e_N\right)\right)}\right)\end{aligned}$$

$$\mathscr{L}=D\tilde{\mathscr{L}}.$$

Additionally, I_s and e_s are the identity matrix and the vector of ones of dimension s, $\circ$ stands for the Hadamard product, $\diamond$ stands for the element-wise power, ψ, ϕ, θ, $\tilde{\alpha}$, α^K, Ω, γ, τ, L, and K and are the corresponding vectors of dimension N for ψ_{ri}, ϕ_{ri}, θ_{ri}, α_{ri}, α^K_{ri}, Ω_{ri}, γ_i, τ_{ri}, L_{ri} and K_i, ι is the corresponding vector of dimension $\tilde{N}$ for ι_i,and λ stands for the vector of dimension R for λ_r.

From the solution to the second stage we can define

$$\begin{aligned}&\Omega=Max\Big\{0_{N\times1};\tau-diag\left(L\right)^{-1}\left(IQ\circ\left(\tilde{\tau}+Q\circ\left(\gamma\ e_R'+b\right)\right)\right)e_R\\ &-\left(\alpha+\beta L\right)\circ\left(diag\left(\alpha\beta L\right)^{-1}e_N\right)\circ\left[\left(S_M^{\diamond\phi}\circ S^{\diamond\left(diag(\theta)^{-1}e_N\right)}\circ\left(e_N-\left(\left(e_N-\alpha^K\right)\circ\phi\right)\right)\right)+\tilde{q}\right]\Big\}.\end{aligned}$$

The matrix form representation for nominal GDP and consumption as defined in corollary 2.1 is given by

$$\begin{aligned}\text{Nominal GDP}&=IR\left(\left(e_N-\left(\tilde{\alpha}\circ\phi\right)\right)\circ S\right);\\ \text{Nominal Con}&=IR\left[\left(\left(e_N-\left(\tilde{\alpha}\circ\phi\right)\right)\circ S\right)+\left(\psi\circ\left(K-k_M\right)\circ\left(IP\,\iota\right)\right)-b^2e_R\right]-\Psi\left(q\,e_R+C\right)\\ &+\left(\tilde{\psi}\circ q\right)'e_N+\left(Q\circ q\circ\left(\gamma e_R'+b\right)\right)'e_R.\end{aligned}$$

The structure of the matrices used is presented in the last subsection of this Appendix.

B.2 First Stage Matrix Form Representation

The equilibrium conditions for the first stage have a representation in matrix form given by

$$\begin{aligned}q=&\beta IQ\circ\left[diag\left(\alpha\ e_N+\beta L\right)^{-1}\left(\alpha\left(\tau-\Omega\right)+\beta\left(IQ\circ\left(\tilde{\tau}+Q\circ\left(\gamma\ e_R'+b\right)\right)\right)e_R\right)e_R'\right.\\ &\left.-\left(\tilde{\tau}+Q\circ\left(\gamma e_R'+b\right)\right)\right];\end{aligned}$$

$$0_{N\times R}=IQ\circ\left(q-\beta\left(\tilde{\psi}+\left(Q\circ\left(\gamma\ e_R'+b\right)\right)\right)\right);$$

$$\begin{aligned}0_{N\times R}=&IQ\circ\Big\{\beta\ diag\left(\alpha\ e_N+\beta L\right)^{-1}\left(\psi\ e_R'\circ Q\right)\left[\left(\alpha+\beta\right)\ e_N\ e_R'\right.\\ &\left.+\left(\left(\alpha\ e_N+\beta\left(L+e_N\right)\right)e_R'\right)\circ\left(\frac{1}{\beta}q+Q\circ\left(\gamma e_R'+b\right)\right)+\beta\left[IQ\circ\left(\frac{1}{\alpha}q-Q\circ\left(\gamma e_R'+b\right)\right)\right]e_Re_R'\right]\\ &-2\,b-\left(I\Omega\,e_R'\right)\circ\left[\left(\left(diag\left(e_N-\tau+\Omega\right)^{-1}e_N\right)e_R'\right)\right.\\ &\left.\circ\left[Q+\left(\alpha^K\circ\left(\theta-e_N\right)\circ\left(diag\left(e_N-\tau+\Omega\right)^{-1}e_N\right)\circ E\circ S\right)e_R'\right]\right]\circ\left(J\,e_R'\right)\Big\}.\end{aligned}$$

Where

$$
\begin{aligned}
A =& (e_N - \tau)^{\diamond\left(\alpha^K \circ (e_N - \theta)\right)} \circ \left[\psi \circ (e_N - \tau + \Omega)^{\diamond\left(\alpha^K \circ (\theta - e_N)\right)} + (e_N - \psi) \circ (e_N - \tau)^{\diamond\left(\alpha^K \circ (\theta - e_N)\right)}\right] \\
Ln\, B =& D^{-1} Ln\, P + (\tilde{\alpha} \circ \epsilon) + \left(e_N - \tilde{\alpha} - \alpha^K\right) \circ Ln\left(e_N - \tilde{\alpha} - \alpha^K\right) - \left(e_N - \tilde{\alpha} - \alpha^K\right) \circ \hat{w}_N \\
&+ \alpha^K \circ Ln\,(e_N - \tau) + \alpha^K \circ Ln\, \alpha^K - \alpha^K \circ Ln\,(IP\,\iota) + \tilde{\alpha} \circ Ln\, \tilde{\alpha} + \tilde{\alpha} \circ ((W \circ Ln\, W)\, e_N) \\
E =& \frac{1}{\alpha\beta}\Bigg[(\alpha e_N + \beta L) \circ \left(e_N - \left(\left(e_N - \alpha^K\right) \circ \phi\right)\right) \circ (e_N - \tau + \Omega)^{\diamond\left(\alpha^K \circ \theta\right)} \circ B^{\diamond\theta} \\
&\circ \left(\psi + (e_N - \psi) \circ \left(diag\,(e_N - \tau + \Omega)^{-1} (e_N - \tau)\right)^{\diamond\left(\alpha^K \circ (\theta - e_N)\right)}\right)^{\diamond\left(diag(\theta - e_N)^{-1} e_N\right)}\Bigg] \\
\frac{\partial S}{\partial \Omega} =& I\Omega \circ \left(diag\left(e_N - \left((e_N - \tau)^{\diamond\left(\alpha^K \circ \Omega\right)} \circ A^{\diamond\left(diag(\phi)^{-1} e_N\right)} \circ B^{\diamond\theta}\right)\right)\right)^{-1} \\
&\times \left(\psi \circ \alpha^K \circ \theta \circ A^{\diamond\left(diag(\theta - e_N)^{-1} e_N\right)} \circ B^{\diamond\theta} \circ (e_N - \tau + \Omega)^{\diamond\left(\left(\alpha^K \circ (\theta - e_N)\right) - e_N\right)} \circ (e_N - \tau)^{\diamond\alpha^K}\right)\Bigg) \circ S \\
J =& \left(\left(e_N - \left(\phi \circ \left(e_N - \alpha^K\right)\right)\right) \circ \frac{\partial S}{\partial \Omega}\right) \\
&+ \left(\alpha^K \circ \phi \circ B^{\diamond(\theta - e_N)} \circ (e_N - \tau + \Omega)^{\diamond\left(\alpha^K \circ (\theta - e_N)\right)} \circ \left(\left((e_N - \tau + \Omega) \circ \frac{\partial S}{\partial \Omega}\right) + \left(\left(e_N + \left(\alpha^K \circ (\theta - e_N)\right)\right) \circ S\right)\right)\right).
\end{aligned}
$$

The structure of the matrices used is presented in the next subsection.

B.3 Matrices

$$
W = \begin{bmatrix}
\omega_{1111} & \cdots & \omega_{111N_1} & \cdots & \omega_{11R1} & \cdots & \omega_{11RN_R} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\omega_{1N_111} & \cdots & \omega_{1N_11N_1} & \cdots & \omega_{1N_1R1} & \cdots & \omega_{1N_1RN_R} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\omega_{R111} & \cdots & \omega_{R11N_1} & \cdots & \omega_{R1R1} & \cdots & \omega_{R1RN_R} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\omega_{RN_R11} & \cdots & \omega_{RN_R1N_1} & \cdots & \omega_{RN_RR1} & \cdots & \omega_{RN_RRN_R}
\end{bmatrix},
\tilde{\beta} = \begin{bmatrix}
\beta_{111} & \beta_{211} & \cdots & \beta_{R11} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{11N_1} & \beta_{21N_1} & \cdots & \beta_{R1N_1} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{1R1} & \beta_{2R1} & \cdots & \beta_{RR1} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{1RN_R} & \beta_{2RN_R} & \cdots & \beta_{RRN_R}
\end{bmatrix},
$$

$$
q = \begin{bmatrix}
q_{111} & q_{112} & \cdots & q_{11R} \\
\vdots & \vdots & \ddots & \vdots \\
q_{1N_11} & q_{1N_12} & \cdots & q_{1N_1R} \\
\vdots & \vdots & \ddots & \vdots \\
q_{R11} & q_{R12} & \cdots & q_{R1R} \\
\vdots & \vdots & \ddots & \vdots \\
q_{RN_R1} & q_{RN_R2} & \cdots & q_{RN_RR}
\end{bmatrix},
Q = \begin{bmatrix}
Q_{111} & Q_{112} & \cdots & Q_{11R} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{1N_11} & Q_{1N_12} & \cdots & Q_{1N_1R} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{R11} & Q_{R12} & \cdots & Q_{R1R} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{RN_R1} & Q_{RN_R2} & \cdots & Q_{RN_RR}
\end{bmatrix},
b = \begin{bmatrix}
b_{111} & b_{112} & \cdots & b_{11R} \\
\vdots & \vdots & \ddots & \vdots \\
b_{1N_11} & b_{1N_12} & \cdots & b_{1N_1R} \\
\vdots & \vdots & \ddots & \vdots \\
b_{R11} & b_{R12} & \cdots & b_{R1R} \\
\vdots & \vdots & \ddots & \vdots \\
b_{RN_R1} & b_{RN_R2} & \cdots & b_{RN_RR}
\end{bmatrix},
$$

$$IR = \begin{bmatrix} \underbrace{1 \ldots 1}_{N_1} & 0 & \ldots & 0 \\ 0 & \underbrace{1 \ldots 1}_{N_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \underbrace{1 \ldots 1}_{N_R} \end{bmatrix}_{R\times N}, IP = \begin{bmatrix} ep'_{11} \\ \vdots \\ ep'_{1N_1} \\ \vdots \\ ep'_{R1} \\ \vdots \\ ep'_{RN_R} \end{bmatrix}_{N\times\tilde{N}}, IK = \begin{bmatrix} ek'_1 \\ \vdots \\ ek'_{\tilde{N}} \end{bmatrix}_{\tilde{N}\times N},$$

$$\tilde{\tau} = \begin{bmatrix} \tau(1,1,1) & \tau(1,1,2) & \ldots & \tau(1,1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \tau(1,N_1,1) & \tau(1,N_1,2) & \ldots & \tau(1,N_1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \tau(R,1,1) & \tau(R,1,2) & \ldots & \tau(R,1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \tau(R,N_R,1) & \tau(R,N_R,2) & \ldots & \tau(R,N_R,R) \end{bmatrix}, \tilde{\psi} = \begin{bmatrix} \psi(1,1,1) & \psi(1,1,2) & \ldots & \psi(1,1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \psi(1,N_1,1) & \psi(1,N_1,2) & \ldots & \psi(1,N_1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \psi(R,1,1) & \psi(R,1,2) & \ldots & \psi(R,1,R) \\ \vdots & \vdots & \ddots & \vdots \\ \psi(R,N_R,1) & \psi(R,N_R,2) & \ldots & \psi(R,N_R,R) \end{bmatrix},$$

$$IQ = \begin{bmatrix} 1\{\mathscr{O}_{11}(1) \le L_{11}\} & 1\{\mathscr{O}_{11}(2) \le L_{11}\} & \ldots & 1\{\mathscr{O}_{11}(R) \le L_{11}\} \\ \vdots & \vdots & \ddots & \vdots \\ 1\{\mathscr{O}_{1N_1}(1) \le L_{1N_1}\} & 1\{\mathscr{O}_{1N_1}(2) \le L_{1N_1}\} & \ldots & 1\{\mathscr{O}_{1N_1}(R) \le L_{1N_1}\} \\ \vdots & \vdots & \ddots & \vdots \\ 1\{\mathscr{O}_{R1}(1) \le L_{R1}\} & 1\{\mathscr{O}_{R1}(2) \le L_{R1}\} & \ldots & 1\{\mathscr{O}_{R1}(R) \le L_{R1}\} \\ \vdots & \vdots & \ddots & \vdots \\ 1\{\mathscr{O}_{RN_R}(1) \le L_{RN_R}\} & 1\{\mathscr{O}_{RN_R}(2) \le L_{RN_R}\} & \ldots & 1\{\mathscr{O}_{RN_R}(R) \le L_{RN_R}\} \end{bmatrix}, I\Omega = \begin{bmatrix} 1\{\Omega_{11} > 0\} \\ \vdots \\ 1\{\Omega_{1N_1} > 0\} \\ \vdots \\ 1\{\Omega_{R1} > 0\} \\ \vdots \\ 1\{\Omega_{RN_R} > 0\} \end{bmatrix},$$

$$\tilde{q} = \begin{bmatrix} \begin{bmatrix} I_{N_1} & & 0_{N_1\times(NR-N_1)} \end{bmatrix} & Vec(IP\,IK\,q) \\ \begin{bmatrix} 0_{N_2\times(N+N_1)} & I_{N_2} & 0_{N_2\times\left(N(R-1)-\sum_{r=1}^{2} N_r\right)} \end{bmatrix} & Vec(IP\,IK\,q) \\ \vdots & \vdots \\ \begin{bmatrix} 0_{N_{R-1}\times\left(N(R-2)+\sum_{r=1}^{R-2} N_r\right)} & I_{N_{R-1}} & 0_{N_{R-1}\times\left(2N-\sum_{r=1}^{R-1} N_r\right)} \end{bmatrix} & Vec(IP\,IK\,q) \\ \begin{bmatrix} 0_{N_R\times\left(N(R-1)+\sum_{r=1}^{R-1} N_r\right)} & & I_{N_R} \end{bmatrix} & Vec(IP\,IK\,q) \end{bmatrix},$$

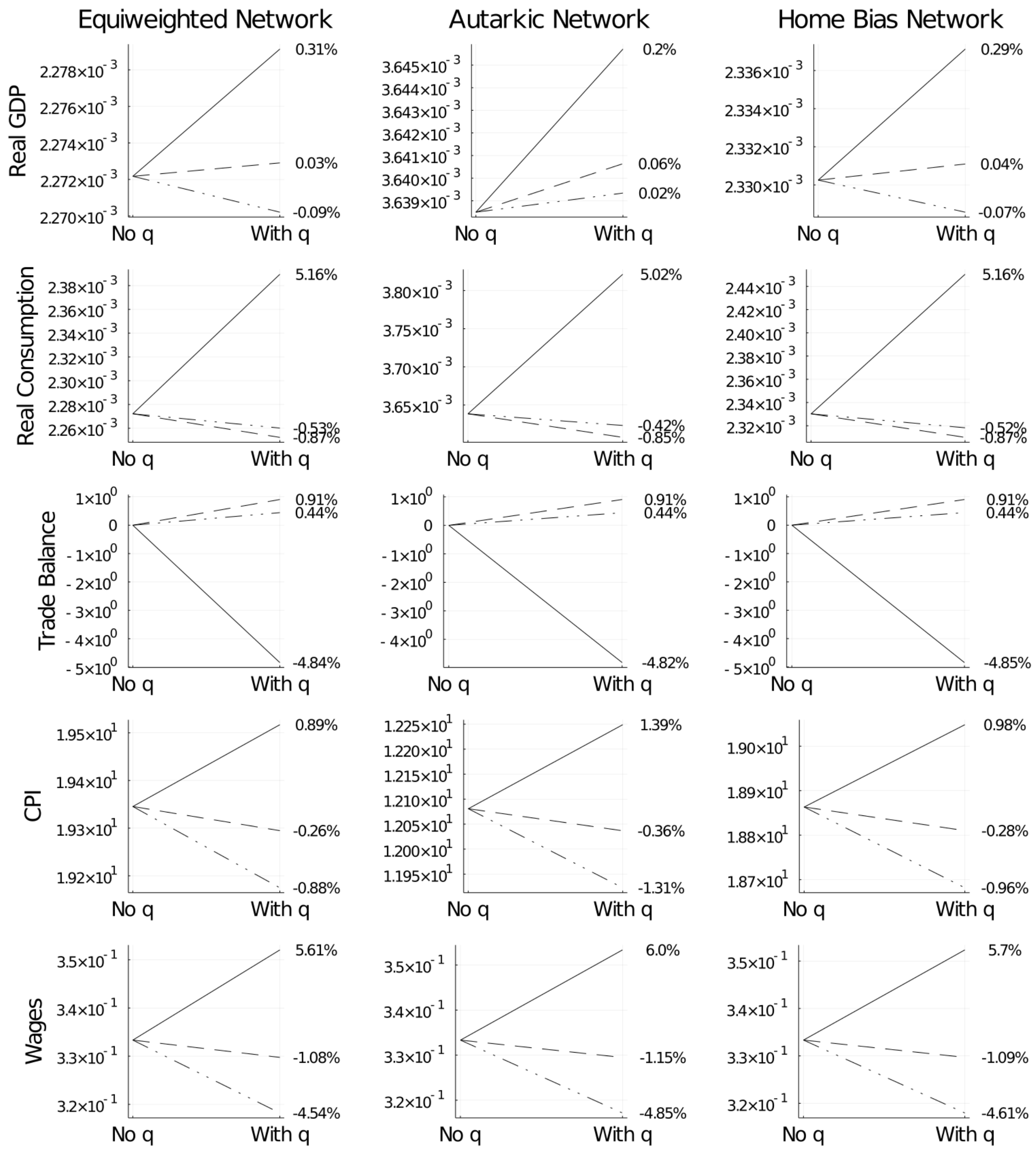
$\hat{w}_N = \Big[\underbrace{Ln\frac{w_1}{n_1} \ldots Ln\frac{w_1}{n_1}}_{N_1} \ldots \underbrace{Ln\frac{w_R}{n_R} \ldots Ln\frac{w_R}{n_R}}_{N_R}\Big]'$, $\Psi = IR\,diag(\psi)$, $\Phi = diag(\phi)$, $\Gamma = IR\,diag\left(e_N - \tilde{\alpha} - \alpha^K\right)$, $\Gamma_M = IR\,diag(\tilde{\alpha})$, $\Gamma_K = IR\,diag\left(\alpha^K\right)$, $\Gamma_M = IR\,diag(\tilde{\alpha})$ and $\tilde{\lambda} = diag(\lambda)$. Moreover, $\tau(r,i,m)$ and $\psi(r,i,m)$ stand for the equivalent τ and ψ in country m of country-sector ri.[19]

Finally, ep_{ri} is a zero vector of size $\tilde{N}$ with a one in the position of the industry to which country-sector ri belongs and ek_i is a zero vector of size N with ones in the positions for those country-sector that belong to industry i. For example, if country-sector ri belongs to industry 1 $ep'_{ri} = \begin{bmatrix}1 & 0 & \ldots & 0\end{bmatrix}$, and in a global economy with three country-sectors in which country-sector 1 and 3 belong to industry 1 $ek'_1 = \begin{bmatrix}1 & 0 & 1\end{bmatrix}$ and $ek'_2 = \begin{bmatrix}0 & 1 & 0\end{bmatrix}$.

[19] For example, if sector index 1 in countries 1 and 2 refer to the same sector, then $\tau(2,1,1) = \tau_{11}$.

C Results

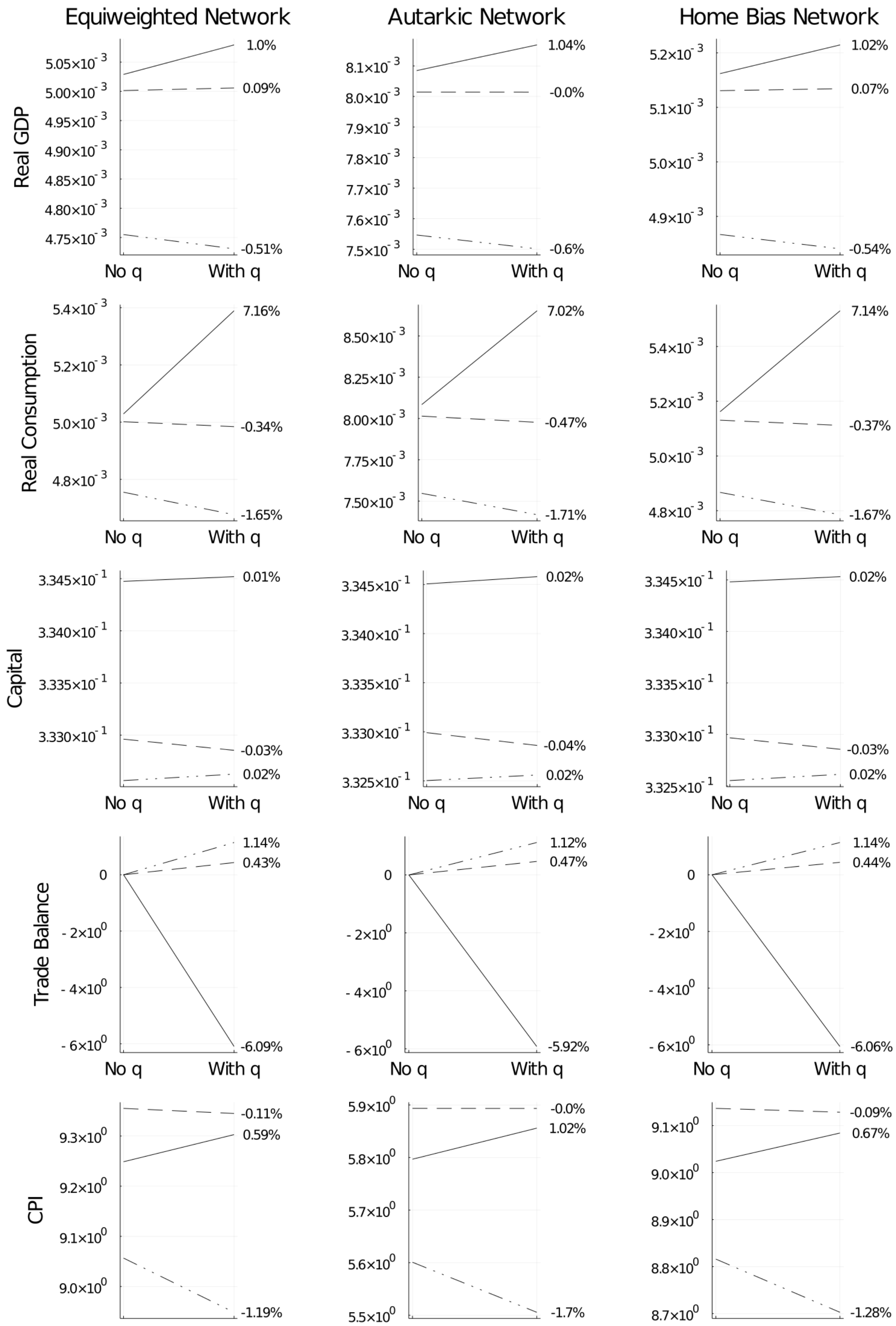
Table 1: Competitive Equilibrium Under Different Networks without K

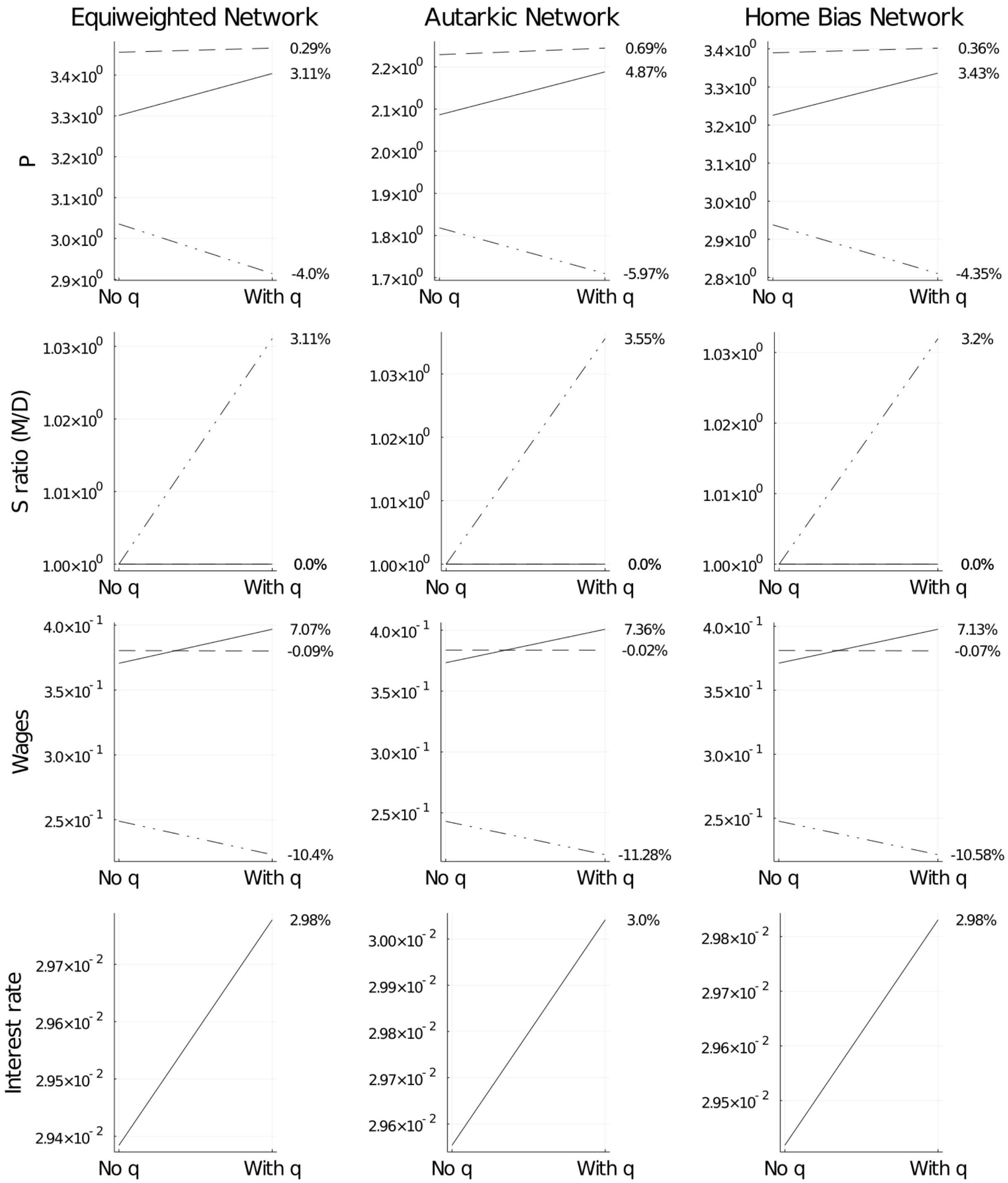


	Equiweighted Network			Autarkic Network			Home Bias Network		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$31.55e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$
$3 \to 1$	$16.91e^{-3}$	$16.75e^{-4}$	$15.15e^{-1}$	$16.87e^{-3}$	$16.57e^{-4}$	$14.99e^{-1}$	$16.89e^{-3}$	$16.64e^{-4}$	$15.04e^{-1}$
$3 \to 2$	$1.29e^{-3}$	$-24.77e^{-4}$	$-25.98e^{-1}$	$1.24e^{-3}$	$-25.15e^{-4}$	$-26.39e^{-1}$	$1.26e^{-3}$	$-25.5e^{-4}$	$-26.77e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	8.44%	0.0%	0.0%	8.48%	0.0%	0.0%	8.47%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The consumption bundle with home bias has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The equiweighted network has $\omega_{rm} = 1/3\ \forall r, m$. The autarkic network has $\omega_{rr} = 1$. The home bias network has $\omega_{rm} = 0.5$ when $r = m$, and $\omega_{rm} = 0.25$ when $r \neq m$.

Table 2: Competitive Equilibrium Under Different Networks with K

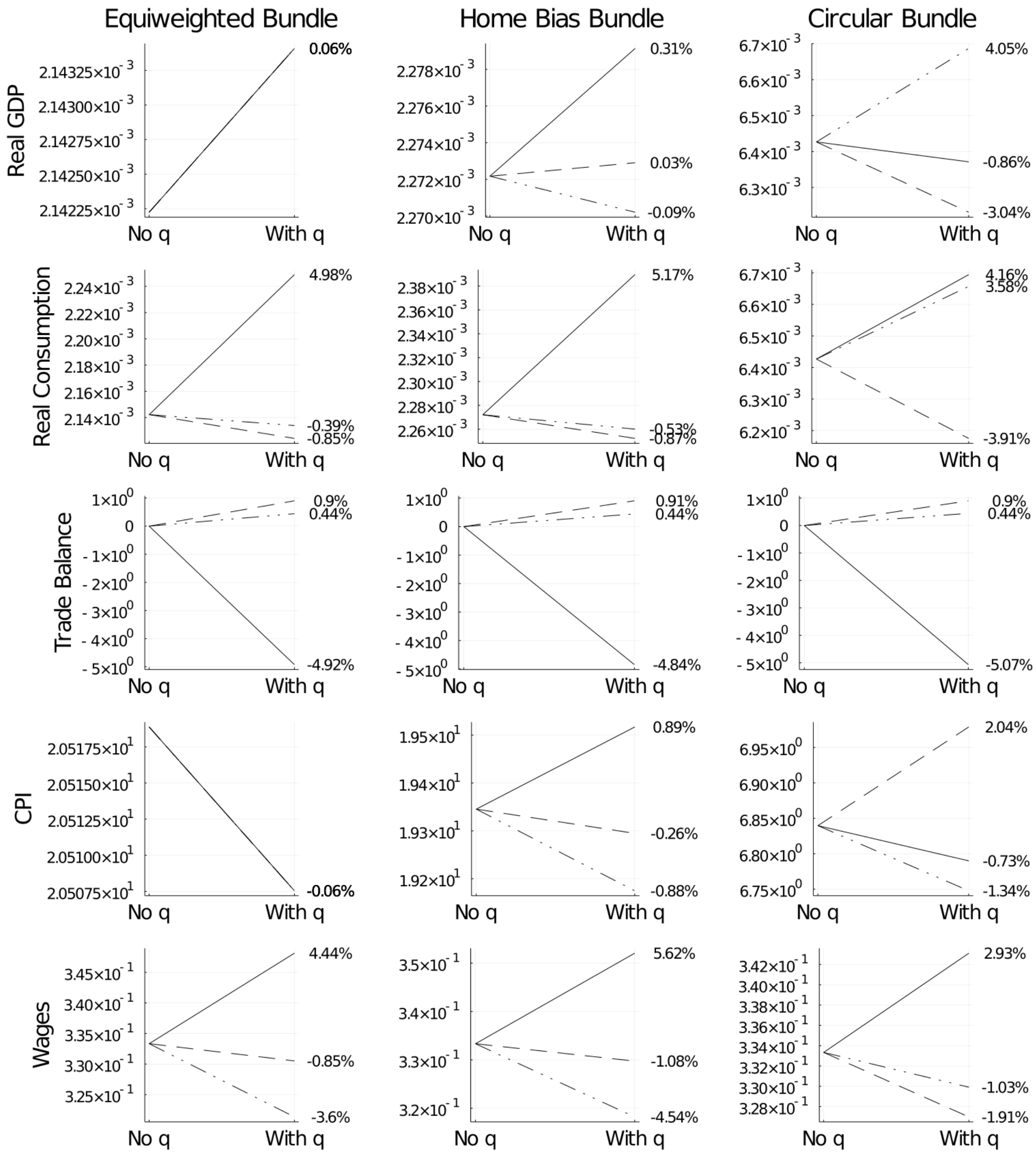




	Equiweighted Network			Autarkic Network			Home Bias Network		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$
$3 \to 1$	$22.22e^{-3}$	$-30.77e^{-4}$	$56.17e^{-1}$	$21.91e^{-3}$	$-29.64e^{-4}$	$53.52e^{-1}$	$22.16e^{-3}$	$-30.54e^{-4}$	$55.64e^{-1}$
$3 \to 2$	$6.6e^{-3}$	$-85.35e^{-4}$	$9.48e^{-1}$	$6.29e^{-3}$	$-89.05e^{-4}$	$1.39e^{-1}$	$6.54e^{-3}$	$-85.87e^{-4}$	$8.08e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	2.43%	0.0%	0.0%	2.78%	0.0%	0.0%	2.5%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The consumption bundle with home bias such that $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The equiweighted network has $\omega_{rm} = 1/3$ $\forall r, m$. The autarkic network has $\omega_{rr} = 1$. The home bias network has $\omega_{rm} = 0.5$ when $r = m$, and $\omega_{rm} = 0.25$ when $r \neq m$.

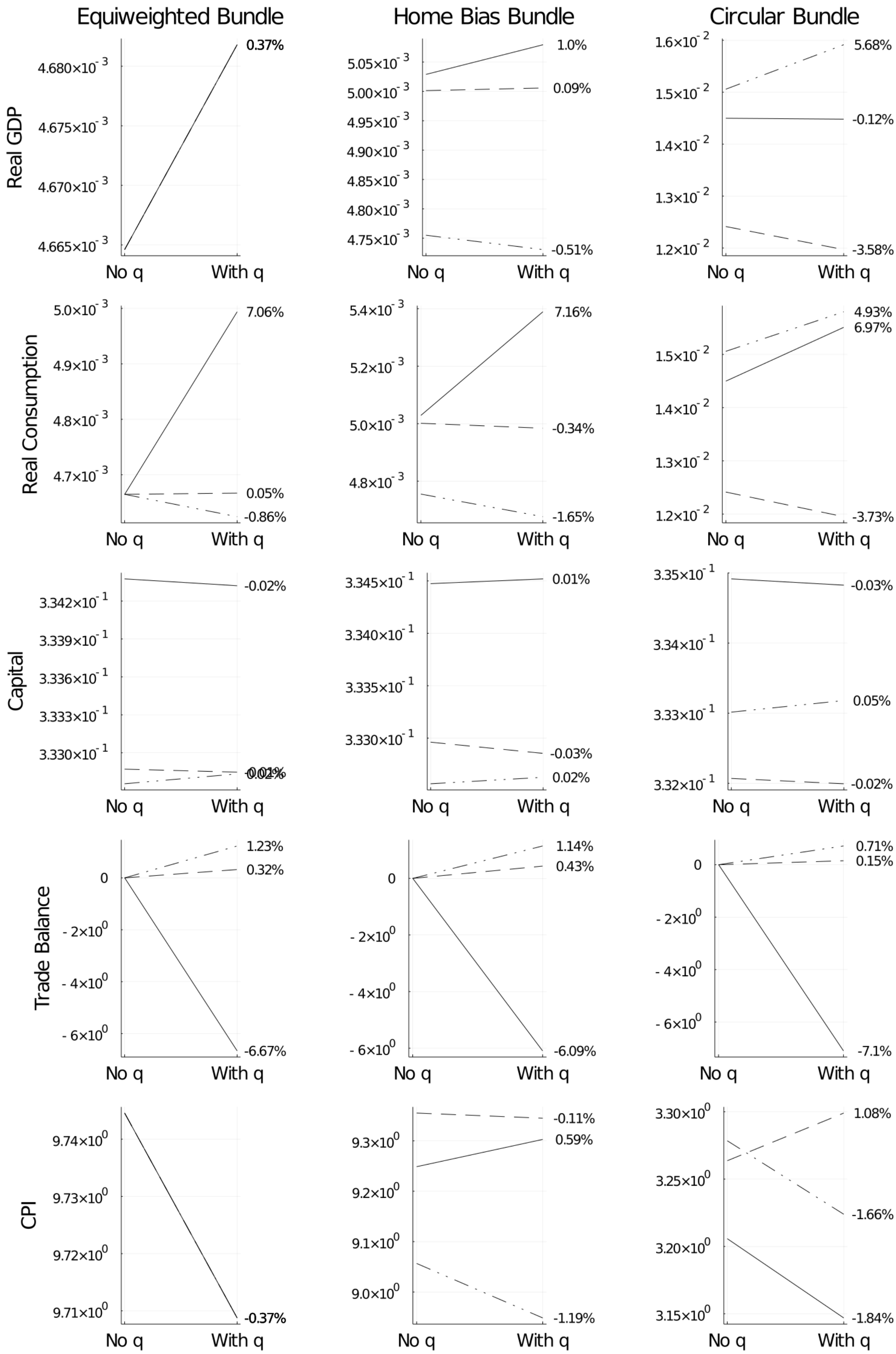
Table 3: Competitive Equilibrium Under Different Bundles and an Equiweighted Network without K

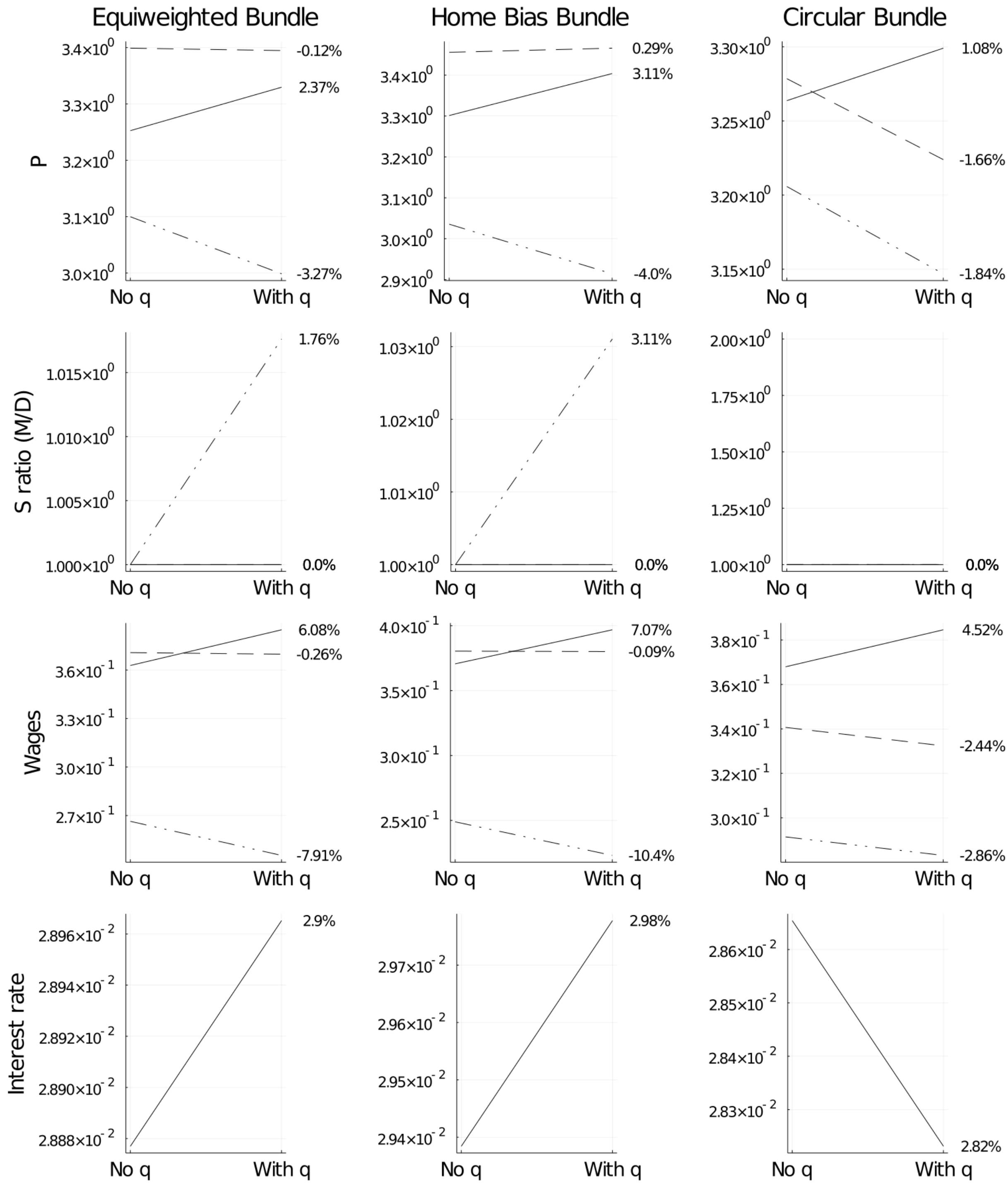


	Equiweighted Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	Q	q	b	Q	q	b	Q
$2 \rightarrow 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \rightarrow 1$	$16.98e^{-3}$	$1.98e^{-4}$	$1.79e^{-1}$	$16.89e^{-3}$	$1.94e^{-4}$	$1.75e^{-1}$	$17.22e^{-3}$	$2.09e^{-4}$	$1.88e^{-1}$
$3 \rightarrow 2$	$1.35e^{-3}$	$-1.87e^{-4}$	$-1.96e^{-1}$	$1.27e^{-3}$	$-1.9e^{-4}$	$-2.0e^{-1}$	$1.59e^{-3}$	$-1.78e^{-4}$	$-1.87e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	8.37%	0.0%	0.0%	8.47%	0.0%	0.0%	8.09%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The equiweighted network has $\omega_{rm} = 1/3\ \forall r, m$. The equiweighted bundle has $\beta_{rm} = 1/3\ \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

Table 4: Competitive Equilibrium Under Different Bundles and an Equiweighted Network with K

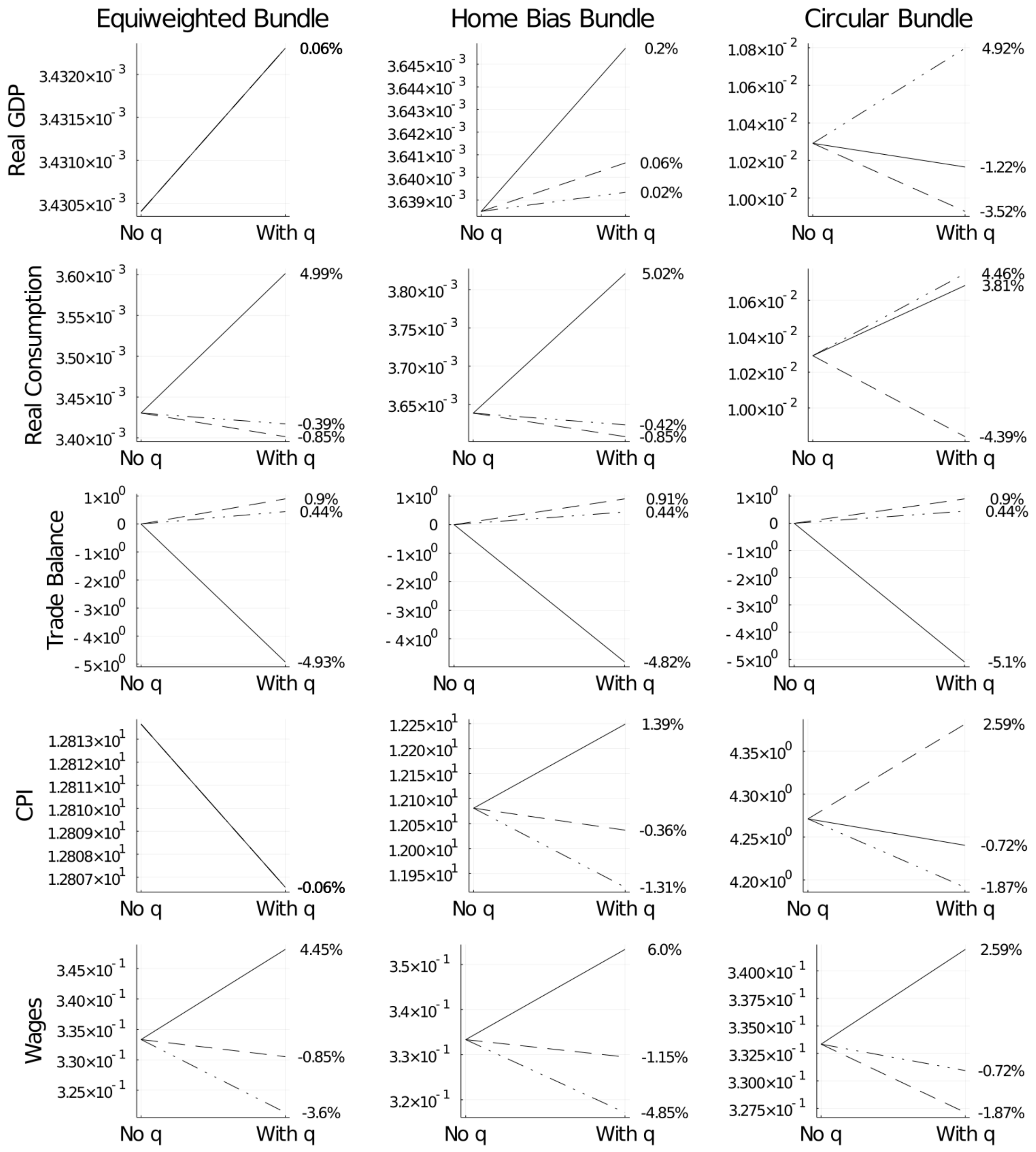




	Equiweighted Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$23.15e^{-3}$	$-102.81e^{-4}$	$4.75e^{-1}$	$22.22e^{-3}$	$-89.84e^{-4}$	$4.27e^{-1}$	$24.36e^{-3}$	$5.6e^{-4}$	$4.72e^{-1}$
$3 \to 2$	$7.52e^{-3}$	$-107.69e^{-4}$	$0.57e^{-1}$	$6.6e^{-3}$	$-94.56e^{-4}$	$0.15e^{-1}$	$8.73e^{-3}$	$1.03e^{-4}$	$0.99e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	1.38%	0.0%	0.0%	2.43%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The equiweighted network has $\omega_{rm} = 1/3\ \forall r, m$. The equiweighted bundle has $\beta_{rm} = 1/3\ \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

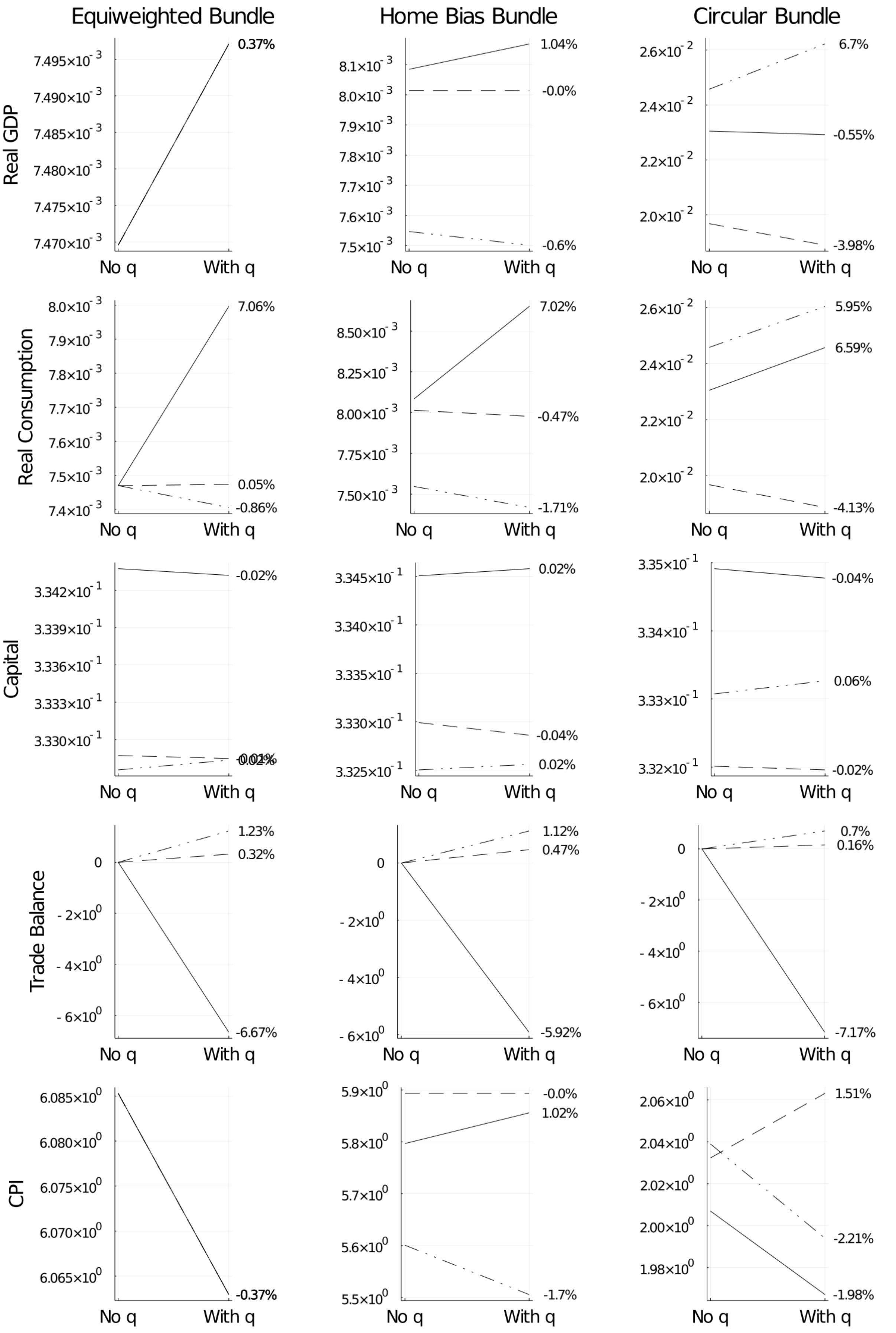
Table 5: Competitive Equilibrium Under Different Bundles and an Autarkic Network without K

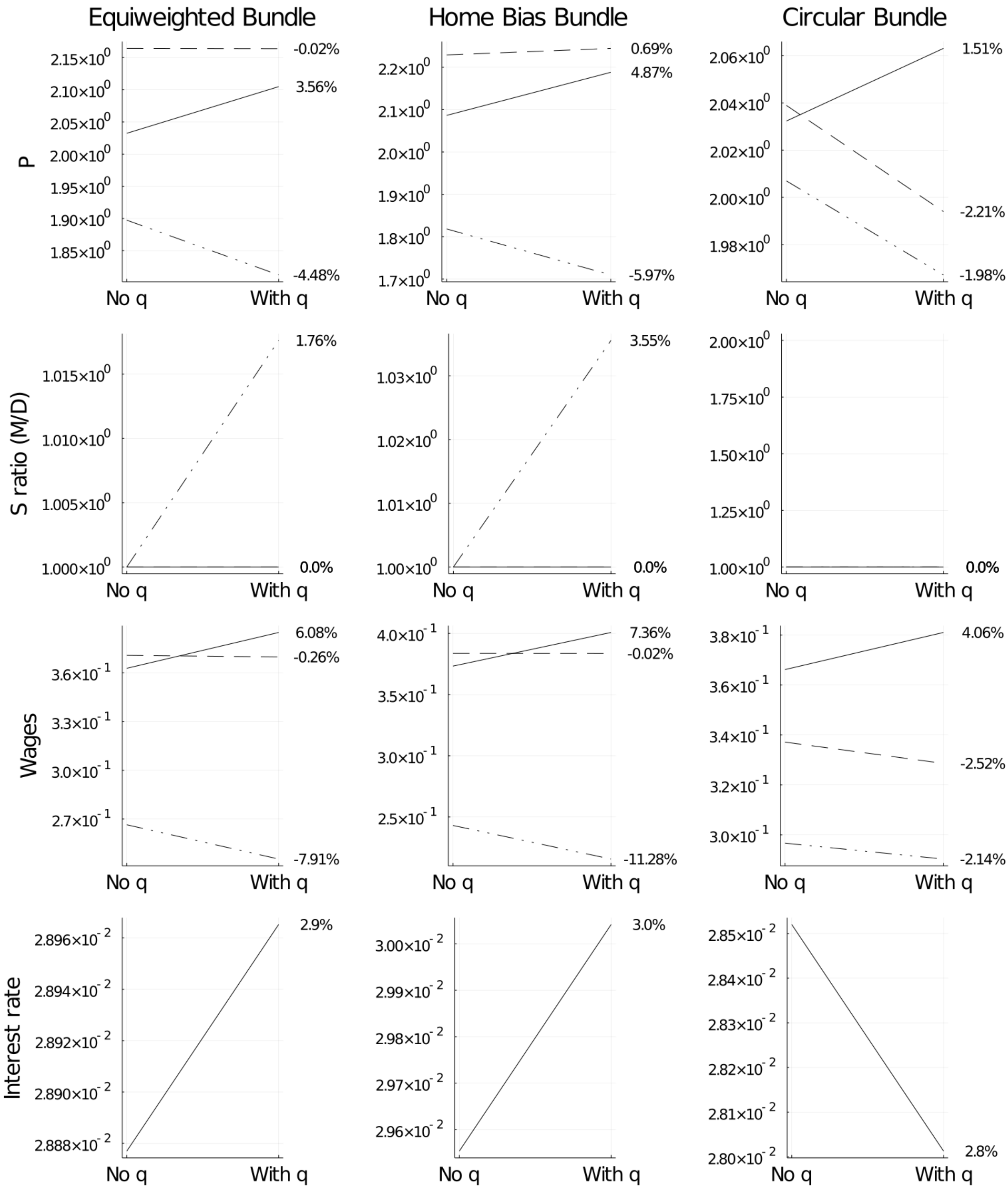


	Autarkic Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	Q	q	b	Q	q	b	Q
$2 \rightarrow 1$	$15.52e^{-3}$	$4.12e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \rightarrow 1$	$17.25e^{-3}$	$1.84e^{-4}$	$1.92e^{-1}$	$16.86e^{-3}$	$1.93e^{-4}$	$1.74e^{-1}$	$17.25e^{-3}$	$2.1e^{-4}$	$1.89e^{-1}$
$3 \rightarrow 2$	$1.62e^{-3}$	$-1.79e^{-4}$	$-1.85e^{-1}$	$1.24e^{-3}$	$-1.91e^{-4}$	$-2.01e^{-1}$	$1.62e^{-3}$	$-1.77e^{-4}$	$-1.85e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	8.05%	0.0%	0.0%	8.5%	0.0%	0.0%	8.06%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The autarkic network has $\omega_{rr} = 1 \; \forall r$. The equiweighted bundle has $\beta_{rm} = 1/3 \; \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

Table 6: Competitive Equilibrium Under Different Bundles and an Autarkic Network with K

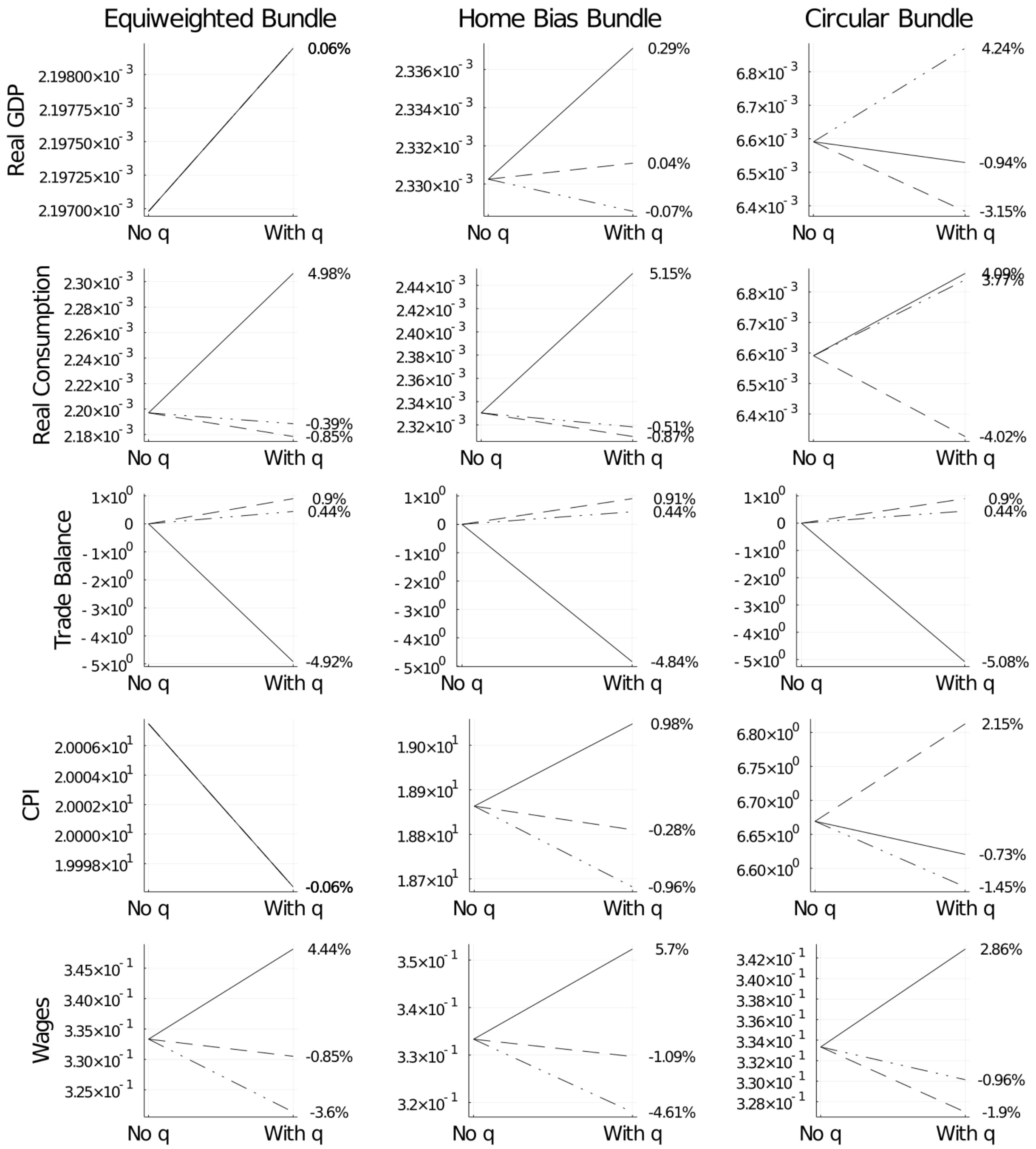




	Autarkic Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	Q	q	b	Q	q	b	Q
$2 \rightarrow 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \rightarrow 1$	$23.15e^{-3}$	$-102.81e^{-4}$	$4.75e^{-1}$	$21.91e^{-3}$	$-85.73e^{-4}$	$4.12e^{-1}$	$24.36e^{-3}$	$5.6e^{-4}$	$4.72e^{-1}$
$3 \rightarrow 2$	$7.52e^{-3}$	$-107.69e^{-4}$	$0.57e^{-1}$	$6.29e^{-3}$	$-90.39e^{-4}$	$0.02e^{-1}$	$8.73e^{-3}$	$1.03e^{-4}$	$0.99e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	1.38%	0.0%	0.0%	2.78%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The autarkic network has $\omega_{rr} = 1\ \forall r$. The equiweighted bundle has $\beta_{rm} = 1/3\ \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

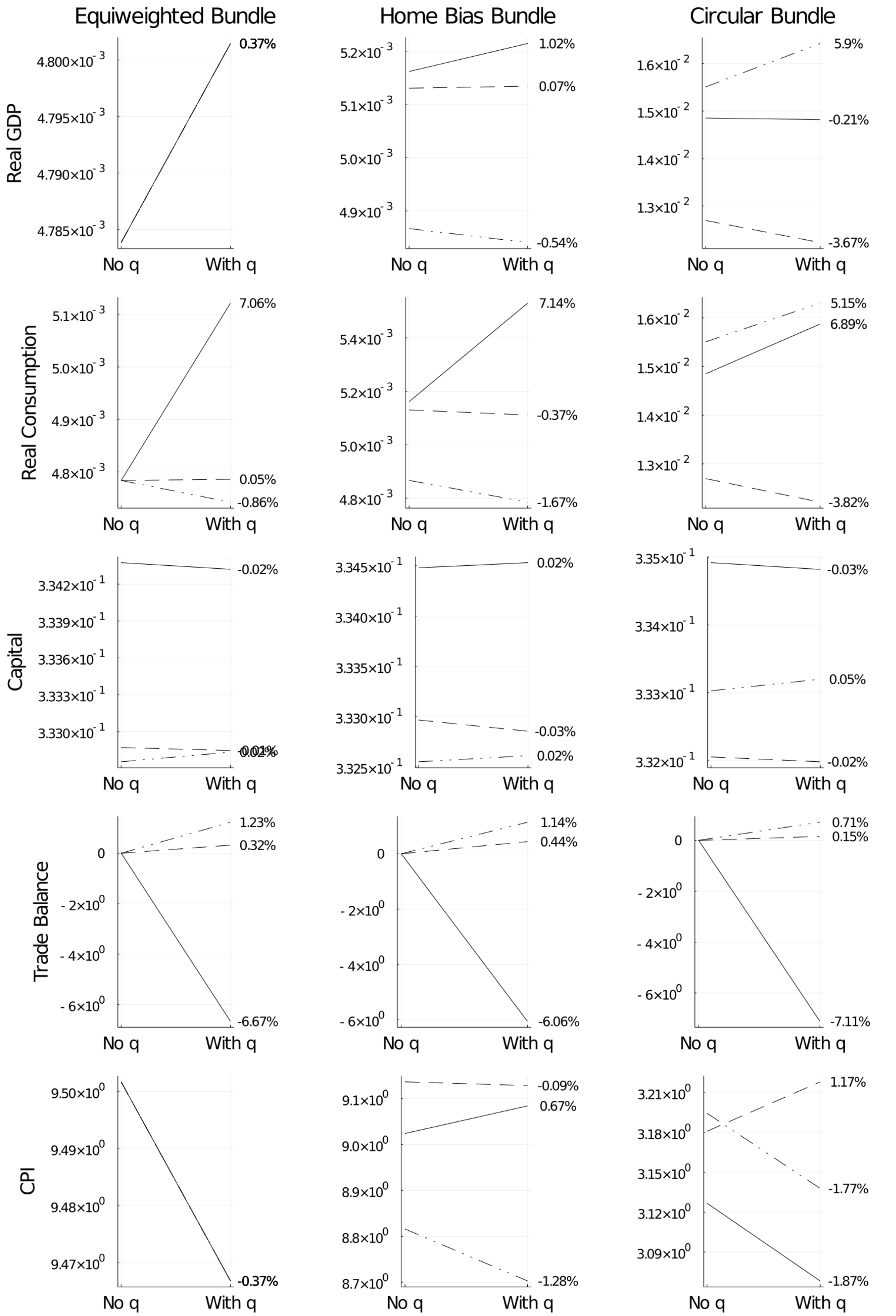
Table 7: Competitive Equilibrium Under Different Bundles and a Home Bias Network without K

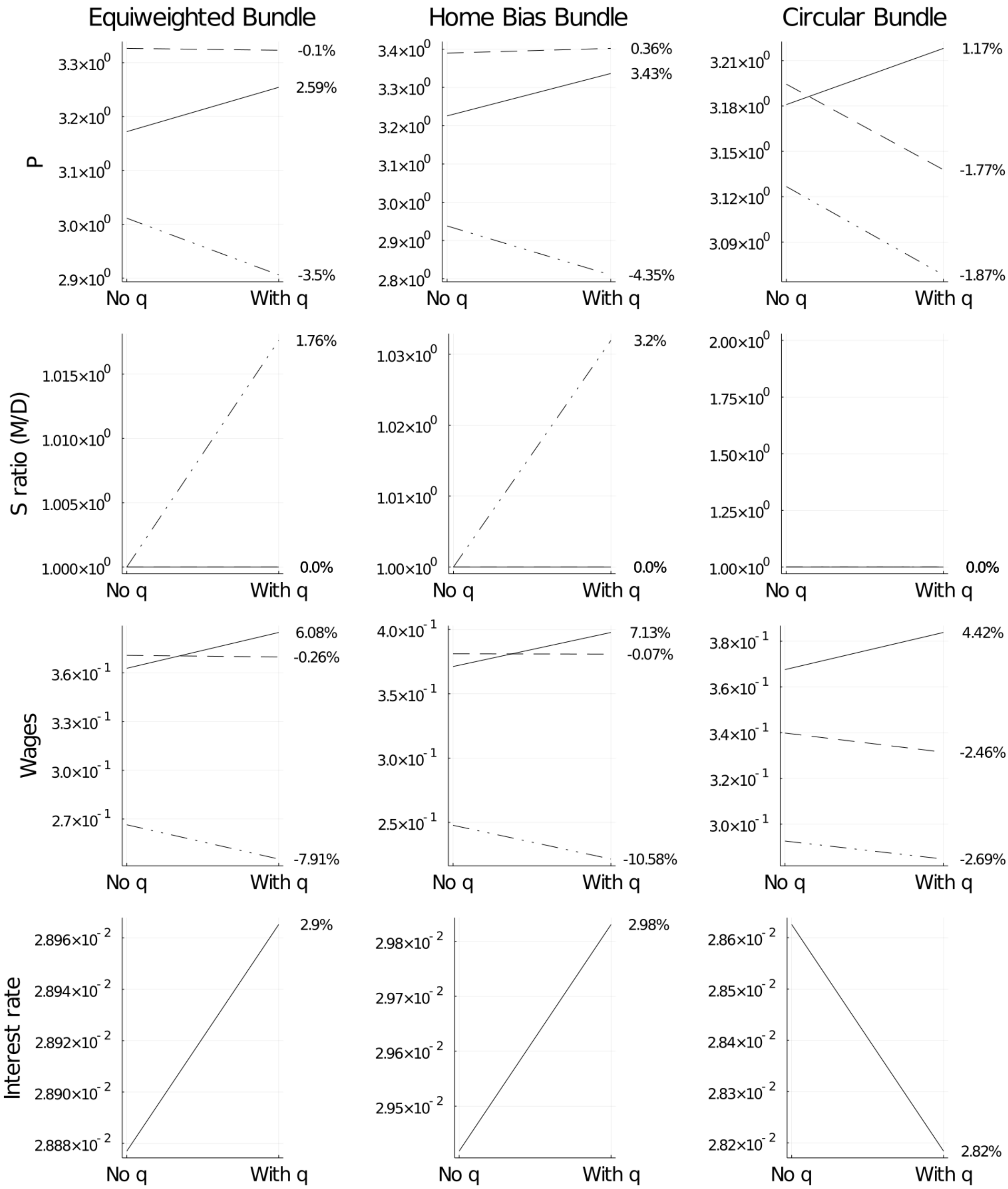


	Home Bias Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.11e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$3.93e^{-4}$	$1.23e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$16.98e^{-3}$	$1.72e^{-4}$	$1.79e^{-1}$	$16.88e^{-3}$	$1.89e^{-4}$	$1.75e^{-1}$	$17.23e^{-3}$	$2.09e^{-4}$	$1.89e^{-1}$
$3 \to 2$	$1.35e^{-3}$	$-1.92e^{-4}$	$-1.96e^{-1}$	$1.26e^{-3}$	$-2.11e^{-4}$	$-1.99e^{-1}$	$1.6e^{-3}$	$-1.78e^{-4}$	$-1.86e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	8.37%	0.0%	0.0%	8.47%	0.0%	0.0%	8.08%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The network with home bias has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4\ \forall r \neq m$. The equiweighted bundle has $\beta_{rm} = 1/3\ \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

Table 8: Competitive Equilibrium Under Different Bundles and a Home Bias Network with K

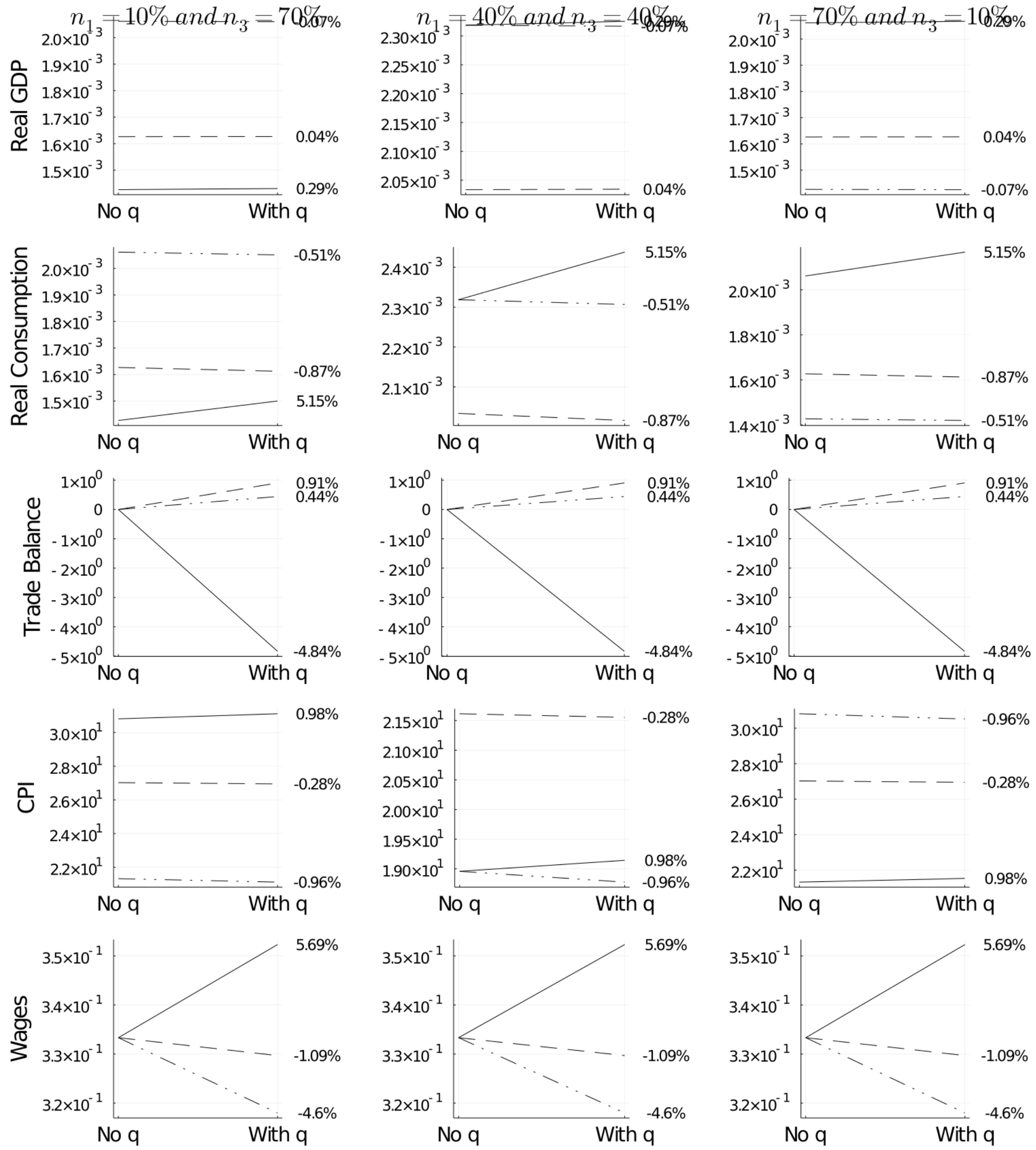




	Home Bias Network								
	Equiweighted Bundle			Home Bias Bundle			Circular Bundle		
	q	b	x Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$23.15e^{-3}$	-$102.81e^{-4}$	$4.75e^{-1}$	$22.16e^{-3}$	-$89.01e^{-4}$	$4.24e^{-1}$	$24.36e^{-3}$	$5.54e^{-4}$	$4.71e^{-1}$
$3 \to 2$	$7.52e^{-3}$	-$107.69e^{-4}$	$0.57e^{-1}$	$6.53e^{-3}$	-$93.71e^{-4}$	$0.13e^{-1}$	$8.73e^{-3}$	$0.83e^{-4}$	$1.0e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	1.38%	0.0%	0.0%	2.5%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The home bias network has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4\ \forall r \neq m$. The equiweighted bundle has $\beta_{rm} = 1/3\ \forall r, m$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$. The circular bundle has $\beta_{13} = 1$, $\beta_{21} = 1$, and $\beta_{32} = 1$.

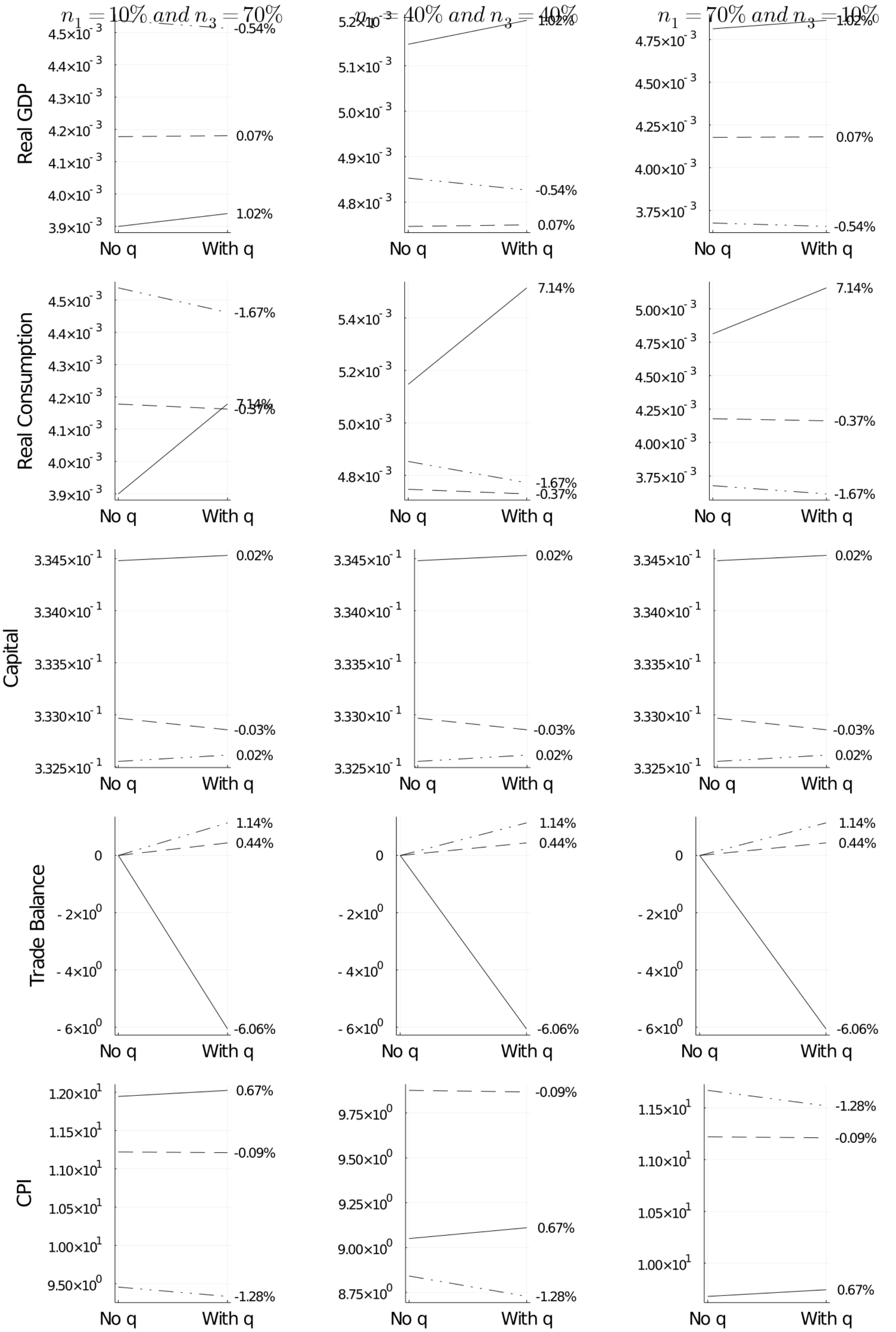
Table 9: Competitive Equilibrium Under Different Population Sizes without K

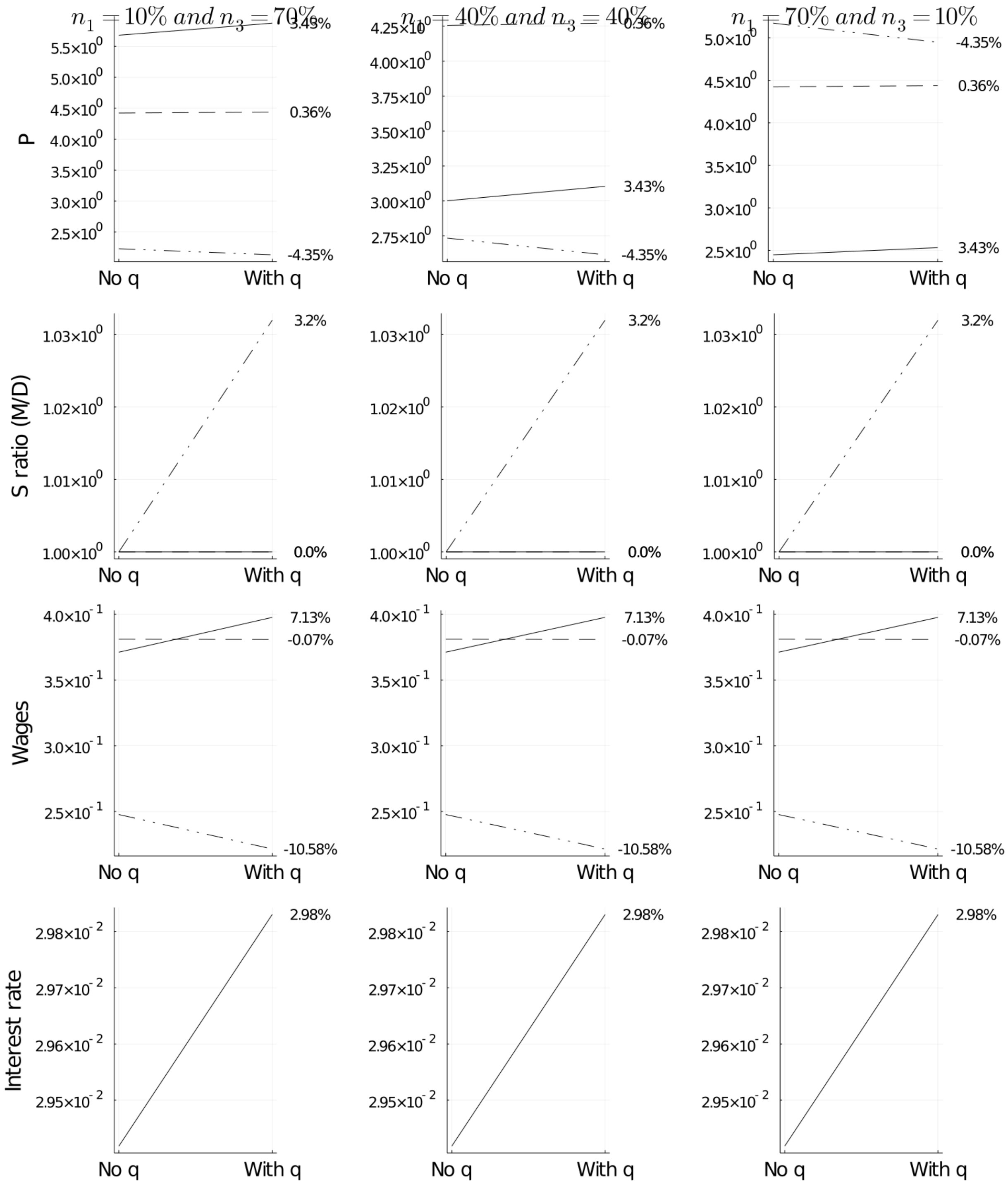


	Home Bias Network and Bundle								
	Small Size Low Tax			Intermediate Size Low Tax			Large Size Low Tax		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.12e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$16.88e^{-3}$	$1.93e^{-4}$	$1.75e^{-1}$	$16.89e^{-3}$	$1.94e^{-4}$	$1.75e^{-1}$	$16.88e^{-3}$	$1.93e^{-4}$	$1.75e^{-1}$
$3 \to 2$	$1.26e^{-3}$	$-1.91e^{-4}$	$-2.0e^{-1}$	$1.26e^{-3}$	$-1.9e^{-4}$	$-2.0e^{-1}$	$1.26e^{-3}$	$-1.94e^{-4}$	$-2.01e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	8.48%	0.0%	0.0%	8.47%	0.0%	0.0%	8.48%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The network with home bias has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 10: Competitive Equilibrium Under Different Population Sizes with K

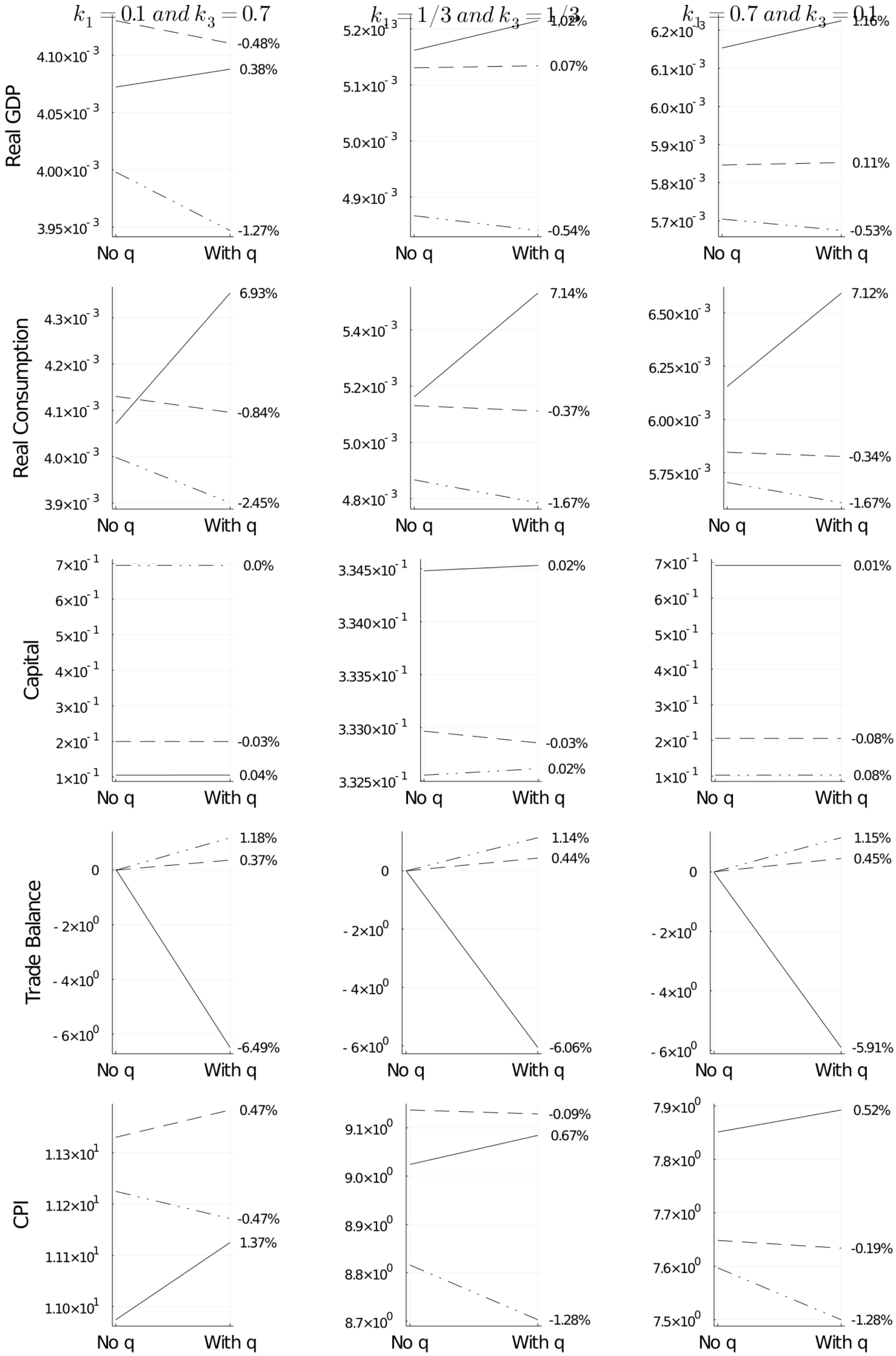


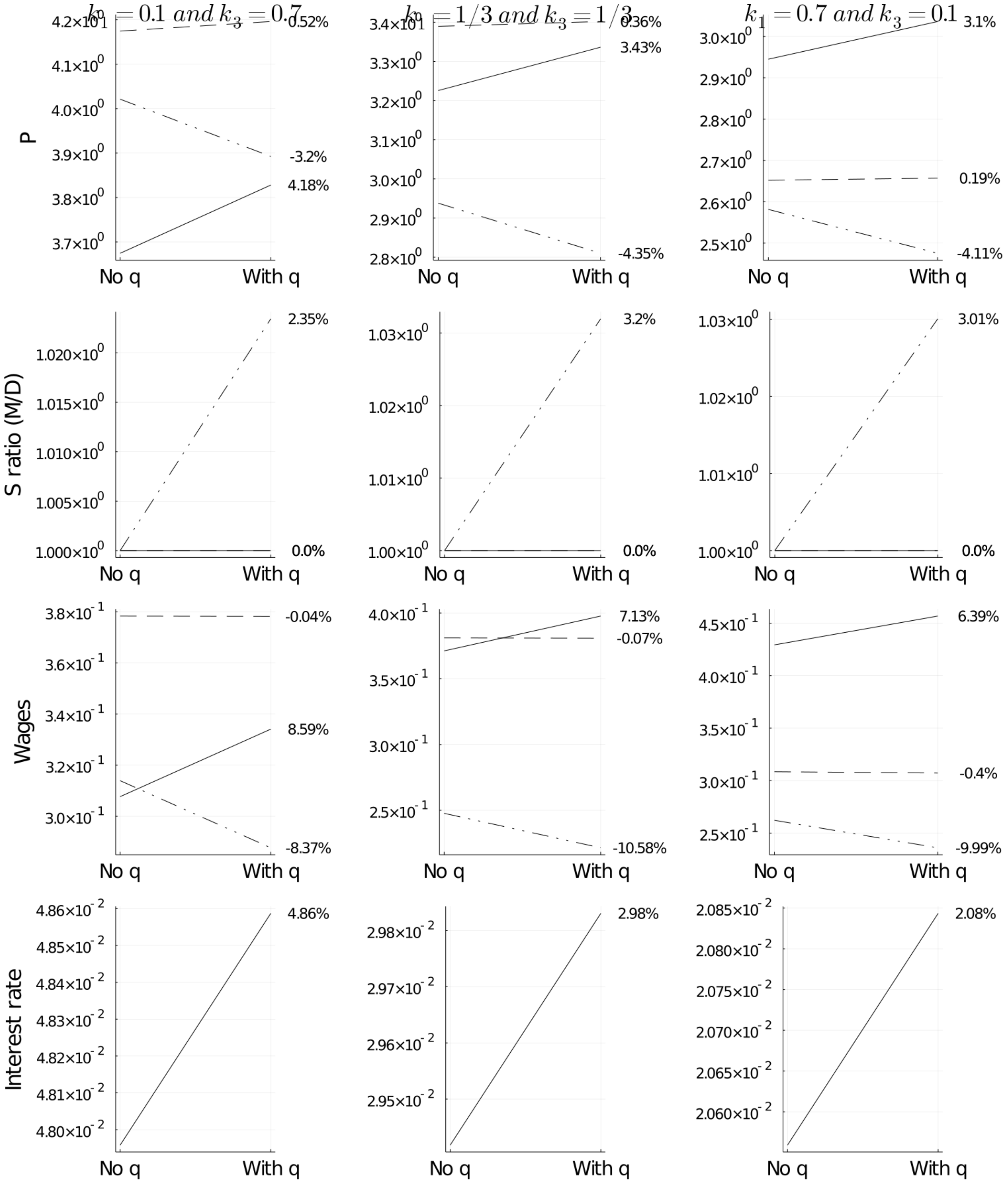


	Home Bias Network and Bundle								
	Small Size Low Tax			Intermediate Size Low Tax			Large Size Low Tax		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$
$3 \to 2$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	2.5%	0.0%	0.0%	2.5%	0.0%	0.0%	2.5%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The home bias network has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 11: Competitive Equilibrium Under Different Capital Allocations

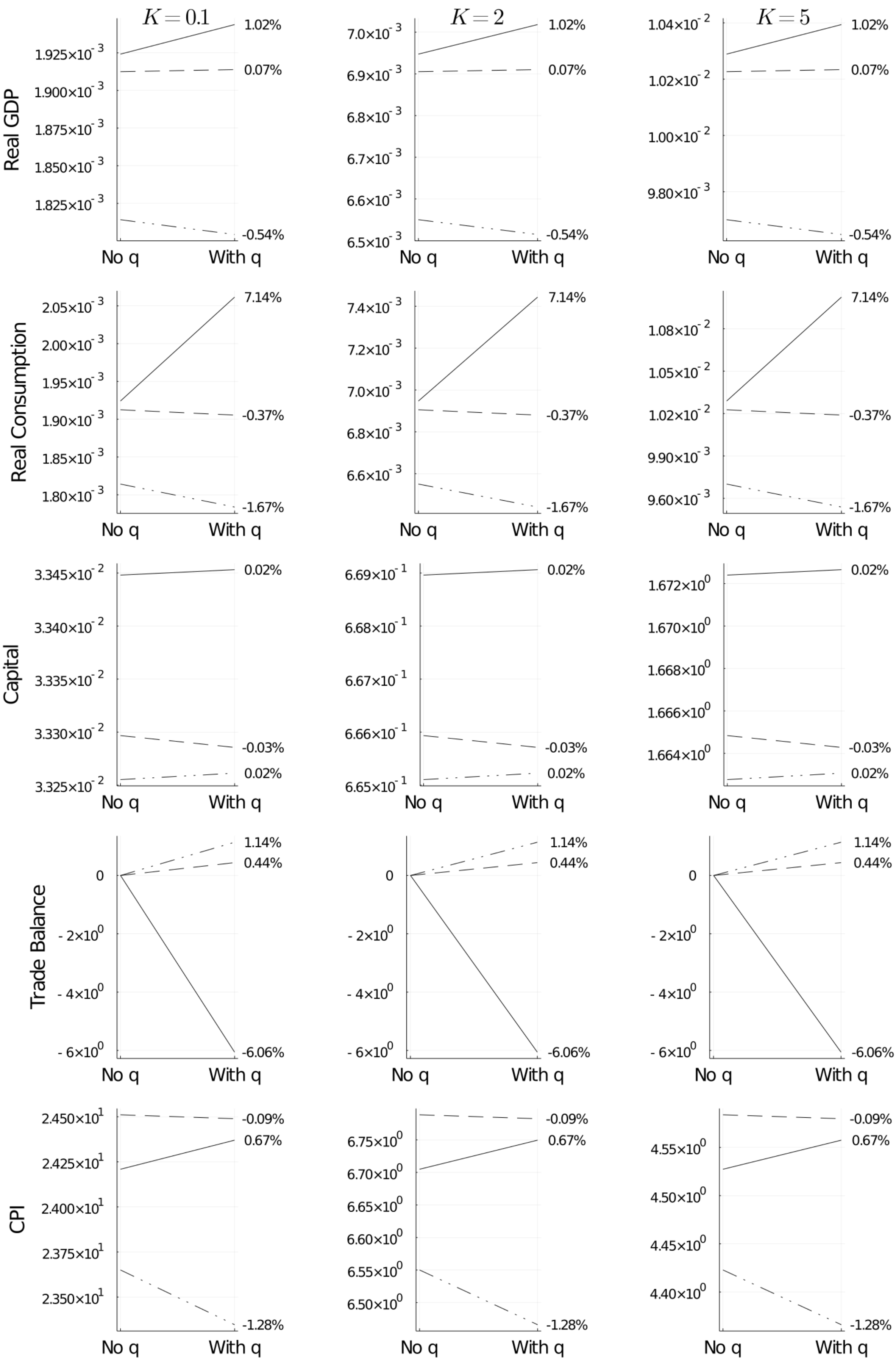


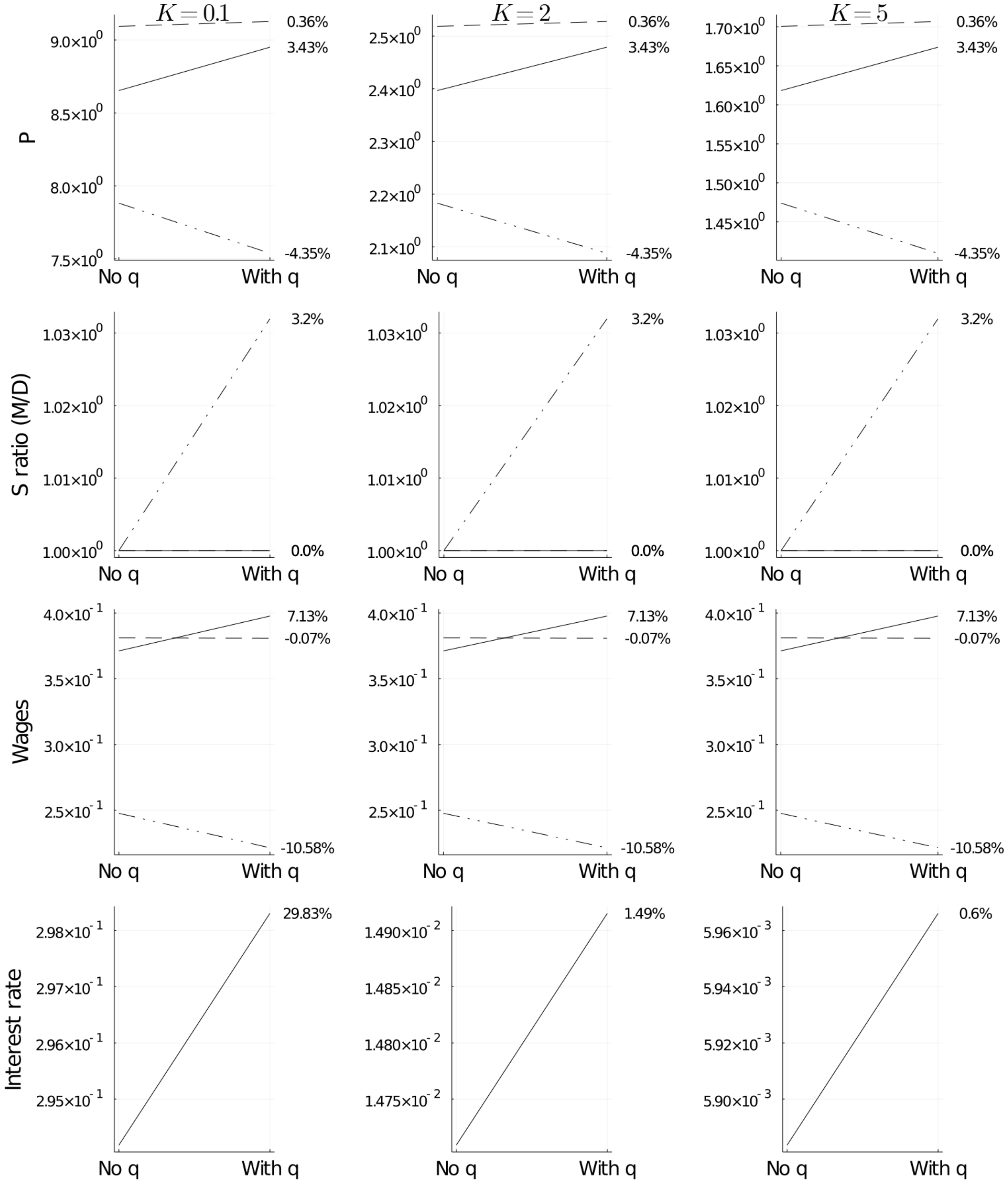


	Home Bias Network and Bundle								
	Low K_1 and High K_3			Intermediate K_1 and K_3			High K_1 and Low K_3		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$22.74e^{-3}$	$-97.07e^{-4}$	$4.54e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$	$22.29e^{-3}$	$-90.75e^{-4}$	$4.31e^{-1}$
$3 \to 2$	$7.12e^{-3}$	$-101.87e^{-4}$	$0.39e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$	$6.66e^{-3}$	$-95.48e^{-4}$	$0.18e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	1.83%	0.0%	0.0%	2.5%	0.0%	0.0%	2.35%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The home bias network has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 12: Competitive Equilibrium Under Different Global Supply of K

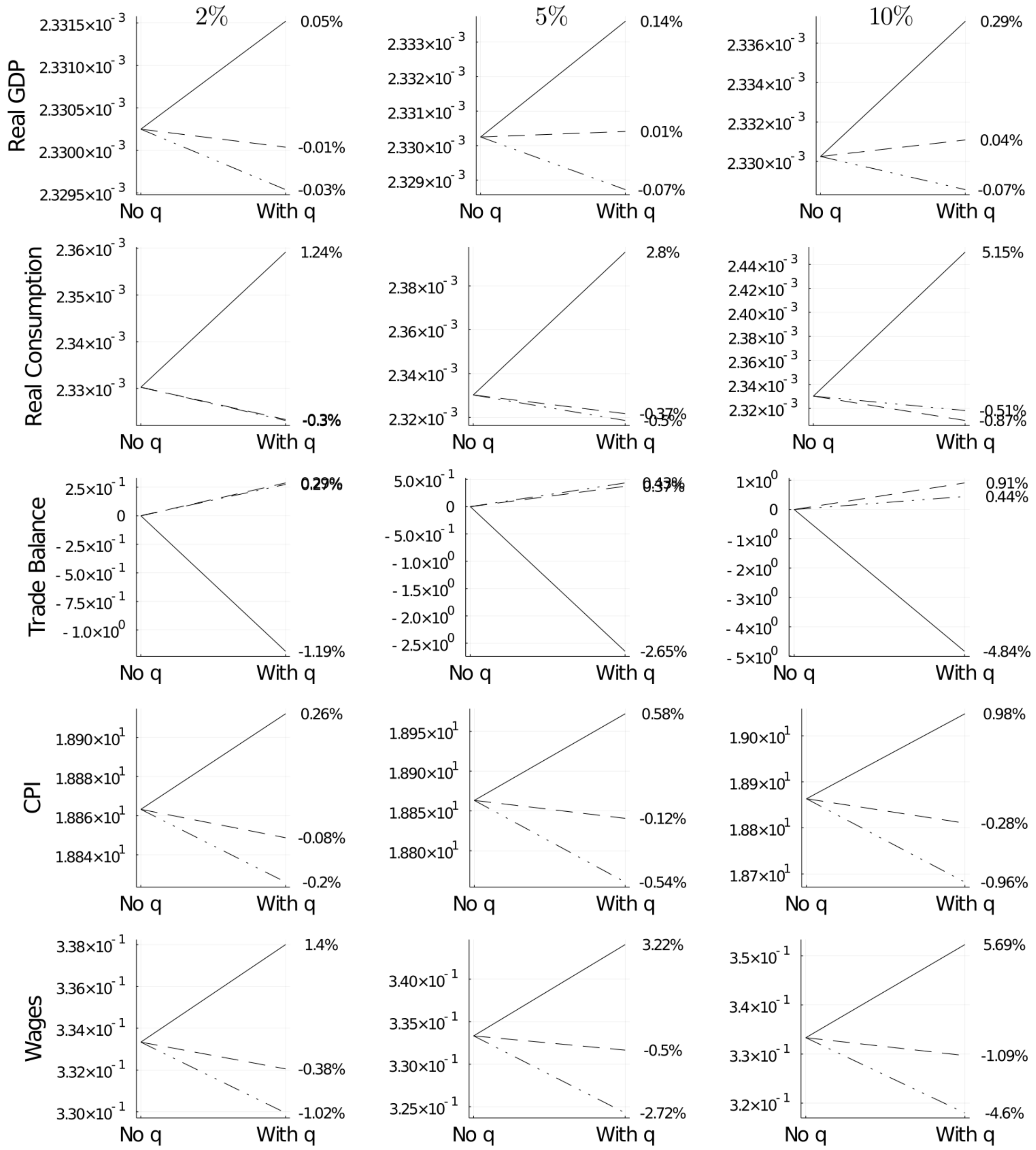




	Home Bias Network and Bundle								
	$\sum_{r=1}^{3} K_r = 0.1$			$\sum_{r=1}^{3} K_r = 2$			$\sum_{r=1}^{3} K_r = 5$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$
$3 \to 2$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	2.5%	0.0%	0.0%	2.5%	0.0%	0.0%	2.5%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. The home bias network has $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. The home bias bundle has $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

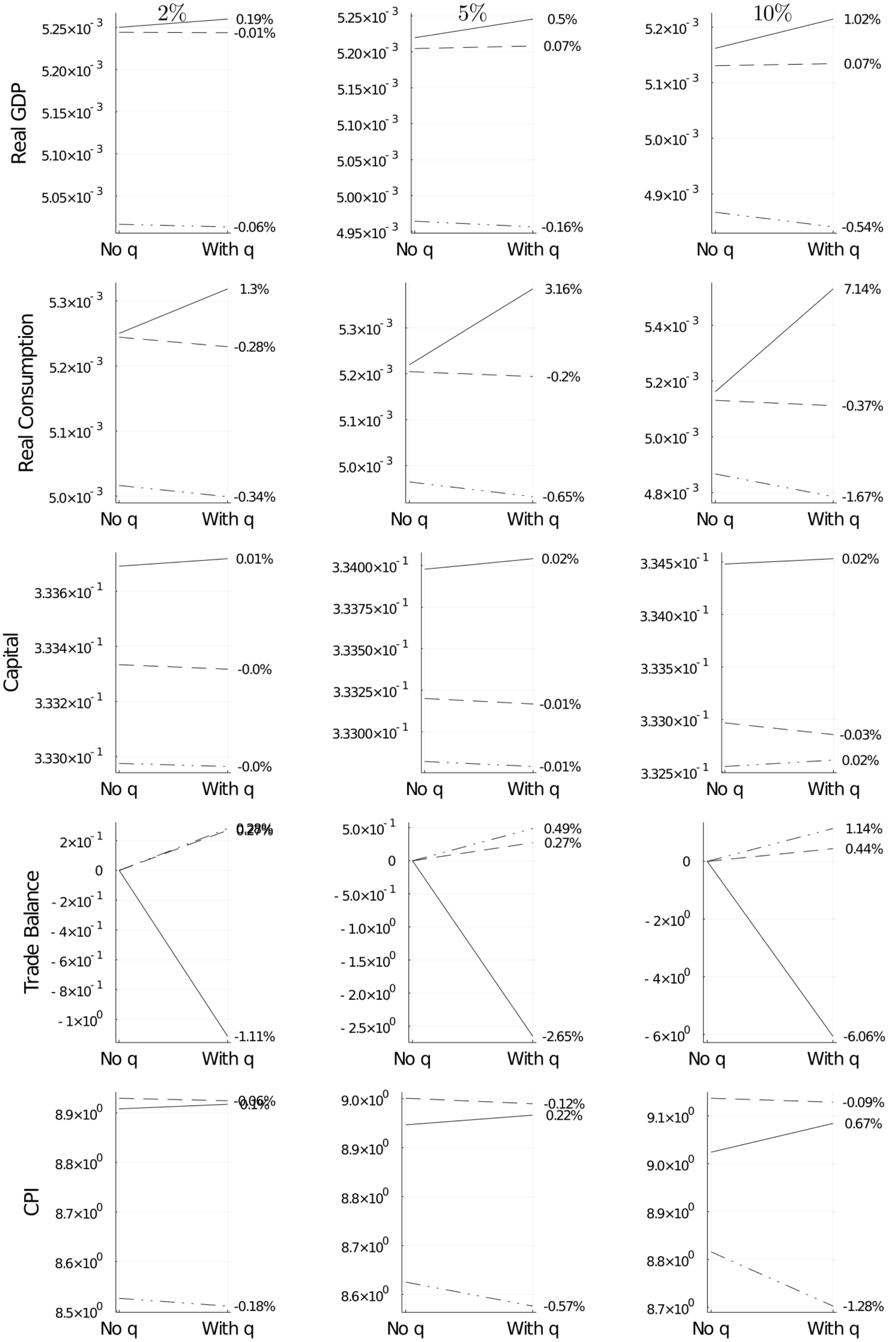
Table 13: Competitive Equilibrium under different tax differentials without K

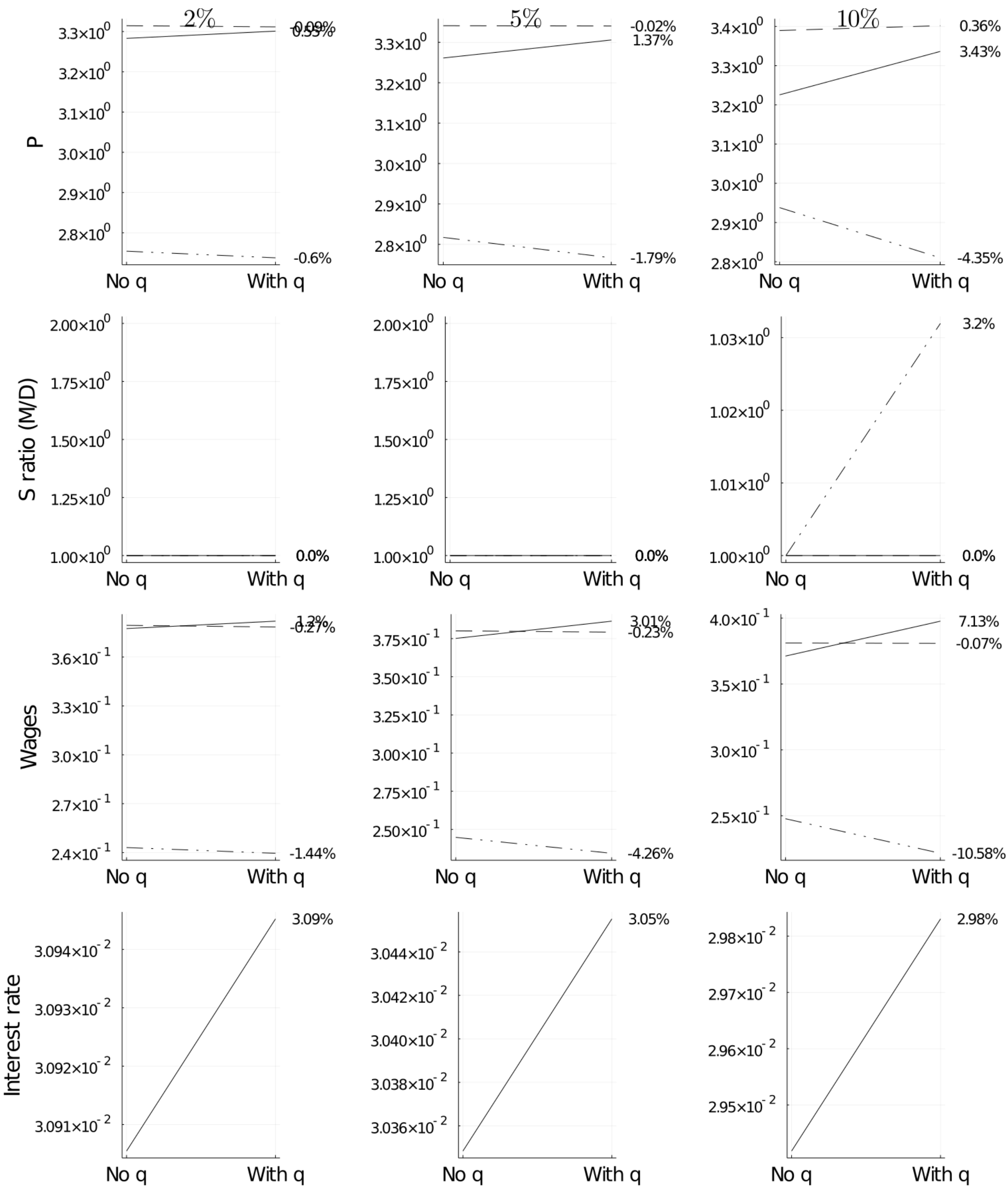


	Home Bias Network and Bundle								
	Tax gap of 2%			Tax Gap of 5%			Tax Gap of 10%		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$10.34e^{-3}$	$-2.84e^{-4}$	$-0.86e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$13.25e^{-3}$	$1.02e^{-4}$	$0.3e^{-1}$
$3 \to 1$	$13.8e^{-3}$	$0.56e^{-4}$	$0.52e^{-1}$	$16.88e^{-3}$	$1.76e^{-4}$	$1.76e^{-1}$	$23.07e^{-3}$	$4.97e^{-4}$	$4.21e^{-1}$
$3 \to 2$	$4.43e^{-3}$	$-0.72e^{-4}$	$-0.73e^{-1}$	$1.26e^{-3}$	$-1.91e^{-4}$	$-2.0e^{-1}$	$-5.05e^{-3}$	$-3.97e^{-4}$	$-4.54e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	0.93%	0.0%	0.0%	8.47%	0.0%	12.2%	23.54%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 14: Competitive Equilibrium under different tax differentials with K

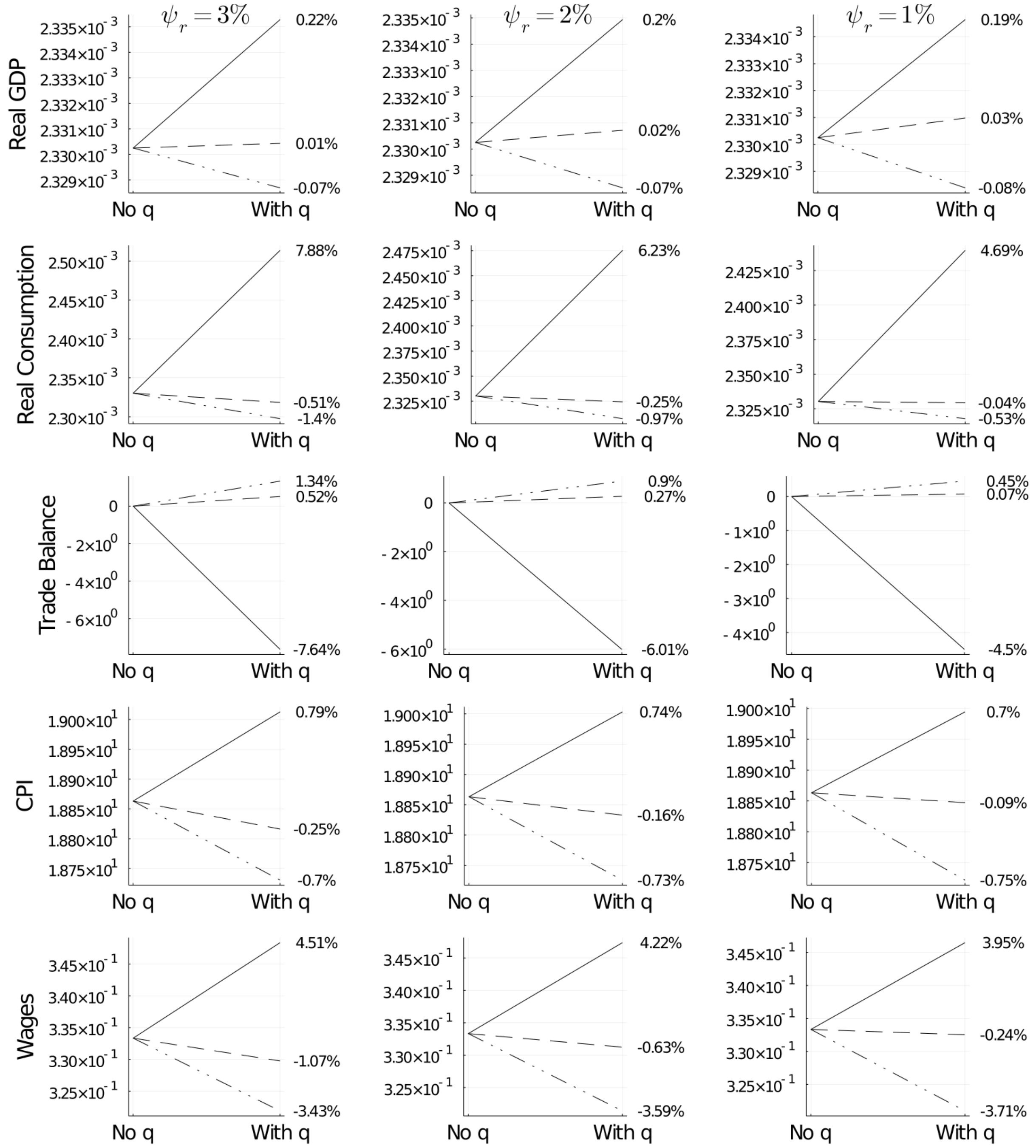




	Home Bias Network and Bundle								
	Tax gap of 2%			Tax Gap of 5%			Tax Gap of 10%		
	q	b	Q	q	b	Q	q	b	Q
$2 \rightarrow 1$	$10.34e^{-3}$	-$2.84e^{-4}$	-$0.86e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$	$25.86e^{-3}$	$19.66e^{-4}$	$5.24e^{-1}$
$3 \rightarrow 1$	$14.61e^{-3}$	$0.91e^{-4}$	$0.84e^{-1}$	$22.16e^{-3}$	-$89.01e^{-4}$	$4.24e^{-1}$	$30.21e^{-3}$	-$67.84e^{-4}$	$7.6e^{-1}$
$3 \rightarrow 2$	$5.24e^{-3}$	-$0.4e^{-4}$	-$0.4e^{-1}$	$6.53e^{-3}$	-$93.71e^{-4}$	$0.13e^{-1}$	$2.09e^{-3}$	-$78.91e^{-4}$	-$1.81e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.0%	0.0%	15.46%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2 \; \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

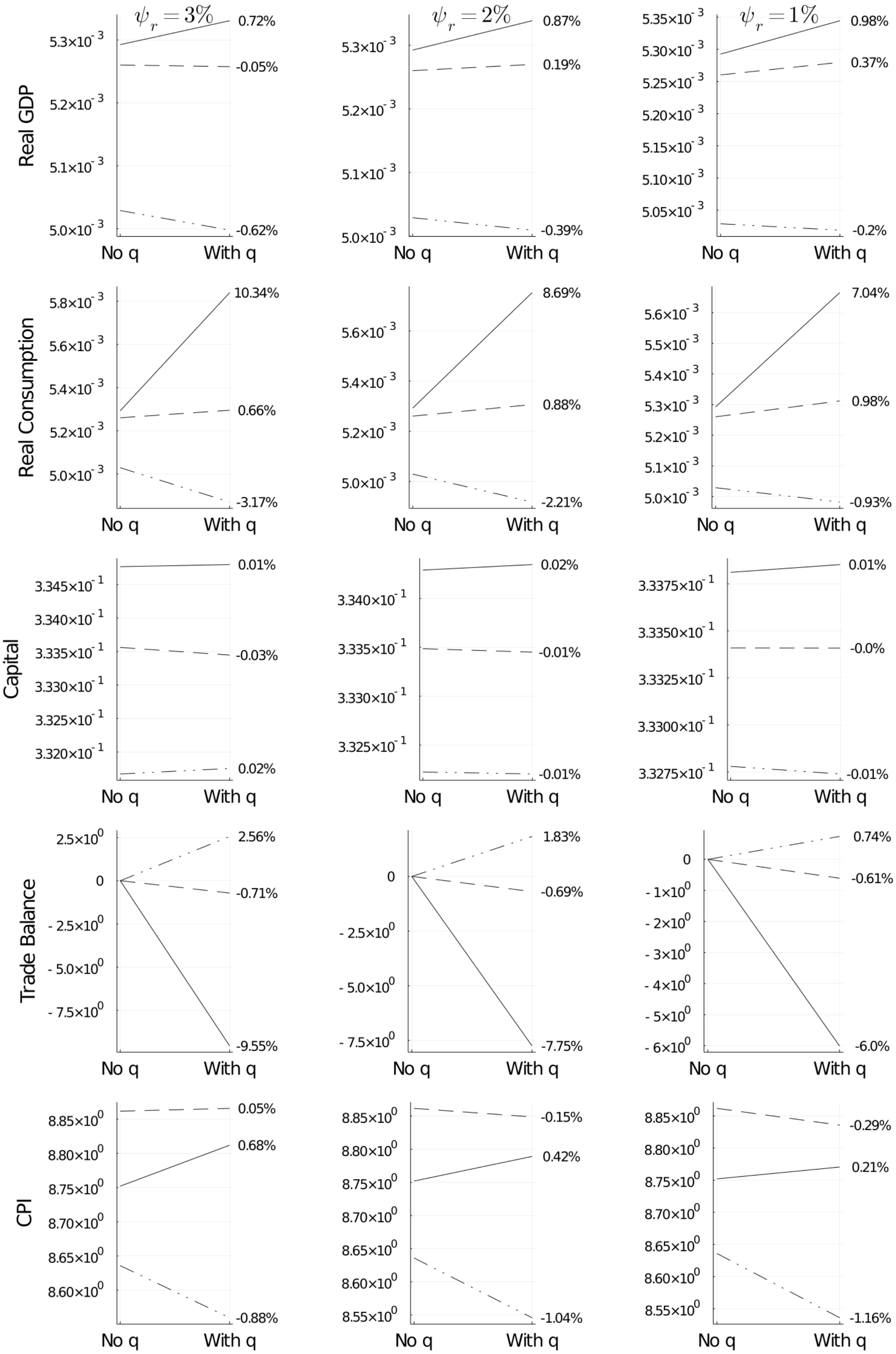
Table 15: Competitive Equilibrium under different ψ without K

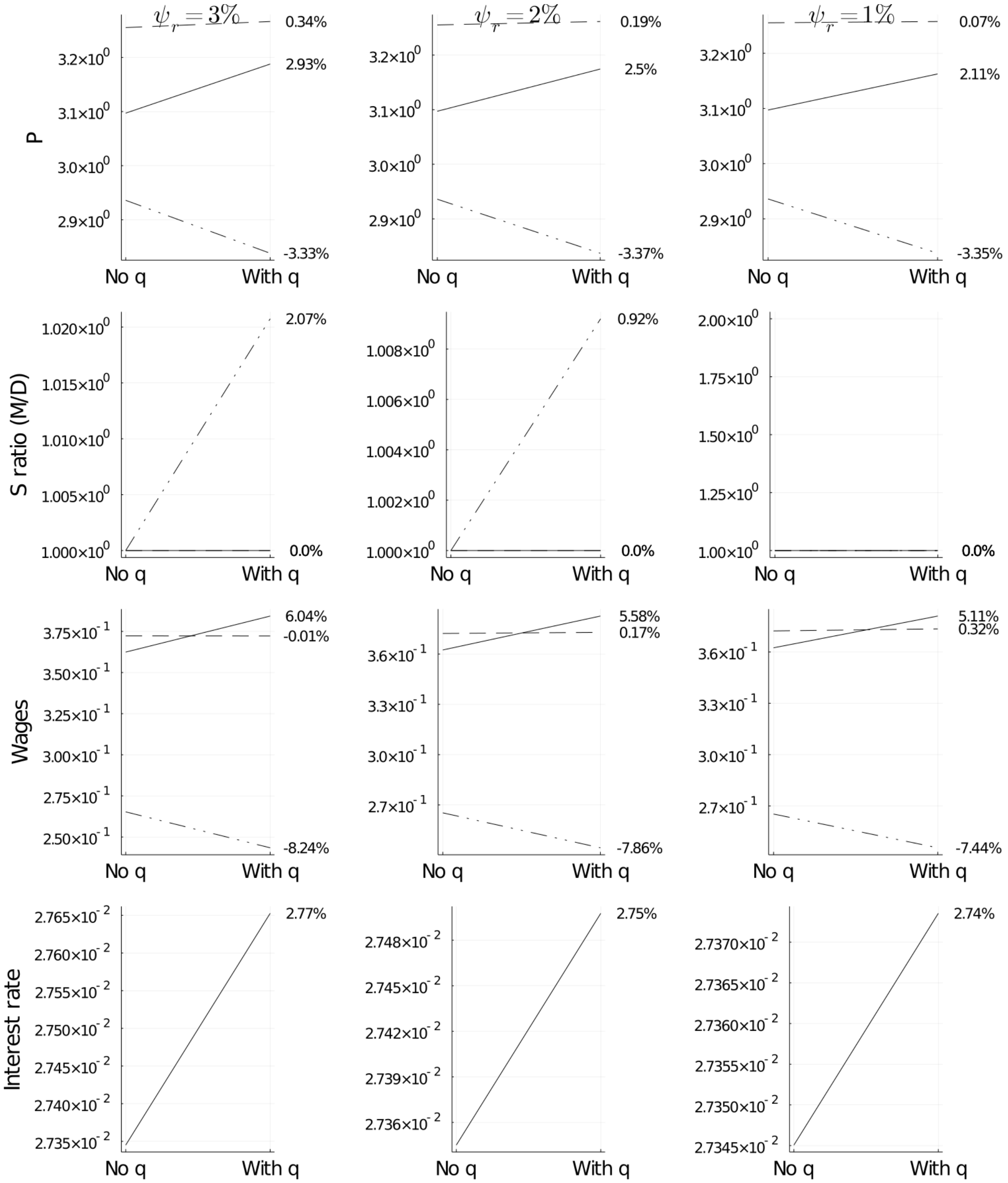


	Home Bias Network and Bundle								
	$\psi = 3\%$			$\psi = 2\%$			$\psi = 1\%$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$13.45e^{-3}$	$9.44e^{-4}$	$2.39e^{-1}$	$12.41e^{-3}$	$8.01e^{-4}$	$2.95e^{-1}$	$11.38e^{-3}$	$4.86e^{-4}$	$3.53e^{-1}$
$3 \to 1$	$15.34e^{-3}$	$10.34e^{-4}$	$3.1e^{-1}$	$15.34e^{-3}$	$9.15e^{-4}$	$4.11e^{-1}$	$15.34e^{-3}$	$5.78e^{-4}$	$5.11e^{-1}$
$3 \to 2$	$2.84e^{-3}$	$-5.68e^{-4}$	$-1.84e^{-1}$	$2.84e^{-3}$	$-1.78e^{-4}$	$-0.86e^{-1}$	$2.84e^{-3}$	$0.13e^{-4}$	$0.14e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	7.69%	0.0%	0.0%	6.69%	0.0%	0.0%	5.7%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, and $\Upsilon = 0$. We assume a network with home bias with $\omega_{rr} = 1/2$ $\forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 16: Competitive Equilibrium under different ψ with K

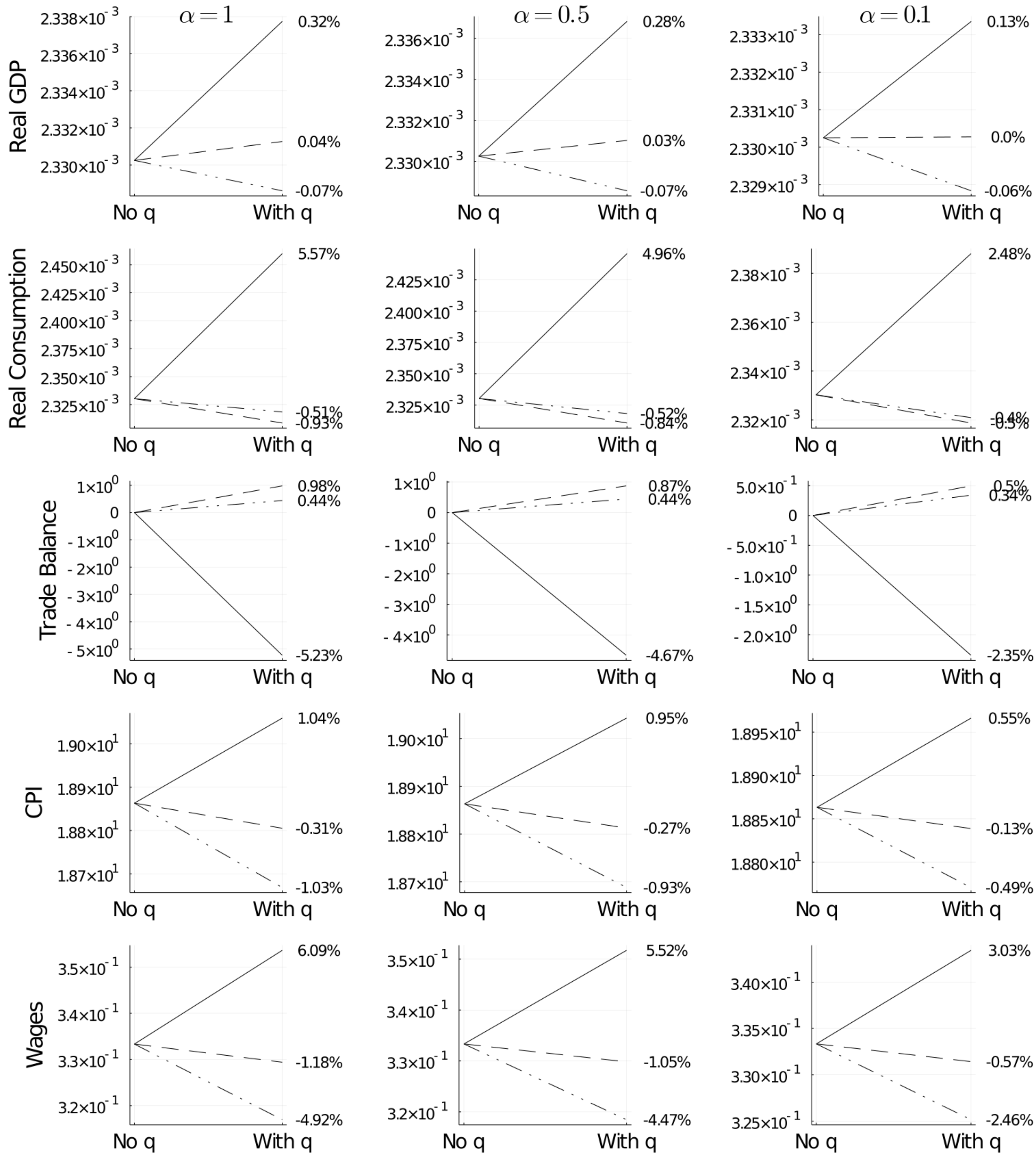




	Home Bias Network and Bundle								
	$\psi = 3\%$			$\psi = 2\%$			$\psi = 1\%$		
	q	b	Q	q	b	Q	q	b	Q
$2 \rightarrow 1$	$13.45e^{-3}$	$9.72e^{-4}$	$2.36e^{-1}$	$12.41e^{-3}$	$8.09e^{-4}$	$2.94e^{-1}$	$11.38e^{-3}$	$4.77e^{-4}$	$3.5e^{-1}$
$3 \rightarrow 1$	$20.71e^{-3}$	$-74.91e^{-4}$	$5.71e^{-1}$	$20.62e^{-3}$	$-79.24e^{-4}$	$6.78e^{-1}$	$20.37e^{-3}$	$8.42e^{-4}$	$7.09e^{-1}$
$3 \rightarrow 2$	$8.21e^{-3}$	$-93.62e^{-4}$	$0.31e^{-1}$	$8.12e^{-3}$	$-92.15e^{-4}$	$1.37e^{-1}$	$7.87e^{-3}$	$2.15e^{-4}$	$2.14e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	1.62%	0.0%	0.0%	0.73%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, and $k_r = 1/3$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

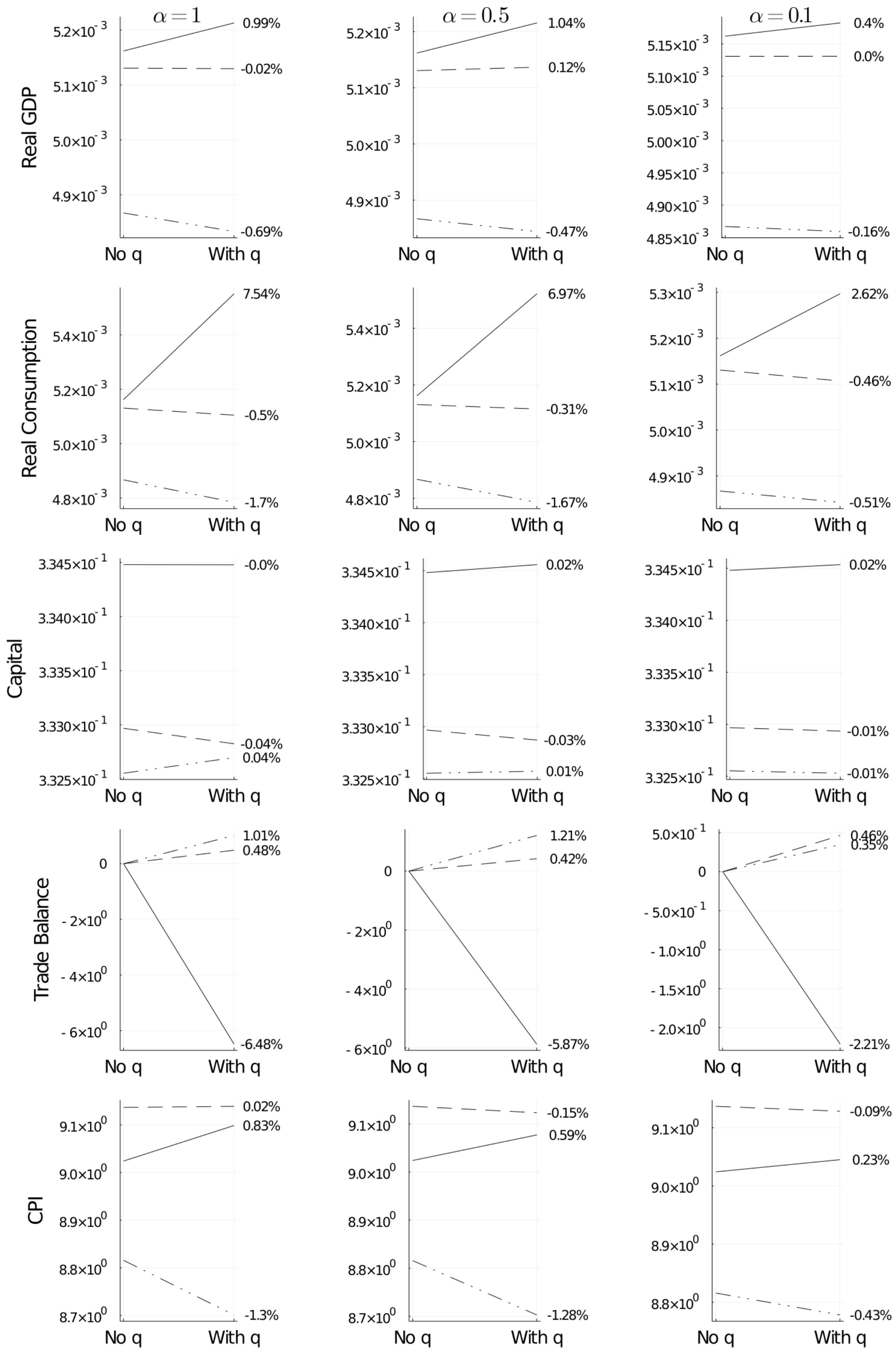
Table 17: Competitive Equilibrium under different α without K

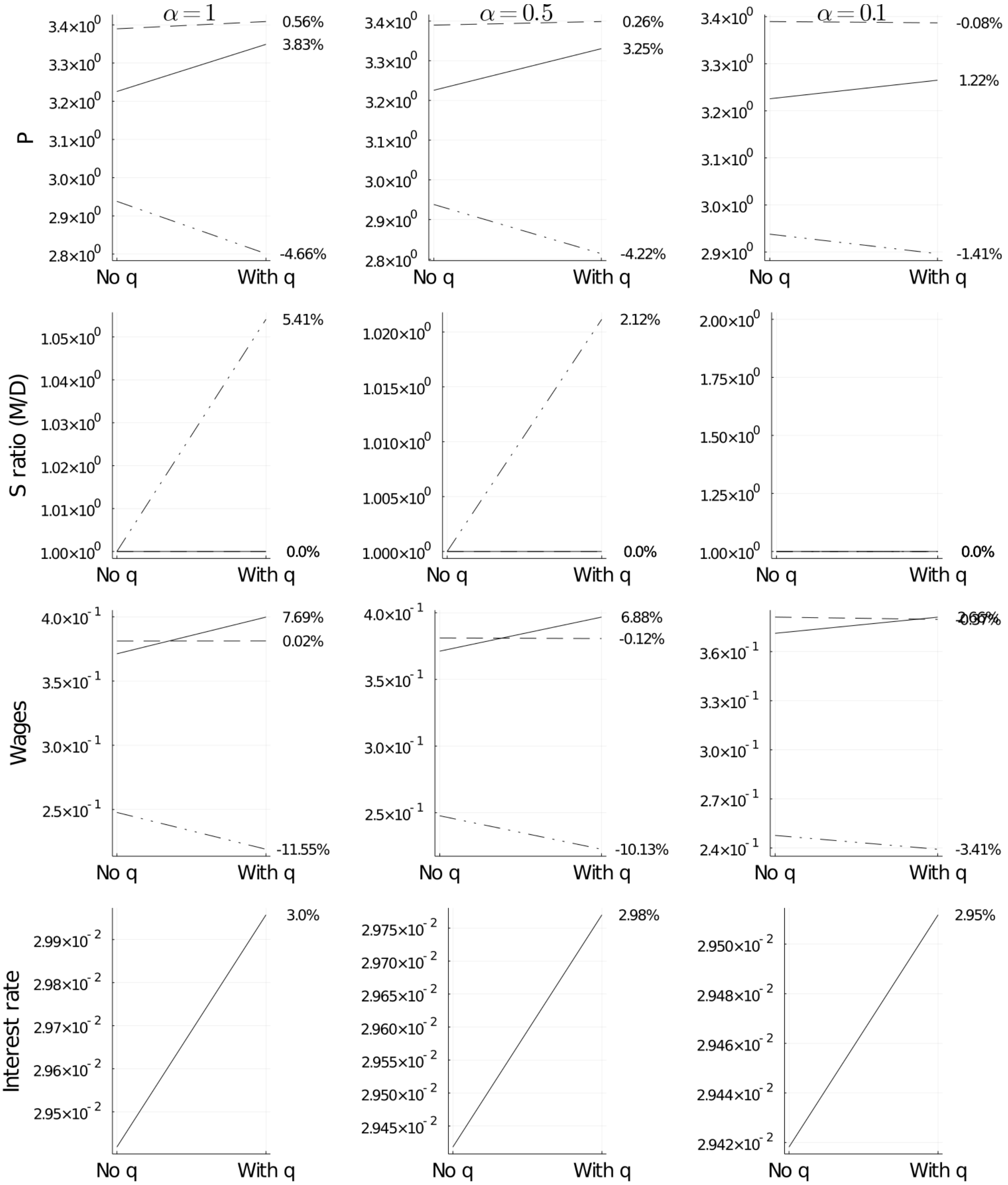


	Home Bias Network and Bundle								
	$\alpha = 1$			$\alpha = 0.5$			$\alpha = 0.1$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$16.67e^{-3}$	$5.54e^{-4}$	$1.67e^{-1}$	$15.0e^{-3}$	$3.37e^{-4}$	$0.99e^{-1}$	$8.33e^{-3}$	$-5.76e^{-4}$	$-1.68e^{-1}$
$3 \to 1$	$16.89e^{-3}$	$2.04e^{-4}$	$1.75e^{-1}$	$16.89e^{-3}$	$1.88e^{-4}$	$1.75e^{-1}$	$14.51e^{-3}$	$0.74e^{-4}$	$0.8e^{-1}$
$3 \to 2$	$1.26e^{-3}$	$-2.46e^{-4}$	$-1.99e^{-1}$	$1.26e^{-3}$	$-2.06e^{-4}$	$-1.99e^{-1}$	$-1.12e^{-3}$	$-2.21e^{-4}$	$-2.95e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	9.68%	0.0%	0.0%	7.86%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 18: Competitive Equilibrium under different α with K

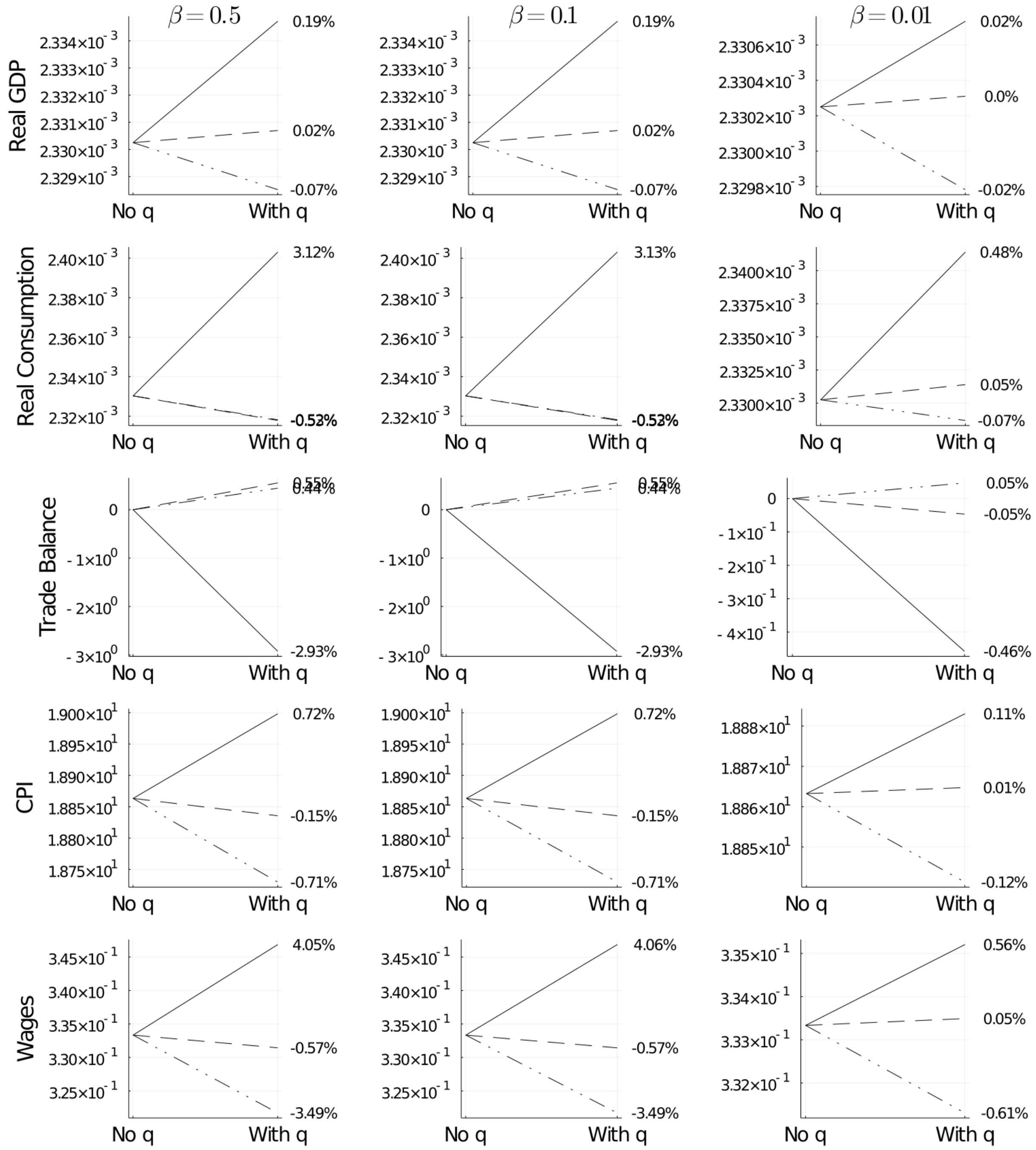




	Home Bias Network and Bundle								
	$\alpha = 1$			$\alpha = 0.5$			$\alpha = 0.1$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$16.67e^{-3}$	$5.7e^{-4}$	$1.66e^{-1}$	$15.0e^{-3}$	$3.43e^{-4}$	$1.0e^{-1}$	$8.33e^{-3}$	-$5.76e^{-4}$	-$1.68e^{-1}$
$3 \to 1$	$22.34e^{-3}$	-$72.32e^{-4}$	$4.24e^{-1}$	$22.06e^{-3}$	-$97.22e^{-4}$	$4.24e^{-1}$	$14.51e^{-3}$	$0.74e^{-4}$	$0.8e^{-1}$
$3 \to 2$	$6.72e^{-3}$	-$77.19e^{-4}$	$0.2e^{-1}$	$6.44e^{-3}$	-$101.86e^{-4}$	$0.08e^{-1}$	-$1.12e^{-3}$	-$2.21e^{-4}$	-$2.95e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	4.23%	0.0%	0.0%	1.65%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

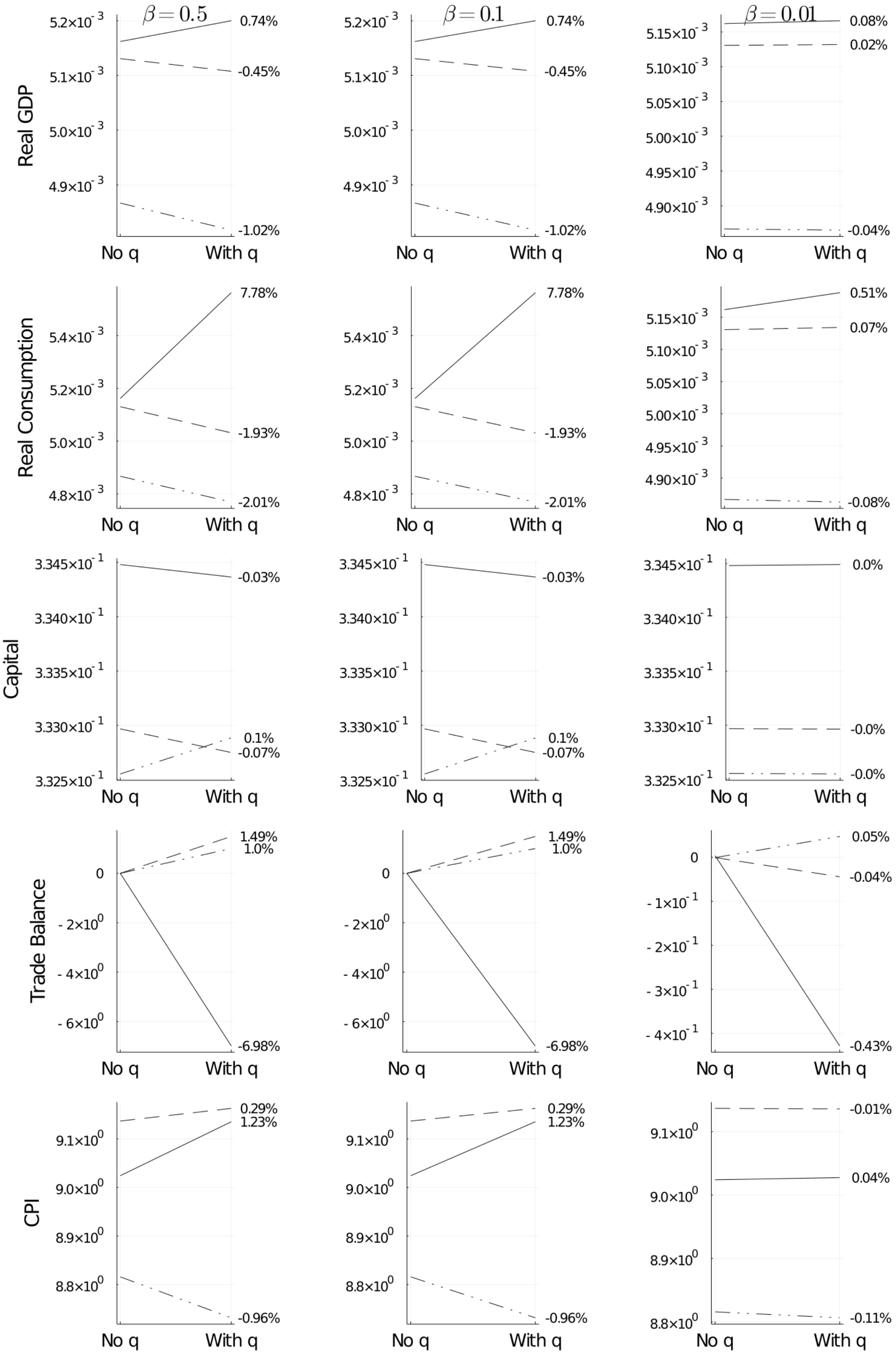
Table 19: Competitive Equilibrium under different β without K

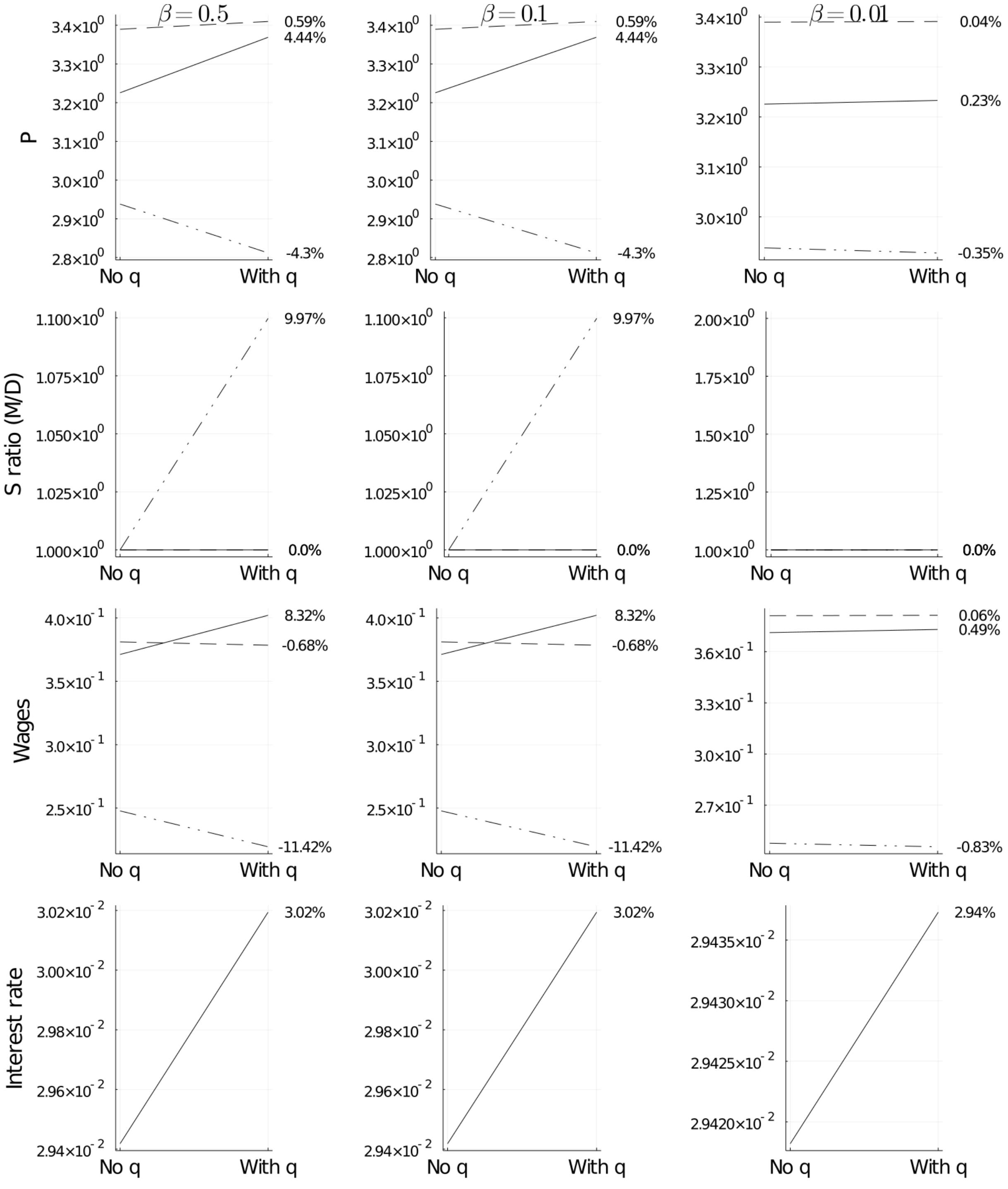


	Home Bias Network and Bundle								
	$\beta = 0.5$			$\beta = 0.1$			$\beta = 0.01$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$11.76e^{-3}$	$-17.12e^{-4}$	$-2.69e^{-1}$	$11.78e^{-3}$	$-17.1e^{-4}$	$-2.69e^{-1}$	$0.74e^{-3}$	$0.3e^{-4}$	$2.43e^{-1}$
$3 \to 1$	$24.71e^{-3}$	$-0.11e^{-4}$	$-0.06e^{-1}$	$24.71e^{-3}$	$-0.11e^{-4}$	$-0.06e^{-1}$	$1.23e^{-3}$	$0.43e^{-4}$	$7.34e^{-1}$
$3 \to 2$	$-6.54e^{-3}$	$-6.15e^{-4}$	$-3.83e^{-1}$	$-6.54e^{-3}$	$-6.15e^{-4}$	$-3.83e^{-1}$	$0.61e^{-3}$	$0.19e^{-4}$	$3.57e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	8.34%	12.09%	0.0%	8.34%	12.09%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\gamma = 0.1$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 20: Competitive Equilibrium under different β with K

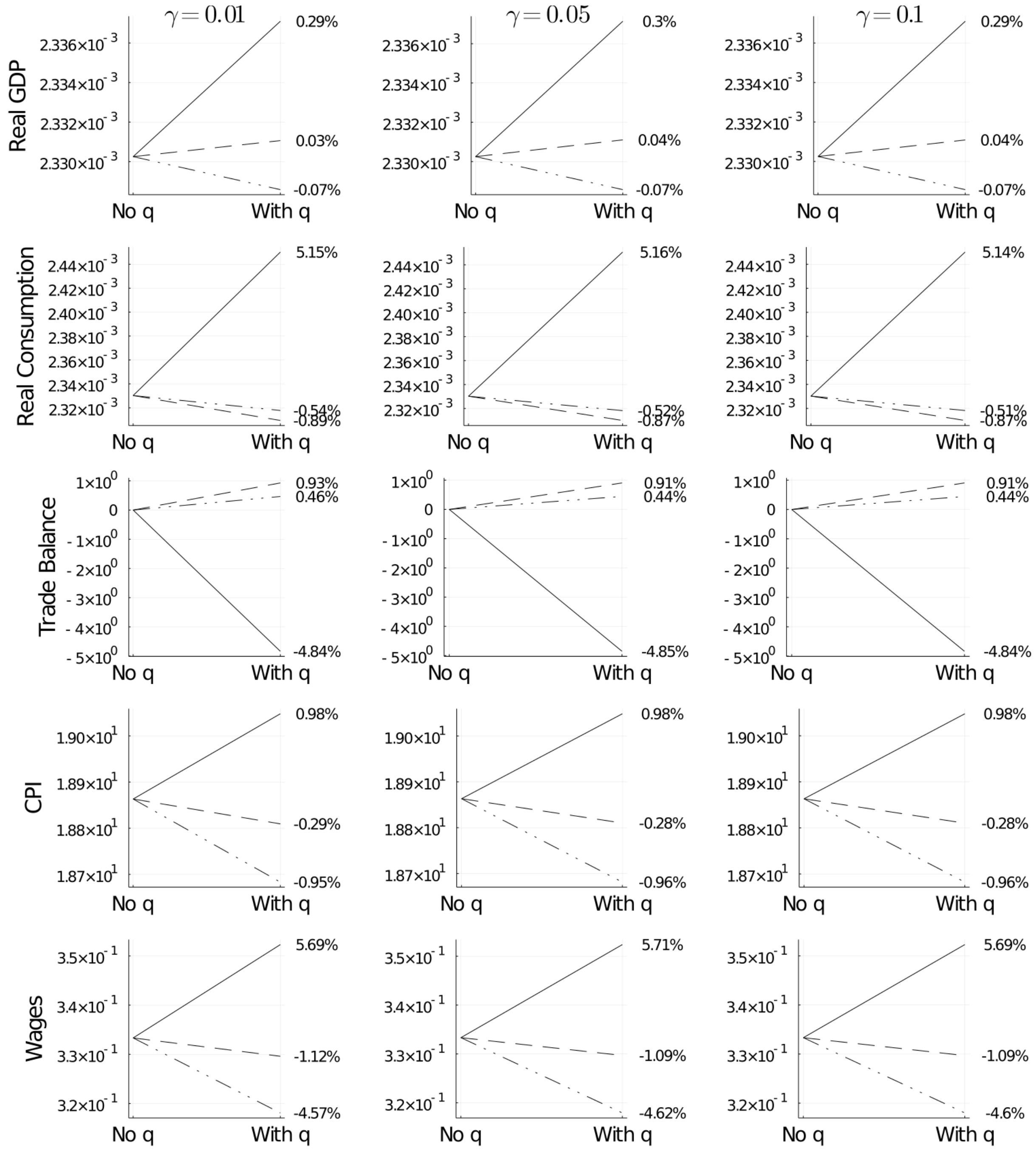




	Home Bias Network and Bundle								
	$\beta = 0.5$			$\beta = 0.1$			$\beta = 0.01$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$26.47e^{-3}$	$2.02e^{-4}$	$0.29e^{-1}$	$26.47e^{-3}$	$2.02e^{-4}$	$0.29e^{-1}$	$0.74e^{-3}$	$0.33e^{-4}$	$2.42e^{-1}$
$3 \to 1$	$30.56e^{-3}$	$-68.14e^{-4}$	$1.19e^{-1}$	$30.56e^{-3}$	$-68.14e^{-4}$	$1.19e^{-1}$	$1.23e^{-3}$	$0.43e^{-4}$	$7.33e^{-1}$
$3 \to 2$	$-0.69e^{-3}$	$-75.15e^{-4}$	$-2.85e^{-1}$	$-0.69e^{-3}$	$-75.15e^{-4}$	$-2.85e^{-1}$	$0.61e^{-3}$	$0.19e^{-4}$	$3.6e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	7.8%	0.0%	0.0%	7.8%	0.0%	0.0%	0.0%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\beta = 0.25$, $\gamma = 0.1$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2 \; \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

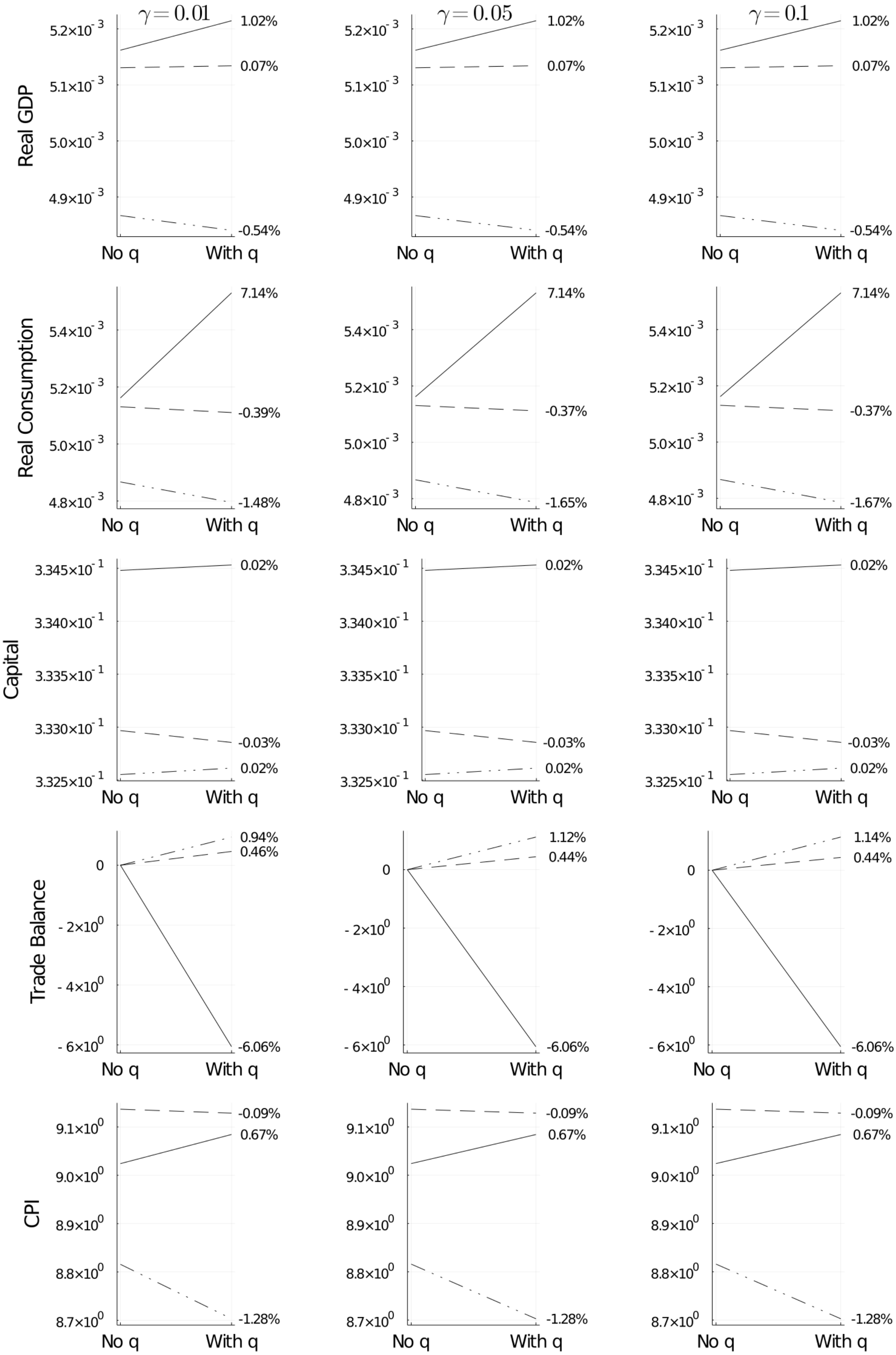
Table 21: Competitive Equilibrium under different γ without K

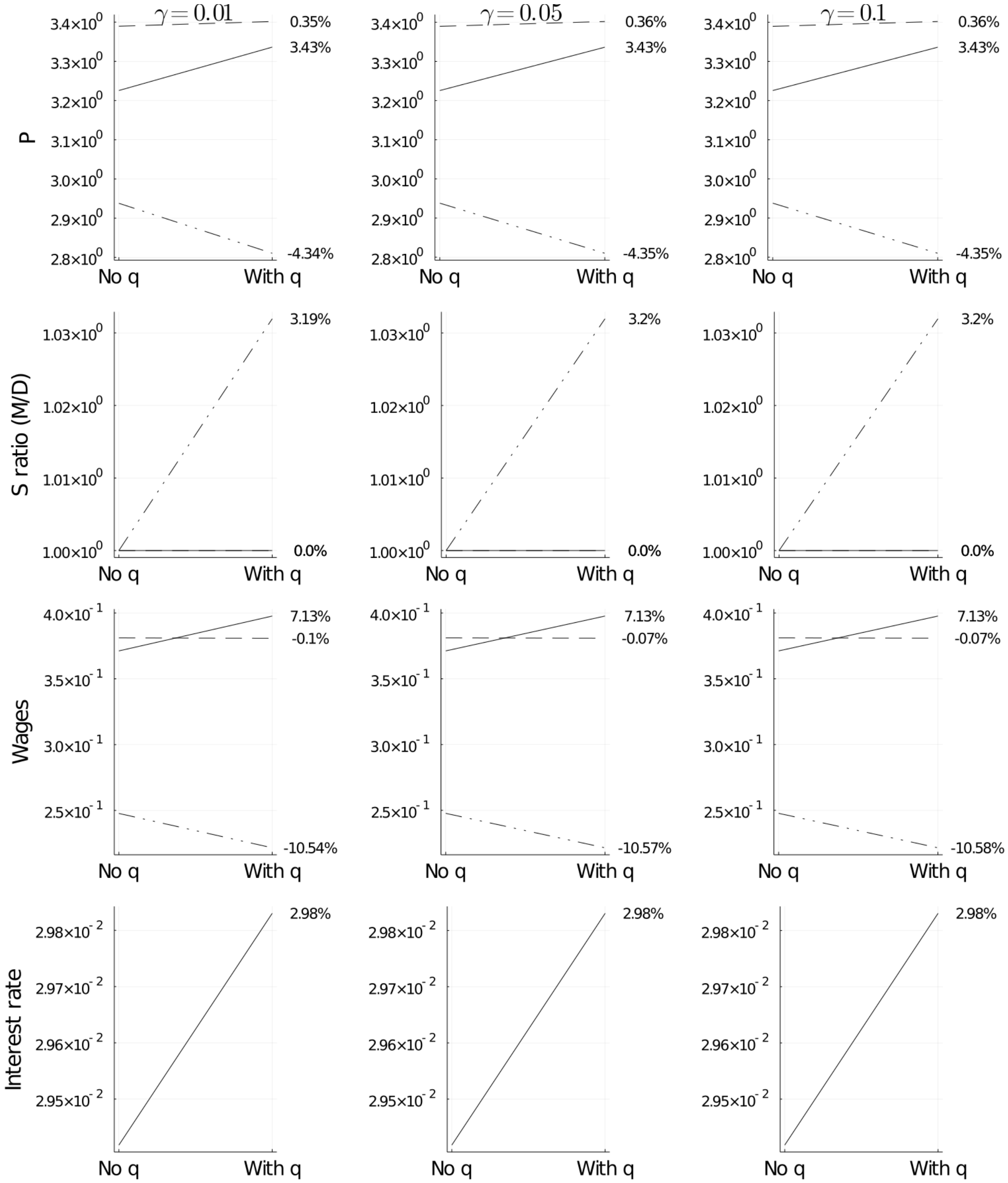


	Home Bias Network and Bundle								
	$\gamma = 0.01$			$\gamma = 0.05$			$\gamma = 0.1$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$59.6e^{-4}$	$17.34e^{-1}$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$3.93e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$18.64e^{-3}$	$49.44e^{-4}$	$44.01e^{-1}$	$16.89e^{-3}$	$16.64e^{-4}$	$15.05e^{-1}$	$16.88e^{-3}$	$1.94e^{-4}$	$1.75e^{-1}$
$3 \to 2$	$3.01e^{-3}$	$-3.95e^{-4}$	$-6.13e^{-1}$	$1.26e^{-3}$	$-25.47e^{-4}$	$-26.74e^{-1}$	$1.26e^{-3}$	$-2.2e^{-4}$	$-1.98e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	7.08%	0.0%	0.0%	8.47%	0.0%	0.0%	8.47%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\Upsilon = 0$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.

Table 22: Competitive Equilibrium under different γ with K





	Home Bias Network and Bundle								
	$\gamma = 0.01$			$\gamma = 0.05$			$\gamma = 0.1$		
	q	b	Q	q	b	Q	q	b	Q
$2 \to 1$	$15.52e^{-3}$	$59.6e^{-4}$	$17.34e^{-1}$	$15.52e^{-3}$	$31.54e^{-4}$	$9.18e^{-1}$	$15.52e^{-3}$	$4.13e^{-4}$	$1.2e^{-1}$
$3 \to 1$	$22.16e^{-3}$	$-130.4e^{-4}$	$-32.1e^{-1}$	$22.16e^{-3}$	$-30.54e^{-4}$	$55.63e^{-1}$	$22.16e^{-3}$	$-89.01e^{-4}$	$4.24e^{-1}$
$3 \to 2$	$6.54e^{-3}$	$-95.21e^{-4}$	$-1.35e^{-1}$	$6.54e^{-3}$	$-85.87e^{-4}$	$8.08e^{-1}$	$6.53e^{-3}$	$-93.71e^{-4}$	$0.13e^{-1}$
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
	0.0%	0.0%	2.49%	0.0%	0.0%	2.5%	0.0%	0.0%	2.5%

Note: The low tax economy is represented by the continuous line, the intermediate tax economy by the dashed line, and the high tax economy by the dashed line with points. The percentage value on the right side margin of each graph represents the percentage change on each variable, except for the trade balance and the interest rate, where it represents the level of these variables. These estimations are solved assuming $\tau_1 = 10\%$, $\tau_2 = 20\%$, $\tau_3 = 30\%$, $\theta_r = 3$, $\lambda_r = 7$, $\alpha_r = 0.3$, $\alpha_r^K = 0.3$, $n_r = 1/3$, $\epsilon_r = 0$, $\alpha = 0.6$, $\beta = 0.25$, $\Upsilon = 0$, $k_r = 1/3$, $\psi_1 = 5\%$, $\psi_2 = 2.5\%$ and $\psi_3 = 1\%$. We assume a network with home bias with $\omega_{rr} = 1/2\ \forall r$ and $\omega_{rm} = 1/4$. We assume a home bias bundle with $\beta_{rm} = 0.5$ when $r = m$, and $\beta_{rm} = 0.25$ when $r \neq m$.