Incidence on Real Value Added in Production Networks

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April 2026 - Midwest International Trade Conference

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This project: First-order decomposition for Y_r and TFP_r in a general production network with

1. Firm & Household Hetero
2. Price/marginal cost **wedges**
3. No market segmentation

Literature Review

Aggregation & Networks

Solow (1957), Domar (1961), Hulten (1978),Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (2012), Baqaee & Farhi (2019, 2020, 2024),

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Open-economy Value-Added Country-Accounting Comovement

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Distortions & Misallocation in Networks

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Distortions & Misallocation in Networks

Liu (2019), Bigio & La'O (2020), Baqaee & Farhi (2020, 2024), Rojas-Bernal (2026)

Heterogeneous Agents

Baqaee & Burnstein (2025), Davila-Schaab (2025), Rojas-Bernal (2026)

Two Closest Environments

Huo, Levchenko & Pandalai-Nayar (2025)

Production network general equilibrium model **without rent generating distortions** and **sector-specific factors**

$$GDP_r = \sum_{i \in \mathcal{N}_r} w_i \, \ell_i$$

GDP comovement in response to foreign shocks occurs through endogenous adjustments in factor supply, i.e., there are no cross-country effects on TFP.

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Baqaee & Farhi (2024)

Production network general equilibrium model **with rent generating distortions** and **country-specific factors**

$$GDP_r = \sum_{i \in \mathcal{N}_r} \Bigg(\overbrace{\sum_{f \in \mathcal{F}_r} w_f \, \ell_{if}}^{\text{Primary Factor Costs}} + \overbrace{(1 - \mu_i)}^{\text{Profit Margin}} \overbrace{p_i \, y_i}^{\text{Revenue}} \Bigg)$$

Foreign shocks affect country GDP through imported intermediate linkages and open-economy reallocation effects

Contributions

Theoretical Contribution

1. Non-parametric first-order decomposition for cluster-level real value added in production networks.

2. Applies to arbitrary firm partitions. There is no need for factor market segmentation.

3. Delivers sufficient statistics and centralities that map into cluster real value-added.

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Empirical Contribution

1. Implementation with U.S. data. Showing which sectors lead U.S. TFP growth.

2. Large and relatively upstream sectors are negatively correlated with the rest of the economy.

3. Sufficient statistics in response to productivity shocks are a good measure of general equilibrium effects.

Environment

- Non-parametric CRS economy in production and consumption

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- **Production:** Firm $i \in \mathscr{N}$ produces using

$$y_i = A_i\, Q_i(L_i, X_i) \qquad \begin{cases} L_i : \text{ Primary factor bundle} \\ X_i : \text{ Intermediate input bundle} \end{cases}$$

$$\begin{aligned} L_i &= Q_i^{\ell}\left(\{\ell_{ih}\}_{h \in \mathscr{H}} \right) \\ X_i &= Q_i^{\ell}\left(\{x_{ij}\}_{j \in \mathscr{N}} \right) \end{aligned} \qquad \begin{cases} \ell_{ih} : \text{ Factor supplied by household } h \\ x_{ij} : \text{ Intermediate input supplied by firm } j \end{cases}$$

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Firm i solves

$$\underset{y_i,\{\ell_{ih}\}_{h\in\mathscr{H}},\{x_{ij}\}_{j\in\mathscr{N}}}{Min} \quad Cost_i = \sum_{h\in\mathscr{H}} w_h\,\ell_{ih} + \sum_{j\in\mathscr{N}} p_j\, x_{ij} \qquad \text{subject to}$$

$$1 = A_i\, Q_i(L_i, X_i) \quad \text{and} \quad mc_i = \boldsymbol{\mu_i}\, p_i \qquad \rightarrow \qquad Cost_i = \boldsymbol{\mu_i}\, p_i\, y_i$$

where $\boldsymbol{\mu_i}$ stands for the **inverse markup** with $0 < \mu_i \leq 1$

Environment

- **Consumption:** Household $h \in \mathscr{H}$

$$\underset{\left\{C_h, \{C_{hi}\}_{i\in\mathscr{N}}, L_h\right\}}{Max} \quad U_h\left(C_h, L_h\right) \qquad \text{subject to}$$

$$C_h = Q_h^c\left(\{C_{hi}\}_{i\in\mathscr{N}}\right) \quad \text{and} \quad \underbrace{p_h^c\, C_h = \sum_{i\in\mathscr{N}} p_i\, C_{hi}}_{\text{Expenditure}} \leq \underbrace{w_h\, L_h}_{\substack{\text{Factor}\\ \text{Income}}} + \underbrace{\sum_{i\in\mathscr{N}} \kappa_{ih}\, \pi_i}_{\text{Corporate Income}} + \underbrace{T_h}_{\substack{\text{Lump Sum}\\ \text{Transfers}}}$$

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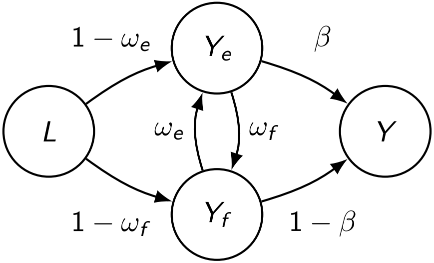
- **Market Clearing:**

$$y_i = \sum_{h \in \mathscr{H}} C_{hi} + \sum_{j \in \mathscr{N}} x_{ji} \qquad L_h = \sum_{i \in \mathscr{N}} \ell_{ih} \qquad \sum_{h \in \mathscr{H}} T_h = 0$$

Goods Market Factor Market Balanced Budget

Circular Network

electricity (e) and food (f)



$$y_e = A_e Q_e(\ell_e, x_{ef}) \qquad y_f = Q_f(\ell_f, x_{fe})$$

$$\omega_e = \frac{p_f\, x_{ef}}{\mu_e\, p_e\, y_e} \qquad \omega_f = \frac{p_e\, x_{fe}}{\mu_f\, p_f\, y_f}$$

$$Y = Q_Y(C_e, C_f) \qquad \beta = \frac{p_e\, C_e}{P_Y\, Y}$$

We already have general decompositions for Y, not for Y_r, which is given by $p_{Y_r} Y_r = (1-\mu_r\,\omega_r) p_r\, y_r$. **Parameters:** A_e, A_f, β, ω_e, ω_f, μ_e, μ_f.

Circular Network

$$d\log Y_e = \overbrace{\ddot\lambda_e^e\, d\log A_e + \ddot\lambda_f^e\, d\log A_f}^{\text{Productivity Shocks}} + \overbrace{\left(\ddot\lambda_e^e + \lambda_e\left(\frac{\psi_{e|e} - \mu_e\omega_e}{\Phi_e} - \frac{\psi_e^\ell}{\Lambda}\right)\right)d\log\mu_e}^{\text{Electricity Cost Share } \mu_e}$$

$$+ \overbrace{\left(\ddot\lambda_f^e + \lambda_f\left(\frac{\psi_{f|e} + \mu_f\omega_f}{\Phi_e} - \frac{\psi_f^\ell}{\Lambda}\right)\right)d\log\mu_f}^{\text{Food Cost Share } \mu_f} + \overbrace{\mu_e\lambda_e\left(\frac{\psi_{f|e} - 1}{\Phi_e} + \frac{1 - \psi_f^\ell}{\Lambda}\right)d\omega_e}^{\text{Intermediate Intensity } \omega_e}$$

$$+ \overbrace{\mu_f\lambda_f\left(\frac{\psi_{e|e} + 1}{\Phi_e} + \frac{1 - \psi_e^\ell}{\Lambda}\right)d\omega_f}^{\text{Intermediate Intensity } \omega_f} + \overbrace{\left(\frac{\psi_{e|e} - \psi_{f|e} + 1}{\Phi_e} + \frac{\psi_f^\ell - \psi_e^\ell}{\Lambda}\right)d\beta}^{\text{Expenditure Intensity } \beta} + d\log L,$$

with

$$\ddot\lambda_e^e = \phi(1 - \mu_e\omega_e\omega_f), \quad \ddot\lambda_f^e = \phi(1 - \mu_e)\omega_e, \quad \lambda_e = \frac{\beta + (1-\beta)\mu_f\omega_f}{1 - \mu_e\omega_e\mu_f\omega_f}, \quad \lambda_f = \frac{1 - \beta + \beta\mu_e\omega_e}{1 - \mu_e\omega_e\mu_f\omega_f}$$

$$\psi_{f|e} = \frac{\mu_f\omega_f(1 - \mu_e\omega_e)}{1 - \mu_e\omega_e\mu_f\omega_f}, \quad \psi_{e|e} = -\frac{\mu_e\omega_e(1 - \mu_f\omega_f)}{1 - \mu_e\omega_e\mu_f\omega_f}, \quad \psi_e^\ell = \frac{\mu_e(1 - \omega_e(1 - \mu_f(1 - \omega_f)))}{1 - \mu_e\omega_e\mu_f\omega_f},$$

$$\psi_f^\ell = \frac{\mu_f(1 - \omega_f(1 - \mu_e(1 - \omega_e)))}{1 - \mu_e\omega_e\mu_f\omega_f}, \quad \Lambda = (1 - \omega_e)\mu_e\lambda_e + (1 - \omega_f)\mu_f\lambda_f, \quad \phi = (1 - \mu_e\omega_e)^{-1}(1 - \mu_f\omega_f)^{-1}.$$

Baqaee & Farhi (2024) - Country-Level Y_r

For an open-economy with **country-specific factors $\mathscr{F}_r$**: Ownership of factors and firms is not restricted: ***factor markets are local, but income claims need not be local.***

$$d\log Y_r = \overbrace{\sum_{j\in\mathcal{N}_r} \widetilde{\lambda}_{Y_{rj}} (d \log A_j + d \log \mu_j)}^{\text{Domestic Firm Shocks}} + \overbrace{\sum_{f\in\mathscr{F}_r} \widetilde{\Lambda}_{Y_r f}\, d\log L_f}^{\text{Domestic Factor Supply Shocks}}$$

$$\underbrace{-\sum_{f\in\mathscr{F}_r} \widetilde{\Lambda}_{Y_r f}\, d\log \Lambda_{Y_r h}}_{\text{Domestic Factor Income Distribution Variations}} + \underbrace{\sum_{i\notin\mathcal{N}_r} \left(\widetilde{\Lambda}_{Y_r i} - \Lambda_{Y_r i}\right)\left(\overbrace{d \log q_{ri}}^{\text{Net quantity of i imports to r (REAL)}} - d\log \Lambda_{Y_r i}\right)}_{\text{Foreign Firm Net Import Variations}}$$

1. $p \to q$ or q
2. **Sufficient statistics** only for **domestic shocks**. Foreign shocks require L_f, $\Lambda_{Y_r h}$, q_{ri}, $\Lambda_{Y_r i}$
3. $\mathscr{F}_r$ segmentation

New Sufficient Statistics for Cluster $\mathscr{N}_r$

Exposure of r to firm j

Share of r's value added passing through j

$$\ddot{\lambda}_j^r = \mathbf{1}_{\{j \in \mathscr{N}_r\}} \frac{\lambda_j}{\Phi_r} + \sum_{i \in \mathscr{N}_r} \frac{\lambda_i}{\Phi_r} \widetilde{\psi}^x_{ij,\Delta\Omega}$$

$\lambda_j = S_j / GDP$
Domar weight of firm j

$\Phi_r = GDP_r / GDP$
cluster-r value-added share

$$\widetilde{\psi}^x_{ij,\Delta\Omega} = (1 - \mu_i) \sum_{m \in \mathscr{N}} \widetilde{\Omega}^x_{im} \widetilde{\psi}^x_{mj}$$

$\widetilde{\Omega}^x_{im}$: i's cost share use for input m

$\widetilde{\psi}^x_{mj}$: downstream cost exposure of m to j

$$\widetilde{\Psi}_x = (I - \widetilde{\Omega}_x)^{-1}$$

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Exposure of r to factor h

Share of r's value added attributed to h

$$\ddot{\Lambda}_h^r = \sum_{i \in \mathcal{N}_r} \frac{\lambda_i}{\Phi_r} \left(\widetilde{\Omega}^\ell_{ih} + \widetilde{\psi}^\ell_{ih,\Delta\Omega} \right)$$

λ_i / Φ_r
firm i's weight inside cluster r

$\widetilde{\Omega}^\ell_{ih}$
direct cost share of factor h in firm i

$$\widetilde{\psi}^\ell_{ih,\Delta\Omega} = (1 - \mu_i) \sum_{j \in \mathcal{N}} \widetilde{\Omega}^x_{ij} \widetilde{\psi}^\ell_{jh}$$

$\widetilde{\psi}^\ell_{jh}$: downstream cost exposure of j to h
$\widetilde{\Psi}_\ell = \widetilde{\Psi}_x \widetilde{\Omega}_\ell$

Proposition 1. Network-Adjusted Price Pass-Through

Network-adjusted pass-through for cluster deflator.

$$d \log p_{Y_r} = - \sum_{i \in \mathcal{N}} \ddot{\lambda}_i^r \, d \log(A_i \, \mu_i) + \sum_{h \in \mathscr{H}} \ddot{\Lambda}_h^r \, d \log w_h$$

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Aggregating across clusters using Φ recovers the economy-wide pass-through:

$$d \log p_Y = \sum_{r \in \mathcal{R}} \Phi_r \, d \log p_{Y_r} = -\sum_{i \in \mathcal{N}} \tilde{\lambda}_i \, d \log(A_i \, \mu_i) + \sum_{h \in \mathcal{H}} \tilde{\Lambda}_h \, d \log w_h$$

where $\tilde{\lambda}_i$ and $\tilde{\Lambda}_h$ stand for the cost-based Domar weights.

Theorem 1. $d \log Y_r$ with Heterogeneous Households

Cluster-level real value added satisfies

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2. **Statistics** for **all** shocks
3. Removes $\mathscr{F}_r$ segmentation $\rightarrow$ works for any cluster

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We can still push $d\log\Phi_r$ and $d\log\Lambda_h$ further...

Theorem 1. Expanded Decomposition for Distributional Reallocation

From **Rojas-Bernal (2026)** the **sales λ** and **factor income Λ** distributions variation:

$$d\,\lambda_i = \underbrace{\sum_{h \in \mathscr{H}} \mathscr{B}_{hi}\, d\chi_h}_{}$$

Expenditure $\chi_h = \frac{p_h^c C_h}{GDP}$

$$d\,\Lambda_h = \overbrace{\sum_{b \in \mathscr{H}} \mathscr{C}_{bh}\, d\chi_b}^{}$$

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Expenditure $\chi_h = \frac{p^c_h C_h}{GDP}$ **Wedge (Cost-Share) Effect**

$$d\,\Lambda_h = \overbrace{\sum_{b\in\mathcal{H}} \mathscr{C}_{bh}\, d\chi_b}^{} + \overbrace{\sum_{i\in\mathcal{N}} \psi^{\ell}_{ih}\,\lambda_i\, d\log\mu_i}^{}$$

Theorem 1. Expanded Decomposition for Distributional Reallocation

From **Rojas-Bernal (2026)** the **sales λ** and **factor income Λ** distributions variation:

$$d\lambda_i = \underbrace{\sum_{h\in\mathcal{H}} \mathscr{B}_{hi}\, d\chi_h}_{\text{Expenditure } \chi_h = \frac{p_h^c c_h}{GDP}} + \underbrace{\sum_{j\in\mathcal{N}} \psi^x_{ji}\, \lambda_j\, d\log\mu_j}_{\text{Wedge (Cost-Share) Effect}} + \underbrace{\sum_{j\in\mathcal{N}} \psi^x_{ji}\left(\sum_{h\in\mathcal{H}} \chi_h\, d\beta_{hj} + \sum_{m\in\mathcal{N}} \mu_m\, \lambda_m\, d\widetilde{\Omega}^x_{mj}\right)}_{\text{Final \& Intermediate Cost Recomposition Effect}}$$

$$d\Lambda_h = \overbrace{\sum_{b\in\mathcal{H}} \mathscr{C}_{bh}\, d\chi_b} + \overbrace{\sum_{i\in\mathcal{N}} \psi^\ell_{ih}\, \lambda_i\, d\log\mu_i} + \overbrace{\sum_{j\in\mathcal{N}} \psi^\ell_{jh}\left(\sum_{b\in\mathcal{H}} \chi_b\, d\beta_{bj} + \sum_{i\in\mathcal{N}} \mu_i\, \lambda_i\, d\widetilde{\Omega}^x_{ij}\right)} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\, \lambda_i\, d\widetilde{\Omega}^\ell_{ih}}^{\text{Factor Cost Recomposition}}$$

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From **Rojas-Bernal (2026)** the **sales λ** and **factor income Λ** distributions variation:

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Expenditure $\chi_h = \frac{p^c_h C_h}{GDP}$ | Wedge (Cost-Share) Effect | Final & Intermediate Cost Recomposition Effect | Factor Cost Recomposition

$$d\,\Lambda_h = \overbrace{\sum_{b\in\mathcal{H}} \mathscr{C}_{bh}\, d\chi_b}^{} \; + \overbrace{\sum_{i\in\mathcal{N}} \psi^\ell_{ih}\,\lambda_i\, d\log\mu_i}^{} + \overbrace{\sum_{j\in\mathcal{N}} \psi^\ell_{jh}\left(\sum_{b\in\mathcal{H}} \chi_b\, d\beta_{bj} + \sum_{i\in\mathcal{N}} \mu_i\,\lambda_i\, d\widetilde{\Omega}^x_{ij}\right)}^{} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\,\lambda_i\, d\widetilde{\Omega}^\ell_{ih}}^{}$$

We can further decompose **Distributional Reallocation**

$$d\log\Phi_r - \sum_{h\in\mathcal{H}} \ddot{\Lambda}^r_h\, d\log\Lambda_h$$

$$d\,\Phi_r = \overbrace{\sum_{h\in\mathcal{H}} \left(\beta_{h|r} + \mathscr{B}_{h|r}\right) d\,\chi_h}^{\text{Expenditure}}$$

$$\beta_{h|r} = \sum_{i\in\mathcal{N}_r} \beta_{hi}, \quad \Omega^x_{i|r} = \sum_{j\in\mathcal{N}_r} \Omega^x_{ij}, \quad \mathscr{B}_{h|r} = \sum_{i\notin\mathcal{N}_r} \mathscr{B}_{hi}\,\Omega^x_{i|r} - \sum_{i\in\mathcal{N}_r} \mathscr{B}_{hi} \sum_{j\notin\mathcal{N}_r} \Omega^x_{ij}, \quad \psi_{i|r} = \sum_{m\notin\mathcal{N}_r} \psi^x_{im}\,\Omega^x_{m|r} - \sum_{m\in\mathcal{N}_r} \psi^x_{im} \sum_{f\notin\mathcal{N}_r} \Omega^x_{mf},$$

$$\mathscr{B}_{hi} = \sum_{j\in\mathcal{N}} \beta_{hj}\psi^x_{ji}, \qquad \delta^r_h = \frac{\ddot{\Lambda}^r_h}{\Lambda_h}, \qquad M^r_h = \sum_{b\in\mathcal{H}} \mathscr{C}_{hb}\,\delta^r_b, \qquad F^r_i = \sum_{h\in\mathcal{H}} \psi^\ell_{ih}\,\delta^r_h.$$

$$\sum_{h\in\mathcal{H}} \ddot{\Lambda}^r_h\, d\log\Lambda_h = \overbrace{\sum_{h\in\mathcal{H}} M^r_h\, d\chi_h}^{\text{Expenditure}}$$

$$d\,\Phi_r = \overbrace{\sum_{h\in\mathcal{H}} \left(\beta_{h|r} + \mathscr{B}_{h|r}\right) d\,\chi_h}^{\textbf{Expenditure}} + \overbrace{\sum_{i\in\mathcal{N}} \lambda_i\,\psi_{i|r}\,d\log\mu_i}^{\textbf{Wedges}}$$

$$\beta_{h|r} = \sum_{i\in\mathcal{N}_r}\beta_{hi}, \quad \Omega^x_{i|r} = \sum_{j\in\mathcal{N}_r}\Omega^x_{ij}, \quad \mathscr{B}_{h|r} = \sum_{i\notin\mathcal{N}_r}\mathscr{B}_{hi}\,\Omega^x_{i|r} - \sum_{i\in\mathcal{N}_r}\mathscr{B}_{hi}\sum_{j\notin\mathcal{N}_r}\Omega^x_{ij}, \quad \psi_{i|r} = \sum_{m\notin\mathcal{N}_r}\psi^x_{im}\,\Omega^x_{m|r} - \sum_{m\in\mathcal{N}_r}\psi^x_{im}\sum_{f\notin\mathcal{N}_r}\Omega^x_{mf},$$

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$$\sum_{h\in\mathcal{H}}\ddot{\Lambda}^r_h\,d\log\Lambda_h = \overbrace{\sum_{h\in\mathcal{H}} M^r_h\,d\chi_h}^{\textbf{Expenditure}} + \overbrace{\sum_{i\in\mathcal{N}}\lambda_i F^r_i\,d\log\mu_i}^{\textbf{Wedges}}$$

$$d\,\Phi_r = \overbrace{\sum_{h\in\mathscr{H}} \left(\beta_{h|r} + \mathscr{B}_{h|r}\right) d\,\chi_h}^{\text{Expenditure}} + \overbrace{\sum_{i\in\mathscr{N}} \lambda_i\,\psi_{i|r}\,d\log\mu_i}^{\text{Wedges}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\lambda_i \sum_{j\in\mathscr{N}} \psi_{j|r}\,d\widetilde{\Omega}^x_{ij}}^{\text{Intermediate Demand}}$$
$$+ \underbrace{\sum_{i\notin\mathscr{N}_r} \left(\lambda_i\,d\Omega^x_{i|r} - \sum_{j\in\mathscr{N}_r} \lambda_j\,d\Omega^x_{ji}\right)}_{\text{Intermediate Demand}}$$

$$\beta_{h|r} = \sum_{i\in\mathscr{N}_r} \beta_{hi}, \quad \Omega^x_{i|r} = \sum_{j\in\mathscr{N}_r} \Omega^x_{ij}, \quad \mathscr{B}_{h|r} = \sum_{i\notin\mathscr{N}_r} \mathscr{B}_{hi}\,\Omega^x_{i|r} - \sum_{i\in\mathscr{N}_r} \mathscr{B}_{hi} \sum_{j\notin\mathscr{N}_r} \Omega^x_{ij}, \quad \psi_{i|r} = \sum_{m\notin\mathscr{N}_r} \psi^x_{im}\,\Omega^x_{m|r} - \sum_{m\in\mathscr{N}_r} \psi^x_{im} \sum_{f\notin\mathscr{N}_r} \Omega^x_{mf},$$
$$\mathscr{B}_{hi} = \sum_{j\in\mathscr{N}} \beta_{hj}\psi^x_{ji}, \qquad \delta^r_h = \frac{\ddot{\Lambda}^r_h}{\Lambda_h}, \qquad M^r_h = \sum_{b\in\mathscr{H}} \mathscr{C}_{hb}\,\delta^r_b, \qquad F^r_i = \sum_{h\in\mathscr{H}} \psi^\ell_{ih}\,\delta^r_h.$$

$$\sum_{h\in\mathscr{H}} \ddot{\Lambda}^r_h\,d\log\Lambda_h = \overbrace{\sum_{h\in\mathscr{H}} M^r_h\,d\chi_h}^{\text{Expenditure}} + \overbrace{\sum_{i\in\mathscr{N}} \lambda_i F^r_i\,d\log\mu_i}^{\text{Wedges}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\lambda_i \sum_{j\in\mathscr{N}} F^r_j\,d\widetilde{\Omega}^x_{ij}}^{\text{Intermediate Demand}}$$

$$d\,\Phi_r = \overbrace{\sum_{h\in\mathcal{H}} \left(\beta_{h|r} + \mathscr{B}_{h|r}\right) d\,\chi_h}^{\text{Expenditure}} + \overbrace{\sum_{i\in\mathcal{N}} \lambda_i\,\psi_{i|r}\,d\log\mu_i}^{\text{Wedges}} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\lambda_i \sum_{j\in\mathcal{N}} \psi_{j|r}\,d\tilde{\Omega}^x_{ij}}^{\text{Intermediate Demand}}$$
$$+ \underbrace{\sum_{i\notin\mathcal{N}_r} \left(\lambda_i\,d\Omega^x_{i|r} - \sum_{j\in\mathcal{N}_r} \lambda_j\,d\Omega^x_{ji}\right)}_{\text{Intermediate Demand}} + \underbrace{\sum_{h\in\mathcal{H}} \chi_h \left(d\beta_{h|r} + \sum_{i\in\mathcal{N}} \psi_{i|r}\,d\beta_{hi}\right)}_{\text{Final Demand}}$$

$$\beta_{h|r} = \sum_{i\in\mathcal{N}_r} \beta_{hi}, \quad \Omega^x_{i|r} = \sum_{j\in\mathcal{N}_r} \Omega^x_{ij}, \quad \mathscr{B}_{h|r} = \sum_{i\notin\mathcal{N}_r} \mathscr{B}_{hi}\,\Omega^x_{i|r} - \sum_{i\in\mathcal{N}_r} \mathscr{B}_{hi} \sum_{j\notin\mathcal{N}_r} \Omega^x_{ij}, \quad \psi_{i|r} = \sum_{m\notin\mathcal{N}_r} \psi^x_{im}\,\Omega^x_{m|r} - \sum_{m\in\mathcal{N}_r} \psi^x_{im} \sum_{f\notin\mathcal{N}_r} \Omega^x_{mf},$$
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$$\sum_{h\in\mathcal{H}} \ddot{\Lambda}^r_h\,d\log\Lambda_h = \overbrace{\sum_{h\in\mathcal{H}} M^r_h\,d\chi_h}^{\text{Expenditure}} + \overbrace{\sum_{i\in\mathcal{N}} \lambda_i F^r_i\,d\log\mu_i}^{\text{Wedges}} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\lambda_i \sum_{j\in\mathcal{N}} F^r_j\,d\tilde{\Omega}^x_{ij}}^{\text{Intermediate Demand}}$$
$$+ \underbrace{\sum_{h\in\mathcal{H}} \chi_h \sum_{i\in\mathcal{N}} F^r_i\,d\beta_{hi}}_{\text{Final Demand}}$$

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$$+ \underbrace{\sum_{i\notin\mathcal{N}_r} \left(\lambda_i\,d\Omega^x_{i|r} - \sum_{j\in\mathcal{N}_r} \lambda_j\,d\Omega^x_{ji}\right)}_{\text{Intermediate Demand}} + \underbrace{\sum_{h\in\mathcal{H}} \chi_h \left(d\beta_{h|r} + \sum_{i\in\mathcal{N}} \psi_{i|r}\,d\beta_{hi}\right)}_{\text{Final Demand}}$$

$$\beta_{h|r} = \sum_{i\in\mathcal{N}_r} \beta_{hi}, \quad \Omega^x_{i|r} = \sum_{j\in\mathcal{N}_r} \Omega^x_{ij}, \quad \mathscr{B}_{h|r} = \sum_{i\notin\mathcal{N}_r} \mathscr{B}_{hi}\,\Omega^x_{i|r} - \sum_{i\in\mathcal{N}_r} \mathscr{B}_{hi} \sum_{j\notin\mathcal{N}_r} \Omega^x_{ij}, \quad \psi_{i|r} = \sum_{m\notin\mathcal{N}_r} \psi^x_{im}\,\Omega^x_{m|r} - \sum_{m\in\mathcal{N}_r} \psi^x_{im} \sum_{f\notin\mathcal{N}_r} \Omega^x_{mf},$$
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$$+ \underbrace{\sum_{h\in\mathcal{H}} \chi_h \sum_{i\in\mathcal{N}} F^r_i\,d\beta_{hi}}_{\text{Final Demand}} + \underbrace{\sum_{i\in\mathcal{N}} \mu_i\lambda_i \sum_{h\in\mathcal{H}} \delta^r_h\,d\tilde{\Omega}^\ell_{ih}}_{\text{Factor Demand}}$$

Corollary 1. Partial Equilibrium Sufficient Statistics

$$d\log TFP_r = \overbrace{\sum_{i\in\mathcal{N}} \ddot{\lambda}_i^r \, d\log A_i}^{\text{Technology}_r} + \underbrace{\overbrace{\sum_{i\in\mathcal{N}} \ddot{\lambda}_i^r \, d\log \mu_i}^{\text{Competitiveness}_r} + \overbrace{d\log \Phi_r - \sum_{h\in\mathcal{H}} \ddot{\Lambda}_h^r \, d\log \Lambda_h}^{\text{Distr. Reallocation}_r \ \Lambda_h = \frac{w_h L_h}{GDP}}}_{\text{Allocative Efficiency}_r}$$

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Wedges. For a 1% increase in μ_i:

$$\ddot{\lambda}_i^r + \frac{\lambda_i}{\Phi_r}\left(\psi_{i|r} + \mu_i\left(\mathbb{1}_{\{i\notin\mathcal{N}_r\}} \sum_{j\in\mathcal{N}_r} \widetilde{\Omega}_{ij}^x - \mathbb{1}_{\{i\in\mathcal{N}_r\}} \sum_{j\notin\mathcal{N}_r} \widetilde{\Omega}_{ij}^x\right)\right) - \lambda_i F_i^r$$

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Expenditure Shares. For a 1pp shift from χ_b to χ_h:

$$\frac{1}{\Phi_r}\left(\beta_{h|r} - \beta_{b|r} + \mathscr{B}_{h|r} - \mathscr{B}_{b|r}\right) + M_b^r - M_h^r$$

Also for factor income variatons, factor, intermediate, and final cost reallocation

Data Sources I

Industry Level Production Accounts

- Sector-level A_i productivitvy by year
- Sector-level capital compensation across 9 forms of capital:

  (1) commercial equipment, (2) computer equipment, (3) R&D capital services, (4) software capital services, (5) artistic originals, (6) institutional equipment, (7) structures, (8) transportation, and (9) other

Input Output Tables

- Labor Costs by sector
- Intermediate Input Costs by sector
- Final consumption by sector
- Using sectoral capital + labor + intermediate costs and sales, I estimate $\mu_{i,t}$

Model with 66 sectors with intermediate inputs + Household has 10 different sources of income (Labor + 9 types of capital + Profits)

Data Sources II

Consumer Expenditure Survey

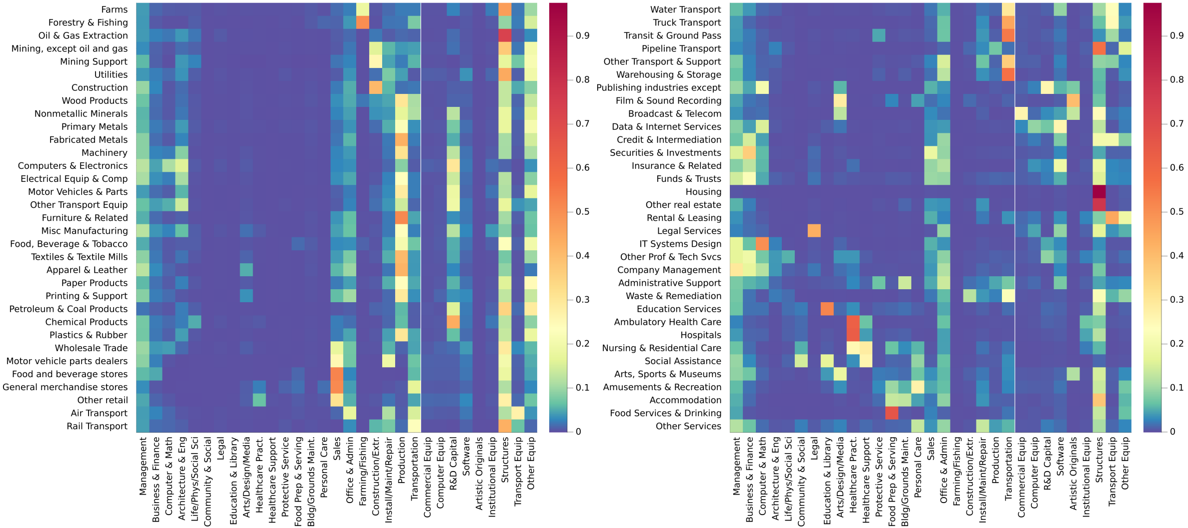
- Map household expenditures (UCC) into NIPA PCE commodities (Markevych, 2025)
- Bridge commodities to NAICS industries
- For each household, use 100 nearest-income neighbors to impute NAICS-level spending
- Apply IO Make matrix to build industry consumption vector
- Generate consumption profiles by income group
- Assign residual spending to capital and firm owners to match aggregates

Occupation and Employment Wage Statistics

- Sector level intensity in labor hiring by occupation
- Occupational variation in labor income exposure by industry

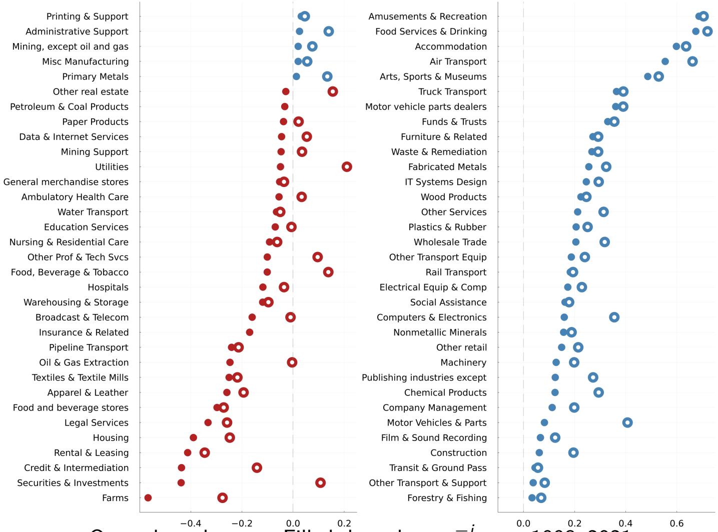
Model with 66 sectors and 843 households (833 occupations + 9 capital owners + firm owners). Labor links from OEWS; consumption based on expenditure vector nearest to average occupational income.

$\ddot{\Lambda}_h^r$ - Value Added by Factor Type



Labor aggregated in 22 SOC categories and 9 types of capital

Sectoral TFP correlation with Aggregate TFP: ρ_i & ρ_i^{-i}



Open dots show ρ_i. Filled dots show ρ_i^{-i} over 1998–2021.

$$\rho_i = corr(d \log TFP_t,\ d \log TFP_{t,i})$$

$$\rho_i^{-i} = corr(d \log TFP_t^{-i},\ d \log TFP_{t,i})$$

ρ_i mechanically positively correlates and ρ_i^{-i} measures correlation with the *rest of the economy*

Most pro-cyclical: Food Services, Recreation, Accommodation, Air and Truck Transport.

Most counter-cyclical: Farms, Housing, Finance, and Legal Services.

Cross-Sectional Regressions of Sectoral Comovement

1. Initial sector size strongly predicts *leave-one-out* comovement: both $\lambda_{i,1997}$ and $\widetilde{\lambda}_{i,1997}$ matter for ρ_i^{-i}.

2. Much weaker for baseline ρ_i, so self-inclusion masks the true cross-sectional pattern.

3. The wedge $\widetilde{\lambda}_i - \lambda_i$ shows that upstream sectors have a more negative correlation with the rest of the economy.

4. Average sectoral TFP growth has limited explanatory power and only with ρ_i.

Panel A: $\lambda_{i,1997}$

	ρ_i	ρ_i^{-i}
$\lambda_{i,1997}$	-0.545	-3.061^{***}
	(1.085)	(1.132)
Cons	0.164^{***}	0.134^{***}
	(0.044)	(0.046)
R^2	0.004	0.103

Panel B: $\widetilde{\lambda}_{i,1997}$

	ρ_i	ρ_i^{-i}
$\widetilde{\lambda}_{i,1997}$	-0.516	-2.973^{***}
	(1.048)	(1.093)
Cons	0.164^{***}	0.135^{***}
	(0.045)	(0.047)
R^2	0.004	0.104

Panel C: $\widetilde{\lambda}_i - \lambda_i$

	ρ_i	ρ_i^{-i}
$\widetilde{\lambda}_i - \lambda_i$	-2.252	-36.955^{*}
	(18.744)	(20.083)
Cons	0.151^{***}	0.098^{**}
	(0.045)	(0.047)
R^2	0.000	0.050

Panel D: $\overline{d \log TFP_i}$

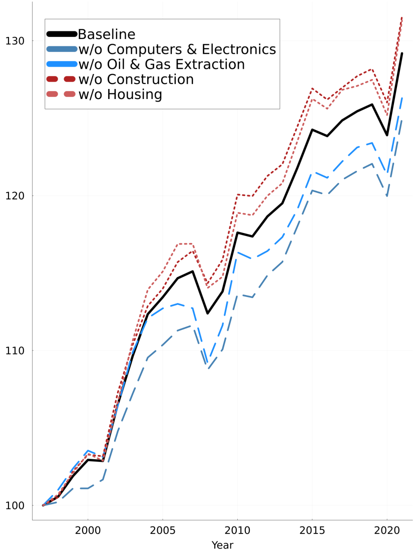
	ρ_i	ρ_i^{-i}
$\overline{d \log TFP_i}$	2.078^{*}	1.514
	(1.055)	(1.179)
Cons	0.114^{***}	0.018
	(0.034)	(0.038)
R^2	0.057	0.025

$N = 66$. Standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Sectoral Leave-One-Out Effects on Aggregate TFP

Table: Top 10 Sectors with Largest + and - Effect on TFP

Sector	Diff	Sector	Diff
Computers & Electronics	4.06%	Farms	-4.26%
Oil & Gas Extraction	3.13%	Construction	-2.76%
Administrative Support	2.96%	Housing	-2.6%
IT Systems Design	2.79%	Chemical Products	-1.85%
Data & Internet Services	1.91%	Other Services	-1.82%
Insurance & Related	1.74%	Rental & Leasing	-1.32%
Ambulatory Health Care	1.62%	Fabricated Metals	-1.29%
Broadcast & Telecom	1.35%	Securities & Investments	-1.17%
Other real estate	1.3%	Credit & Intermediation	-1.12%
Company Management	0.86%	Apparel & Leather	-1.02%



Parametric Model for GE effects from Rojas-Bernal (2026)

Model and Parameterization

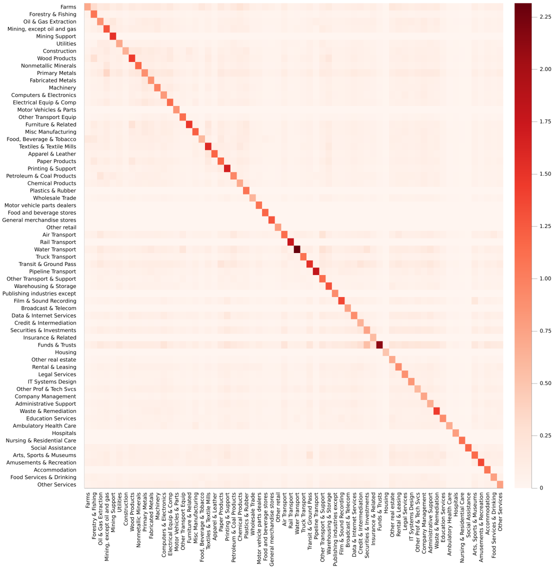
- To capture **general equilibrium** effects on TFP, estimate a **nested CES** economy with:
  1. θ: elasticity between L and X
  2. θ_ℓ: elasticity within L
  3. θ_x: elasticity within X
  4. ρ: elasticity across C
- **Two-stage GMM** to match $\widehat{\text{TFP}}$ treating $\widehat{A}$ and $\widehat{\mu}$ as exogenous. **Stage 1:** global (differential evolution), bounds $[0, 3]$; **Stage 2:** local (L-BFGS, forward diff).
- **Best estimates** have a 0.890 **correlation** and 0.93 **index of agreement**.

GMM Regression Results

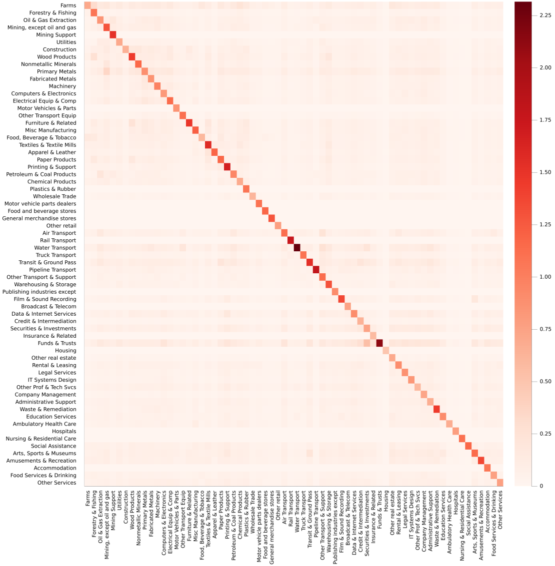
Par	Mean	Sdv	Best	95% CI
θ	0.079	0.072	0.001	[0.001, 0.246]
θ_ℓ	1.466	0.812	0.946	[0.191, 2.894]
θ_x	1.959	0.465	2.319	[1.095, 2.849]
ϱ	0.482	0.370	0.065	[0.014, 1.320]

Complementarity *almost everywhere*. Consistent with Boehm, Flaeen & Nayar (2014) and Atalay (2017) who only estimate θ and θ_x.

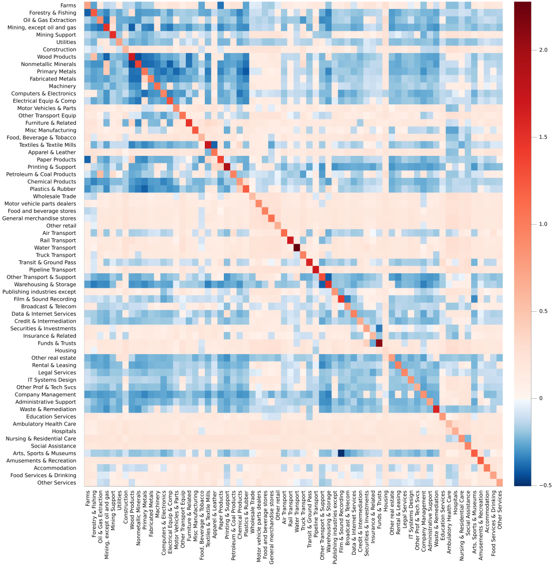
Partial Equilibrium TFP_i to A_j



Partial Equilibrium TFP_i to A_j

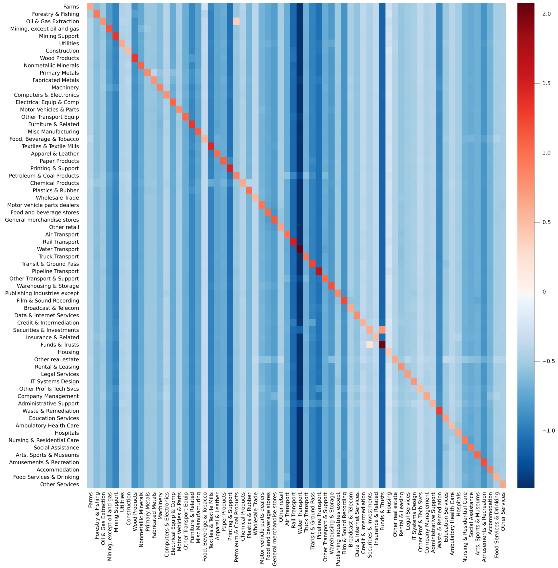


General Equilibrium TFP_i to A_j

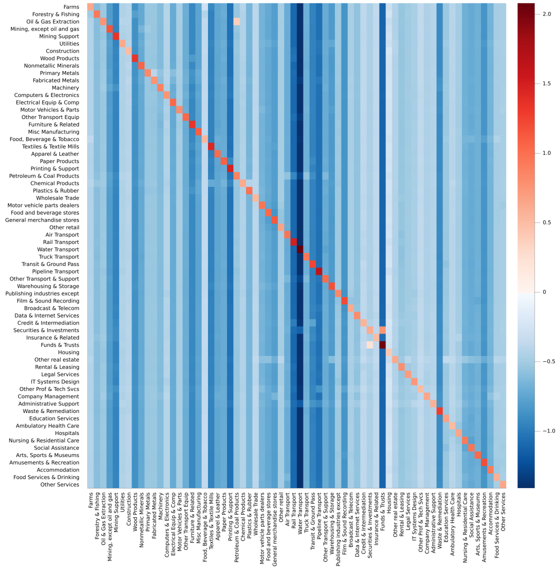


Correlation 0.97 and Index of Agreement 0.98 between PE and GE

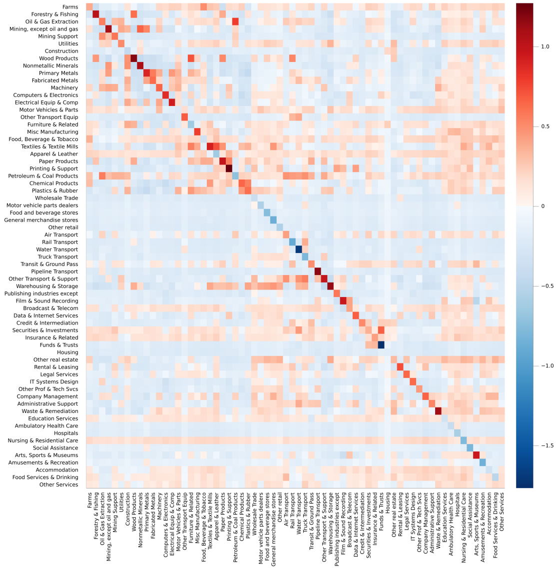
Partial Equilibrium TFP_i to μ_j



Partial Equilibrium TFP_i to μ_j



General Equilibrium TFP_i to μ_j



Correlation -0.17 and Index of Agreement 0.09 between PE and GE

Conclusions

Main theoretical result

1. **Non-parametric first-order decomposition** for Y_r and TFP_r for **any firm cluster**.
2. The framework delivers **sufficient statistics** for productivity, wedges, factor supplies, and distributional reallocation.
3. It **does not require factor-market segmentation**.
4. The new centralities $\ddot{\lambda}_j^r$ and $\ddot{\Lambda}_h^r$ summarize how shocks map into cluster value added.

Main empirical result

1. In U.S. data, **large, relatively upstream sectors** are more negatively correlated with the rest of the economy.
2. Domar weights strongly predict **comovement with the rest of the economy**.
3. **Partial-equilibrium sufficient statistics** closely track **general-equilibrium** effects for A not for μ.
4. Sectoral contributions to aggregate TFP are highly uneven.

Takeaway: Real value added is a network-incidence object shaped jointly by distortions, distribution, and production linkages.