Inequality and Misallocation under Production Networks

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Total Factor Productivity & Aggregation

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    - Solow & Ramsey growth model
    - RBC & New Keynesian models

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   - Solow & Ramsey growth model
   - RBC & New Keynesian models

2. Through Aggregation
   - Multiple Firms ⇒ Allocation
   - Production Networks ⇒ Amplification

Aggregate TFP is endogenous

Research Question & Motivation

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Upper Decile vs The Rest

Higher Expenditure Share in
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Lower Expenditure Share in
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Data from Consumer Expenditure Survey

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Data from Consumer Expenditure Survey

Income share for the top has increased

In this Presentation

In economies with distortions, variations in distributions (labor income & expenditure) can influence misallocation

Novel TFP decomposition that measures aggregate misallocations effects

Implementation of the model with US data

Approach

Δ **Consumption Distribution** & Δ **Demand Structure**

$\Downarrow$

Δ **Income Distribution**

Contribution

Literature Review

Approach

Δ **Consumption Distribution** & Δ **Demand Structure**

$\Downarrow$

Δ Income Distribution

Contribution

Δ **Labor Income Shares**

Literature Review

Approach

Δ **Consumption Distribution** & Δ **Demand Structure**

⇓

Δ **Income Distribution**

Contribution

Bigio & La'O (2020)

(i) Rep Household

(ii) Efficient Equilibrium

↓ **Δ Labor Income Shares**

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My Model

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(ii) Any Equilibrium

Baqaee & Farhi (2020)

(i) Rep Household

(ii) Exogenous L

$\downarrow$

My Model

(i) Het Households

(ii) Endogenous L

Literature Review

Static General Equilibrium Model with...

Two Firms: $\begin{cases} \text{More Competitive: } \boldsymbol{H} \\ \text{Less Competitive: } \boldsymbol{L} \end{cases}$ + **Two Workers**: $\begin{cases} \text{High-Skill: } \boldsymbol{h} \\ \text{Low-Skill: } \boldsymbol{l} \end{cases}$

Caveat: Paper is more general than this case

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$$Cost = \boldsymbol{\mu} \times Revenue$$

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Static General Equilibrium Model with...

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1. Good markets face exogenous distortions

$$Cost = \boldsymbol{\mu} \times Revenue$$

2. Labor markets are competitive

3. Labor supply is endogenous

4. **Correlations:**
   - $\boldsymbol{H}$ has high $\boldsymbol{\mu}$
   - $\boldsymbol{H}$ requires more $\boldsymbol{h}$
   - $\boldsymbol{h}$ have a higher expenditure in $\boldsymbol{H}$

Mechanism's Intuition

1. μ **heterogeneity** $\longrightarrow$ allocates more workers to ***H***
   - ***H*** operates with low marginal productivity
   - ***L*** operates with high marginal productivity

Mechanism's Intuition

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Mechanism's Intuition

1. **μ heterogeneity** $\longrightarrow$ allocates more workers to $\boldsymbol{H}$
   - $\boldsymbol{H}$ operates with low marginal productivity
   - $\boldsymbol{L}$ operates with high marginal productivity
2. **Skill-bias heterogeneity** $\longrightarrow$ asymmetries in the income exposure in response to local perturbations
3. **Preference heterogeneity** expenditure flows
   - As $\boldsymbol{h}$ income increase, expenditure in $\boldsymbol{H}$ rises
   - Workers relocate from $\boldsymbol{L}$ to $\boldsymbol{H}$
   - Misallocation is accentuated

Firm Heterogeneity $i \in \{H, L\}$

- **a. Skill Bias**
- **b. Distortions**

$$\underset{y_i, \ell_{i\,h}, \ell_{i\,l}}{Max} \quad \pi_i = p_i y_i - w_h \ell_{i\,h} - w_l \ell_{i\,l}$$

$$y_i = A_i \, \ell_{i\,h}^{\ \alpha_i} \, \ell_{i\,l}^{1-\alpha_i}$$

Skill Bias

$$\alpha_L \leq \alpha_H$$

Firm Heterogeneity $i \in \{H, L\}$ — **a. Skill Bias**, **b. Distortions**

$$\max_{y_i, \ell_{ih}, \ell_{il}} \quad \pi_i = p_i y_i - w_h \ell_{ih} - w_l \ell_{il}$$

$$y_i = A_i \, \ell_{ih}^{\alpha_i} \, \ell_{il}^{1-\alpha_i}$$

Skill Bias

$\alpha_L \leq \alpha_H$

Markdown

$0 < \mu_L \leq \mu_H \leq 1$

$$\text{Cost}_i = \mu_i \times \text{Revenue}_i$$

Alternative Narrative: Sectoral Dixit-Stiglitz Aggregation

Household Heterogeneity
$r \in \{h, l\}$

- **Preferences** → Aggregate Non-Homotheticity
  Graphic Argument
- **Unique Skill** → Horizontal Income Heterogeneity

Household Heterogeneity
$r \in \{h, l\}$

- **Preferences** → Aggregate Non-Homotheticity (Graphic Argument)
- **Unique Skill** → Horizontal Income Heterogeneity

$$\underset{C_r, L_r}{Max}\ U_r(C_r, L_r) \quad \text{s.t.} \quad \frac{C_r}{\overline{\overline{C}}_r} = \left(\beta_r \left(\frac{C_{rH}}{\overline{\overline{C}}_{rH}} \right)^{\frac{\rho-1}{\rho}} + (1 - \beta_r) \left(\frac{C_{rL}}{\overline{\overline{C}}_{rL}} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$E_r = p_r^c C_r = p_H C_{rH} + p_L C_{rL} \leq w_r L_r + 0.5 \text{ profits}$$

Consumption Bias

$$\beta_l \leq \beta_h$$

Equilibrium Definition

For (A, μ, β, α), prices and allocations:

(i) **Firms'** labor demand and output decisions maximize profits;

(ii) **Households'** consumption and labor supply maximize utility satisfying budget constraints;

(iii) Goods and labor markets clear.

Equilibrium Definition Necessary and sufficient conditions for equilibrium

Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H C_{rH} = \boldsymbol{\beta_r}\, p_r^c\, C_r \qquad\qquad w_h\, \ell_{ih} = \boldsymbol{\alpha_i}\, \boldsymbol{\mu_i}\, p_i\, y_i$$

Solve for Equilibrium Distributions

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In **market clearing conditions**

$$y_i = C_{hi} + C_{li} \qquad\qquad L_r = \ell_{Hr} + \ell_{Lr}$$

Solve for Equilibrium Distributions

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$$p_H\, C_{rH} = \beta_r\; p_r^c\; C_r \qquad\qquad w_h\, \ell_{ih} = \alpha_i\; \mu_i\; p_i\; y_i$$

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Equilibrium in terms of

$$\lambda_i = \frac{p_i\, y_i}{GDP} \qquad\qquad \chi_r = \frac{p_r^c\, C_r}{GDP}$$

Sales (Domar weights) — Expenditure

$$\Lambda_h = \frac{w_h\, L_h}{GDP} = \sum \alpha_i \mu_i \lambda_i \qquad\qquad \tilde{\Lambda}_h = \sum \alpha_i \lambda_i$$

Labor income — Value added

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

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Undistorted Benchmark: $\mu_H = \mu_L = 1$

$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

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$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}$$

Source of Misallocation

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Additional Restriction

$$\mu_H + \mu_L = 1$$

Solution Allocative ≠

What I Don't Do

- Misallocation literature **distorted vs. efficient** equilibrium
- Parametric assumptions (usually CD) $\rightarrow$ analytic TFP
- Evaluate how getting rid of distortions has an effect on TFP

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What I Do

- Local TFP Δ around distorted equilibrium to any perturbation
- Distributional $\Delta \rightarrow$ Misallocation $\Delta \rightarrow \Delta$ TFP
- To illustrate: $d\ log\ A_L = 1\%$

Local Variation to $d\,log\,A_L = 1\%$

$$\frac{d\,\chi_h}{d\,log\,A_L} = \frac{(\alpha_H - \alpha_L)}{2} \frac{d\,\lambda_H}{d\,log\,A_L}$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

First-Order Approximation

Local Variation to $d\log A_L = 1\%$

$$\frac{d\,\chi_h}{d\log A_L} = \frac{(\alpha_H - \alpha_L)}{2}\,\frac{d\,\lambda_H}{d\log A_L}$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

$$\frac{d\,\lambda_H}{d\log A_L} = -\frac{2(\rho-1)\beta_h\beta_l}{2-(\alpha_H-\alpha_L)(\beta_h-\beta_l)+2(\rho-1)\frac{\beta_h\beta_l}{1+\zeta^w}\left(\frac{\alpha_H-\mu_L}{\Lambda_h}+\frac{\alpha_H-\mu_H}{\Lambda_l}+\frac{\zeta^e}{2}\frac{\alpha_H-\alpha_L}{\chi_h\chi_l}\right)}$$

Sales elasticity requires $\rho \neq 1$

First-Order Approximation

Local Variation to $d \log A_L = 1\%$

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Sales elasticity requires $\rho \neq 1$

- **Under $\rho > 1$:** consumers increase expenditure on L & $\lambda_H \downarrow$
- **Under $\rho < 1$:** consumers increase expenditure on H & $\lambda_H \uparrow$ — **Baumol's Cost Disease**

First-Order Approximation

In This Section

- First-order local Δ **Income Distribution**

$$d\,\Lambda_h, \quad d\,\Lambda_l$$

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- First-order local Δ **Income Distribution**

$$d\,\Lambda_h, \quad d\,\Lambda_l$$

- Decomposition of $d\,\Lambda$ in

Δ **Consumption Distribution**	&	Δ **Demand Structure**
Keep β fix & change χ		Keep χ fix & change β

Income Distribution & Bilateral Centralities

$$\Lambda_h = m_{h\to h}\,\chi_h + m_{l\to h}\,\chi_l$$

$m_{l\to h}$ **%** of **expenditure** from l reaching Λ_h

Alternative Definitions for m's

Bilateral Centralities

$$\Lambda_h = \overbrace{\left(\underbrace{\alpha_H \, \mu_H}_{f_{H \to h}} \; \beta_h + \underbrace{\alpha_L \, \mu_L}_{f_{L \to h}} \; (1 - \beta_h) \right)}^{m_{h \to h}} \; \chi_h$$

$$+ \; \underbrace{\left(\overbrace{\alpha_H \, \mu_H} \; \beta_l + \overbrace{\alpha_L \, \mu_L} \; (1 - \beta_l) \right)}_{m_{l \to h}} \; \chi_l$$

$m_{l \to h}$	**%** of	**expenditure** from l	reaching Λ_h
$f_{L \to h}$		**revenue** from L	

Alternative Definitions for m's

Comparative Statics

$$\boldsymbol{\Lambda}_h = m_{h\to h}\, \chi_h + m_{l\to h}\, \chi_l$$

$$m_{r\to h} = \beta_r\, f_{H\to h} + (1-\beta_r)\, f_{L\to h} \qquad f_{i\to h} = \alpha_i\, \mu_i$$

Substitution Effects $d\,\beta_r$ Low skill income share variation $d\,\Lambda_l$ Labor Wedge variation $d\,log\,\Gamma_r$

Comparative Statics

$$\Lambda_h = m_{h\to h}\,\chi_h + m_{l\to h}\,\chi_l$$

$$m_{r\to h} = \beta_r\, f_{H\to h} + (1-\beta_r)\, f_{L\to h} \qquad f_{i\to h} = \alpha_i\,\mu_i$$

Take total derivative

$$d\,\Lambda_h = \underbrace{m_{h\to h}\,d\,\chi_h + m_{l\to h}\,d\,\chi_l}_{\text{Distributive Income}_h} + \underbrace{\chi_h\,d\,m_{h\to h} + \chi_l\,d\,m_{l\to h}}_{\text{Income Centrality}_h}$$

Substitution Effects $d\,\beta_r$ | Low skill income share variation $d\,\Lambda_l$ | Labor Wedge variation $d\,log\,\Gamma_r$

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We know: $d\,\chi_h + d\,\chi_l = 0$ & $d\,m_{r\to h} = (f_{H\to h} - f_{L\to h})\,d\,\beta_r$

Substitution Effects $d\,\beta_r$ | Low skill income share variation $d\,\Lambda_l$ | Labor Wedge variation $d\,log\,\Gamma_r$

Comparative Statics

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Take total derivative

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We know: $d\,\chi_h + d\,\chi_l = 0 \;\&\; d\,m_{r\to h} = (f_{H\to h} - f_{L\to h})\,d\,\beta_r$

$$d\,\Lambda_h = \underbrace{\overbrace{(m_{h\to h} - m_{l\to h})}^{\overbrace{(\beta_h-\beta_l)}^{\geq\ 0}\overbrace{(\alpha_H-\mu_L)}^{\geq\ 0}}\,d\,\chi_h}_{\text{Distributive Income}_h} + \underbrace{\overbrace{(f_{H\to h} - f_{L\to h})}^{\overbrace{(\alpha_H-\mu_L)}^{\geq\ 0}}\sum\chi_r\,d\,\beta_r}_{\text{Income Centrality}_h}$$

Substitution Effects $d\,\beta_r$ | Low skill income share variation $d\,\Lambda_l$ | Labor Wedge variation $d\,log\,\Gamma_r$

Baqaee & Fahri (2020)

A.

$$d\ log\ Y = d\ log\ GDP - d\ log\ P_Y$$
$$= \mathbf{d\ log\ TFP} + \sum_{r \in \{h,l\}} \tilde{\boldsymbol{\Lambda}}_{\boldsymbol{r}}\ d\ log\ L_r$$

Intermediate Steps

Baqaee & Fahri (2020)

A.

$$d\log Y = d\log GDP - d\log P_Y$$
$$= d\log TFP + \sum_{r\in\{h,l\}} \tilde{\Lambda}_r \, d\log L_r$$

B.

Intermediate Steps

$$d\log TFP = \underbrace{\lambda_H \, d\log A_H + \lambda_L \, d\log A_L}_{\text{Technology}}$$
$$+ \underbrace{\lambda_H \, d\log \mu_H + \lambda_L \, d\log \mu_L}_{\text{Competitiveness}}$$
$$- \underbrace{\left(\tilde{\Lambda}_h \, d\log \Lambda_h + \tilde{\Lambda}_l \, d\log \Lambda_l\right)}_{\text{Misallocation}}$$

Without distortions → Hulten (1978)

Distortion Centralities δ

$$\textit{Misallocation} = \tilde{\Lambda}_h \, d \log \Lambda_h + \tilde{\Lambda}_l \, d \log \Lambda_l$$
$$= \frac{\widetilde{\boldsymbol{\Lambda}}_h}{\boldsymbol{\Lambda}_h} \, d \Lambda_h + \frac{\widetilde{\boldsymbol{\Lambda}}_l}{\boldsymbol{\Lambda}_l} \, d \Lambda_l$$
$$= \boldsymbol{\delta}_h \, d \Lambda_h + \boldsymbol{\delta}_l \, d \Lambda_l$$

Distortion Centralities δ

$$\textit{Misallocation} = \tilde{\Lambda}_h \, d \log \Lambda_h + \tilde{\Lambda}_l \, d \log \Lambda_l$$
$$= \frac{\widetilde{\boldsymbol{\Lambda}}_h}{\boldsymbol{\Lambda}_h} \, d \Lambda_h + \frac{\widetilde{\boldsymbol{\Lambda}}_l}{\boldsymbol{\Lambda}_l} \, d \Lambda_l$$
$$= \boldsymbol{\delta}_h \, d \Lambda_h + \boldsymbol{\delta}_l \, d \Lambda_l$$

δ measures how **undervalued** a worker is

$\boldsymbol{\delta_l} - \boldsymbol{\delta_h} > 0$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textit{Misallocation} = \underbrace{(M_h - M_l)\, d\,\chi_h}_{\text{1. Distributive}} + \underbrace{(F_H - F_L) \sum \chi_r\, d\,\beta_r}_{\text{2. Final Demand}}$$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textit{Misallocation} = \underbrace{(M_h - M_l)\, d\,\chi_h}_{\textbf{1. } \textit{Distributive}} + \underbrace{(F_H - F_L) \sum \chi_r\, d\,\beta_r}_{\textbf{2. } \textit{Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M

$$M_r = m_{r\to h}\,\delta_h + m_{r\to l}\,\delta_l$$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\mathit{Misallocation} = \underbrace{(M_{\boldsymbol{h}} - M_{\boldsymbol{l}})\,\boldsymbol{d}\,\boldsymbol{\chi_h}}_{\text{1. Distributive}} + \underbrace{(F_{\boldsymbol{H}} - F_{\boldsymbol{L}})\sum \chi_r\,\boldsymbol{d}\,\boldsymbol{\beta_r}}_{\text{2. Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M

$$M_{\boldsymbol{r}} = m_{r\to h}\,\delta_h + m_{r\to l}\,\delta_l$$

Revenue Centrality F

$$F_{\boldsymbol{i}} = f_{i\to h}\,\delta_h + f_{i\to l}\,\delta_l$$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textit{Misallocation} = \underbrace{(M_h - M_l)\,\mathbf{d}\,\chi_h}_{\text{1. Distributive}} + \underbrace{(F_H - F_L)\sum \chi_r\,\mathbf{d}\,\beta_r}_{\text{2. Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M

$$M_r = m_{r\to h}\,\delta_h + m_{r\to l}\,\delta_l$$

Revenue Centrality F

$$F_i = f_{i\to h}\,\delta_h + f_{i\to l}\,\delta_l$$

1. M_r is high for households that consume from relatively competitive supply chains that demand workers with high δ
2. F_i is high for firms that operate in relatively competitive supply chains and directly or indirectly demand high δ workers

Distributive ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(M_h - M_l)\ d\,\chi_h}_{Distributive} \qquad d\,\chi_h = \frac{(\alpha_H - \alpha_L)}{2}\, d\,\lambda_H$$

Constant a

Distributive ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{\left(M_h - M_l\right)\ d\,\chi_h}_{\text{Distributive}} \qquad d\,\chi_h = \frac{(\alpha_H - \alpha_L)}{2}\ d\,\lambda_H$$

$$\overbrace{M_h - M_l}^{\geq\ 0} = \overbrace{(\mu_H - \mu_L)}^{\geq\ 0}\ \overbrace{(\beta_h - \beta_l)}^{\geq\ 0} \times \underbrace{\left[\delta_l + (\alpha_H - \alpha_L)\,(\alpha_H\mu_H - \alpha_L\mu_L)\,a\right]}_{>\ 0}$$

Constant a

Distributive $\uparrow$ $\rightarrow$ Misallocation $\uparrow$ $\rightarrow$ TFP $\downarrow$

$$\underbrace{\left(M_h - M_l\right) \, d\,\chi_h}_{\text{Distributive}} \qquad d\,\chi_h = \frac{\left(\alpha_H - \alpha_L\right)}{2} \, d\,\lambda_H$$

$$\overbrace{M_h - M_l}^{\geq \ 0} = \overbrace{\left(\mu_H - \mu_L\right)}^{\geq \ 0} \overbrace{\left(\beta_h - \beta_l\right)}^{\geq \ 0} \times \underbrace{\left[\delta_l + \left(\alpha_H - \alpha_L\right)\left(\alpha_H \mu_H - \alpha_L \mu_L\right) \boldsymbol{a}\right]}_{> \ 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Expenditure Redistribution	✗	✗	✗

Constant a

Final Demand $\uparrow$ $\rightarrow$ Misallocation $\uparrow$ $\rightarrow$ TFP $\downarrow$

$$\underbrace{(F_{\boldsymbol{H}} - F_{\boldsymbol{L}}) \sum \chi_r \, \boldsymbol{d} \, \beta_{\boldsymbol{r}}}_{\textit{Final Demand}}$$

Final Demand ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(F_{\boldsymbol{H}} - F_{\boldsymbol{L}}) \sum \chi_r \, \boldsymbol{d} \, \boldsymbol{\beta_r}}_{\text{Final Demand}}$$

$$\overbrace{F_{\boldsymbol{H}} - F_{\boldsymbol{L}}}^{\geq \quad 0} = \overbrace{(\mu_H - \mu_L)}^{\geq \quad 0}$$

$$\times \underbrace{[\delta_I + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \, \boldsymbol{a}]}_{> \quad 0}$$

Final Demand ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{\left(F_H - F_L\right) \sum \chi_r \, \boldsymbol{d} \, \boldsymbol{\beta_r}}_{\textit{Final Demand}}$$

$$\overbrace{F_H - F_L}^{\geq \quad 0} = \overbrace{\left(\mu_H - \mu_L\right)}^{\geq \quad 0}$$

$$\times \underbrace{\left[\delta_l + \left(\alpha_H - \alpha_L\right)\left(\alpha_H \mu_H - \alpha_L \mu_L\right) \boldsymbol{a}\right]}_{> \quad 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Final Demand Recomposition	✗	✓	✓

Requirements in Heterogeneity

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
1. **Expenditure Redistribution**	✗	✗	✗
2. **Final Demand Recomposition**	✗	✓	✓

Representative Household

Assume instead a representative household

$$\underset{Y,L,C_H,C_L,L_h,L_l}{Max} \quad U(Y,L) \quad s.t. \quad Y = Q(C_H, C_L),$$

$$\begin{aligned} p_Y\, Y &= p_H\, C_H + p_L\, C_L \\ &\leq w_h\, L_h + w_l\, L_l + (1-\mu_H)\, p_H\, y_H + (1-\mu_L)\, p_L\, y_L \end{aligned}$$

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The **first-order conditions** imply that

$$\boldsymbol{\delta_h} = \boldsymbol{\delta_l} = \Lambda^{-1} \qquad\qquad \Lambda = \Lambda_h + \Lambda_l$$

The effects from one additional percentage point of labor income share on TFP are equalized

Misallocation under a Representative Household

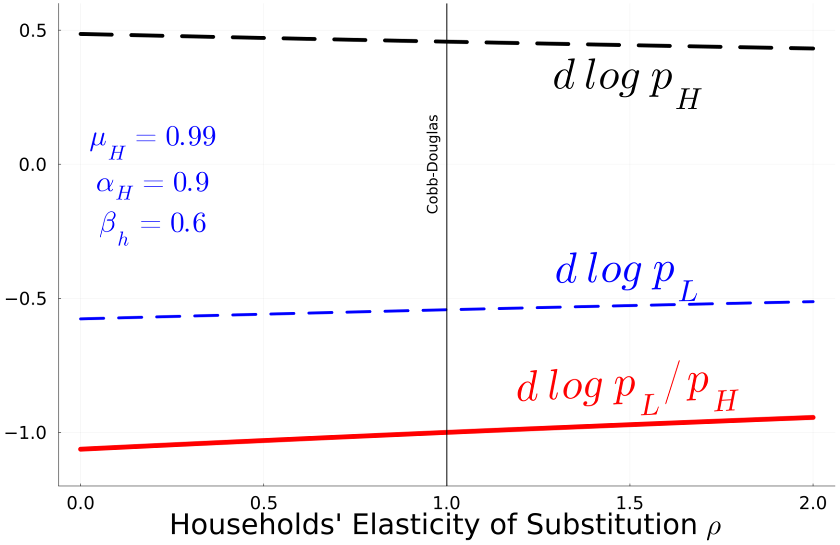
$$\textbf{From } \delta_h = \delta_l = \Lambda^{-1}$$

$$\delta_h \, d\,\Lambda_h + \delta_l \, d\,\Lambda_l = \frac{d\,\Lambda_h + d\,\Lambda_l}{\Lambda} = d\,log\,\Lambda$$

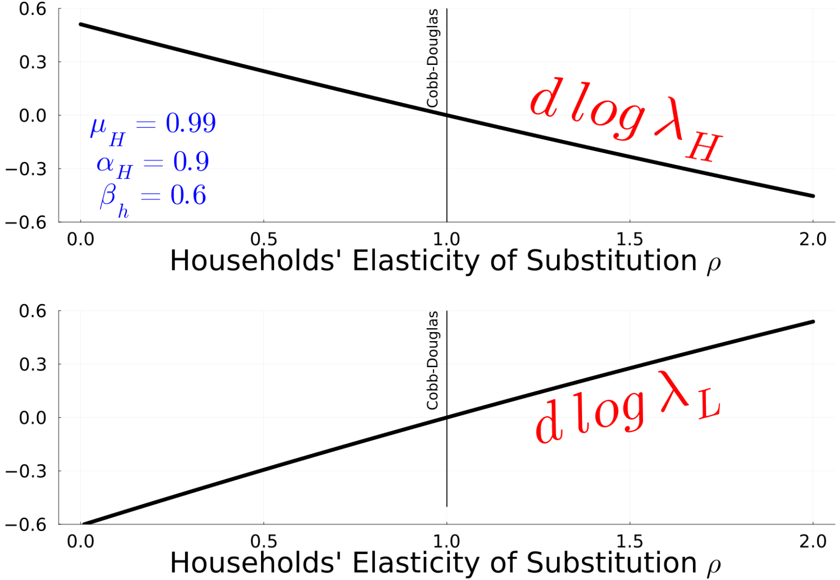
Track only one element of the distribution!

Additional Contribution Relative to HANK models

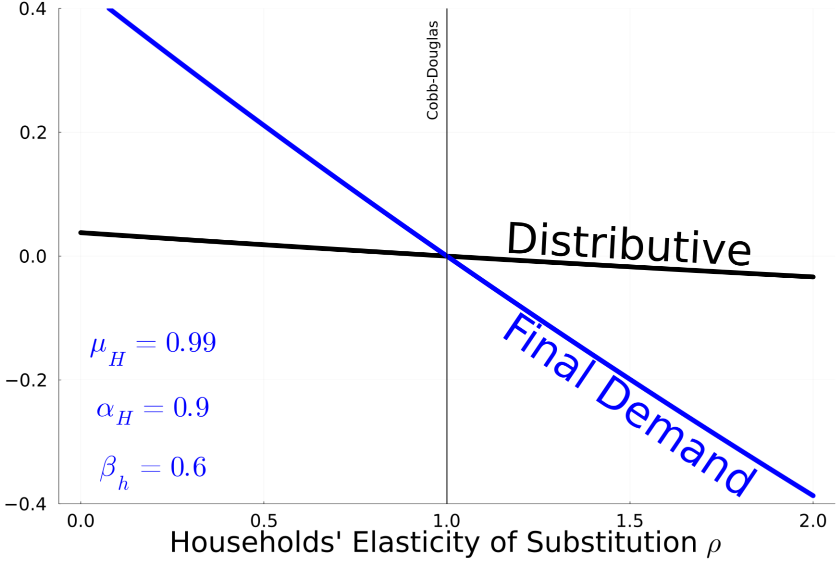
$p_L \downarrow$ & $p_H \uparrow$



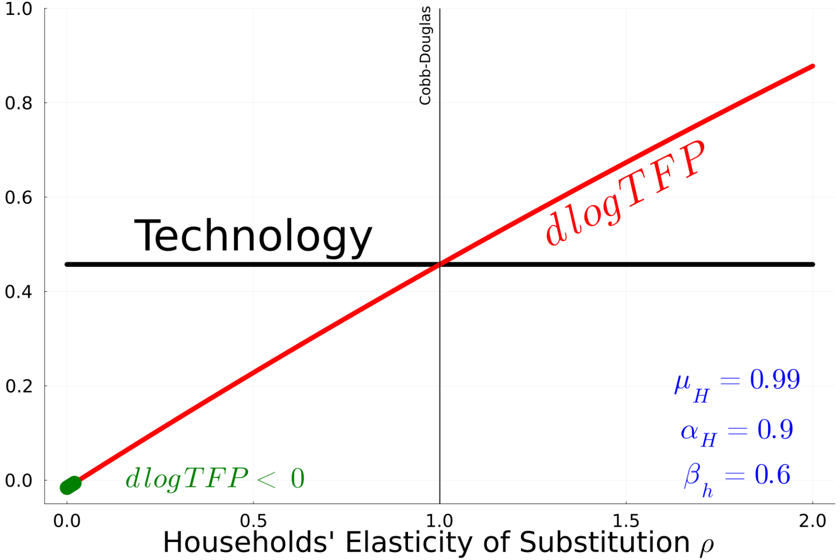
Under $\rho > 1$: $\lambda_L \uparrow$ & $\lambda_H \downarrow$



Under $\rho > 1$: Distributive $\downarrow$ & Final Demand $\downarrow$



Under $\rho > 1$: Misallocation ↓ & *d log TFP* $> \lambda_L$



In the Paper...

- General Non-Parametric CRS model for production & consumption
- General Input-Output Networks
- General Equity Distribution

General Income Centrality General Decomposition for Misallocation

In the Paper...

- General Non-Parametric CRS model for production & consumption
- General Input-Output Networks
- General Equity Distribution

Additional Channels

1. $\mu \uparrow$ & stronger for sectors with high λ_i F_i
2. $\alpha \uparrow$ for high δ workers & stronger if μ_i λ_i high
3. **Intermediate demand** $\uparrow$ on sectors with high F_i

General Income Centrality General Decomposition for Misallocation

Data Requirements

3 Types of Money Flows...

1. Household-to-Firm: Final consumption
2. Firm-to-Firm: Intermediate inputs
3. Firm-to-workers: Labor market

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1. Household-to-Firm: Final consumption
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Measures of Shocks...

A. Productivity shocks
B. Markdown shocks
C. Distributional variations

Data for money flows from 1997 to 2021

Household to Firm

1. State level **P**ersonal **C**onsumption **E**xpenditure (BEA)

$\beta_{state,\, industry}$: $\begin{cases} \text{PCE provides expenditure on types of goods} \\ \text{IO Make matrix: type of good} \rightarrow \text{industry} \end{cases}$

Data for money flows from 1997 to 2021

Household to Firm

1. State level **P**ersonal **C**onsumption **E**xpenditure (BEA)

$\beta_{state,\, industry}$: PCE provides expenditure on types of goods; IO Make matrix: type of good→industry

Firm to Firm

2. **I**nput-**O**utput tables (BEA) for 66 NAICS industries

$$\mu_i = \frac{\text{Total Cost}_i}{\text{Sales}_i}$$

$$\text{Intermediate Intensity}_{ij} = \frac{p_j\, x_{ij}}{\text{Total Cost}_i}$$

$$\text{Total Cost}_i = \text{Labor Costs}_i + \text{Intermediate Cost}_i$$

Firm-to-household

3. County Business Patterns (Census)

Industry specific **geographic (state)** bias in labor

Antisupression Algorithm Missing Private Employment

Firm-to-household

3. County Business Patterns (Census)

Industry specific **geographic (state)** bias in labor

Antisupression Algorithm | Missing Private Employment

4. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

Firm-to-household

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Antisupression Algorithm Missing Private Employment

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From **3** & **4** $\rightarrow$ industry specific heterogeneity by worker type. Worker type comes from State & Occupational interactions H =38,189 ($\approx$ 1.5 bill m's)

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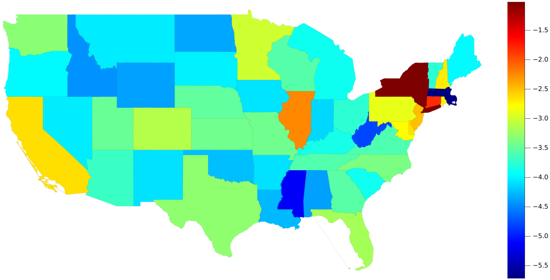
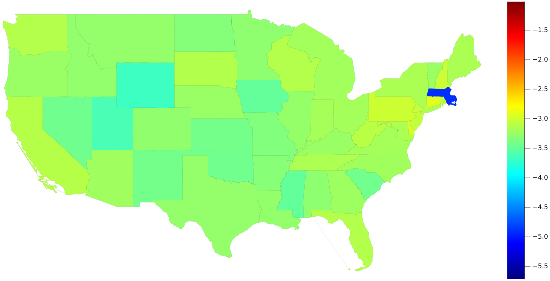
e.g. Finance's labor demand intensity for economists in Maine

$$\alpha_{ir} \propto \underbrace{\text{Finance's share of labor expenditure in Maine}}_{\text{Spatial Demand (CBP)}} \times \underbrace{\text{Finance's share of labor expenditure in economists}}_{\text{Occupational Demand (OEWS)}} \times \underbrace{\text{Maine's share of labor income from economists}}_{\text{Occupational Supply (OEWS)}}.$$

Financial Sector in Economists



Ambulatory Health in Dentists



Data for Shocks

A. **I**ndustry **L**evel **P**roduction **A**ccounts (BEA)

$$d\ log\ A_i$$

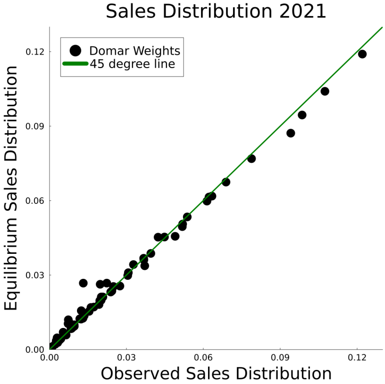
B. **I**nput **O**utput Tables

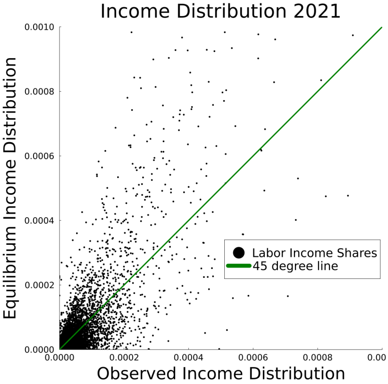
$$\mu_i \longrightarrow d\ log\ \mu_i$$

C. **CBP** + **OEWS**

$$\Lambda_r \longrightarrow d\ log\ \Lambda_r$$

Moments with Heterogeneous Households





	Observed λ on
Model λ	1.021^{***} (0.003)
R^2	0.981
N	1,650

	Observed Λ on
Model Λ	0.435^{***} (0.0015)
R^2	0.682
N	38,189

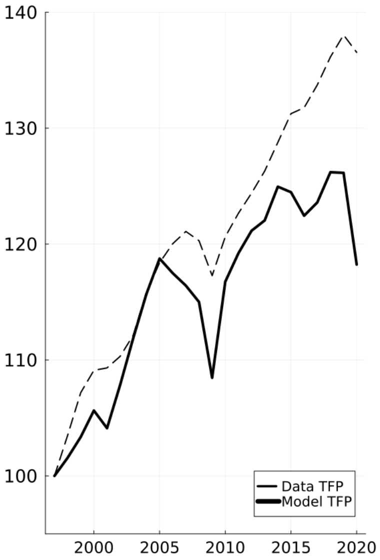
Moments under different RH

Implementation

$$\boldsymbol{d \log TFP_t} = \underbrace{\sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1}\, d \log A_{i,t}}_{\text{Technology}_{\boldsymbol{t}}} + \underbrace{\sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1}\, d \log \mu_{i,t}}_{\text{Competitiveness}_{\boldsymbol{t}}} - \underbrace{\sum_r \widetilde{\boldsymbol{\Lambda}}_{r,t\text{-}1}\, d \log \Lambda_{r,t}}_{\text{Misallocation}_{\boldsymbol{t}}}$$

Growth Decomposition Covariance Decomposition

R^2 rises from 5% to 50% with IO Networks



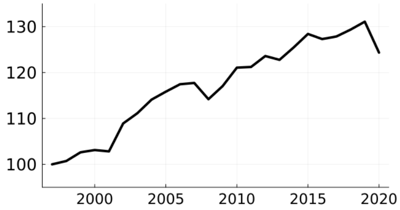
Observed d log TFP on	Without Networks	With Networks
	(1)	**(2)**
$d \log \widehat{TFP}$	-0.265 (0.264)	0.311*** (0.069)
$Adj\ R^2$	0.048	0.499
	(3)	**(4)**
Technology	0.847*** (0.289)	0.413*** (0.082)
Competitive	0.986 (0.695)	0.342*** (0.054)
Misallocation	-0.105 (0.360)	0.0168 (0.125)
$Adj\ R^2$	0.562	0.732

Alternative Without Intermediate Alternative With Intermediate

$$d\ log\ TFP = \text{Technology} + \text{Competitiveness} - \text{Misallocation}$$

Technology ↑

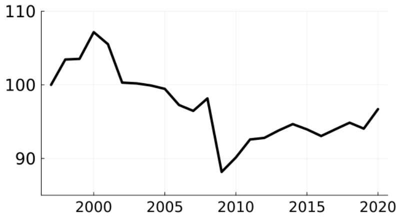
$$\sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \, d \log A_i$$



Between 1997 and 2020	
Oil & gas extraction	-11.1%
Computer & electronic	-6.6%

Competitiveness ↓

$$\sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \, d \log \mu_i$$



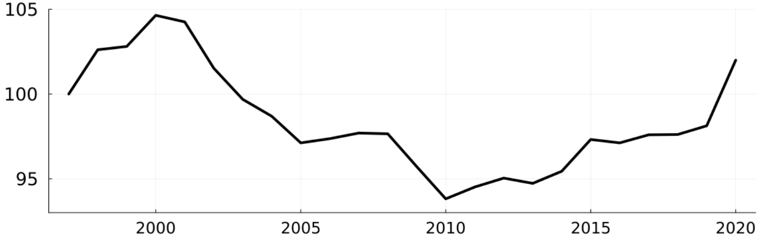
Between 1997 and 2020	
Credit intermediation	4.1%
Between 2002 and 2009	
Oil & gas extraction	6.6%

Technological Sources

Competitiveness Sources

Without Misallocation↑ after 2009, TFP↑ 7.5%

- Misallocation↓ between 2001 and 2010 by **-8.2%**
- Misallocation↑ between 2010 and 2020 by **7.5%**



Increasing profit margins

- Oil & gas extraction: **-1.5%**
- Computer & electronics: **-1.1%**

Increasing labor demand

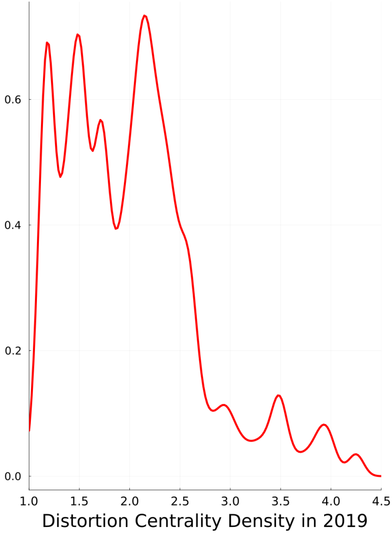
- Credit intermediation: **2.4%**

Final and intermediate demand

- Wholesale trade: **2.2%**

Sources of Misallocation (Graph) Sources of Misallocation (Counterfactual)
Industry variation from μ and Labor Demand Industry variation from Final and Intermediate Demand

Distortion Centralities δ



Lowest δ	
Nursing Assistant	1.05 - 1.08
Residential Advisor	1.06 - 1.22
Rehabilitation Counselor	1.07 - 1.08
Recreational Therapist	1.07 - 1.09
Food Server	1.07 - 1.45

Highest δ	
Teller	4.27 - 4.28
New Accounts Clerk	4.24 - 4.27
Loan Interviewer	4.21 - 4.26
Loan Officer	4.23 - 4.26
Credit Analyst	3.89 - 4.22

Normalized Nested CES

Introduced by **de La Grandville (1989)** and **Klump & de La Grandville (1989)** and as in **Baqaee & Farhi (2019,a,b, 2020, 2022)**

Normalized Nested CES

Parameters - **Atalay (2017)**, **Boehm et al. (2014)**

1. Elasticity of substitution **between worker types**: 1.0
2. Elasticity of substitution **between sectoral intermediate inputs**: 0.2
3. Elasticity of substitution **between labor and intermediate inputs**: 0.5
4. Elasticity of substitution in **final consumption**: 0.9
5. **Substitution effect** in labor supply $\zeta_h^w = 2$
6. **Income effect** in labor supply $\zeta_h^e = 2$

$$d \log TFP = \underbrace{\textbf{Technology}}_{=1\%} - Misallocation$$

Best Sectors	**d log TFP**
1. Nursing & Residential Care	1.041%
2. Social Assistance	1.039%
3. General Merchandise Store	1.029%
4. Ambulatory Health Care	1.027%
5. Hospitals	1.026%

Worst Sectors	**d log TFP**
1. Oil & Gas Extraction	0.587%
2. Primary Metals	0.610%
3. Chemical Products	0.618%
4. Mining, except Oil & Gas	0.630%
5. Utilities	0.647%

	d log TFP on			
μ_i	0.359*** (0.09)			0.207 (0.13)
λ_i		0.170 (0.56)		0.854* (0.50)
F_i			0.212*** (0.05)	0.148** (0.07)
R^2	0.20	$1e^{-3}$	0.21	0.27
N		66		

We want productivity shocks in sectors with high F_i!

$$d \log TFP = \underbrace{\textbf{Competitiveness}}_{=1\%} - Misallocation$$

Best Sectors	**d log TFP**
1. Housing	0.766%
2. Credit Intermediation	0.414%
3. Oil & Gas Extraction	0.384%
4. Furniture	0.370%
5. Mining, except Oil & Gas	0.364%

Worst Sectors	**d log TFP**
1. Nursing & Residential Care	−0.329%
2. Social Assistance	−0.303%
3. General Merchandise Store	−0.274%
4. Hospitals	−0.219%
5. Ambulatory Health Care	−0.201%

	d log TFP on			
μ_i	-0.974*** (0.12)			-0.919*** (0.18)
λ_i		1.351 (0.95)		-0.132 (0.73)
F_i			-0.427*** (0.08)	-0.046 (0.11)
R^2	0.48	0.03	0.29	0.48
N	66			

We want competition shocks in sectors with low μ_i!

Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_l)$$

Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r\left(L_h, L_l\right)$$

$$d\ log\ TFP = \sum \chi_r\ d\ log\ \mathbf{PTT}_r$$

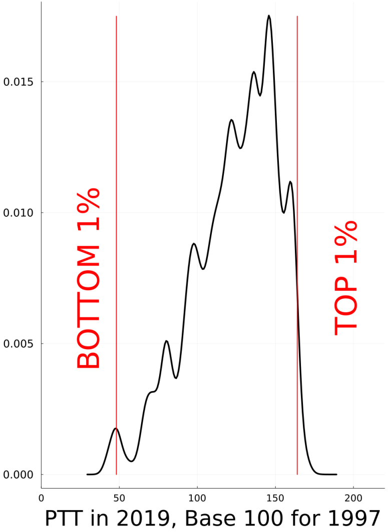
Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_l)$$

$$d \log TFP = \sum \chi_r \, d \log \mathbf{PTT}_r$$

$$d \log \mathbf{PTT}_h = \underbrace{\beta_h \, d \log A_H + (1 - \beta_h) \, d \log A_L}_{\text{Technology}_h} + \underbrace{\beta_h \, d \log \mu_H + (1 - \beta_h) \, d \log \mu_L}_{\text{Competitiveness}_h} - \underbrace{\left(\frac{\widetilde{m}_{h \leftarrow h}}{\Lambda_h} \, d\Lambda_h + \frac{\widetilde{m}_{h \leftarrow l}}{\Lambda_l} \, d\Lambda_l - d \log \chi_h \right)}_{\text{Misallocation}_h}$$

$$C_r = \textbf{Positional Terms of Trade}_r \times f_r(L_h, L_l)$$



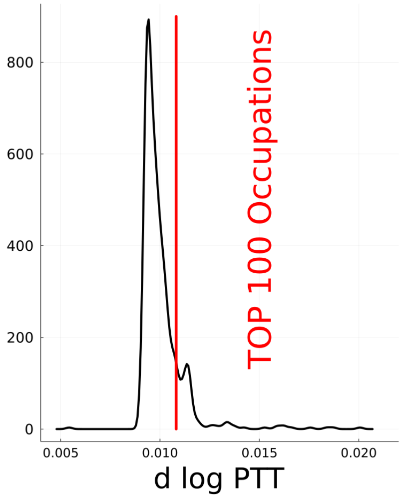
Top 1%

Occupation	
Logging Workers	37%
Computer Occupations	13%
Mathematical Sciences Occupations	10%
Compensation Managers	7%

Bottom 1%

Occupation	
Printing Workers	40%
Shoe & Leather Operator	26%
Textile Machine Operator	15%
Miscellaneous Textile	12%

Effects from more competition in Housing



Top 100 occupations	
Construction Workers	48
Painters, Carpet Installer, Tile Setter, Stonemason, Plasterer, Drywall Installer, Septic Servicer, Construction Supervisor	
Financial Specialist	7
Property appraiser, Loan Officer Credit Analyst, Financial Examiner	
Extraction Workers	7
Rock Splitter, Roof Bolter	
Woodworkers	6
Cabinetmaker, Furnite Finisher	
Installation & Maintenance	5
Heating & AC, Mobile Home Installer	

Conclusion

- First comprehensive study for joint heterogeneity in multisector economies with distortions and input-output networks
- **Theoretical Contribution** in production network + distortions + heterogeneous households:
  - Variation of the income distribution
  - Variations for TFP
  - Variations for PTT
- **Empirical Contribution**: First implementation of a production network model with household heterogeneity for the US
  - In the absence of distributional sources of misallocation, TFP would have grown 7.5% more after Great Recession

Pipeline

Working Papers

1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms

Pipeline

Working Papers

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2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies

Pipeline

Working Papers

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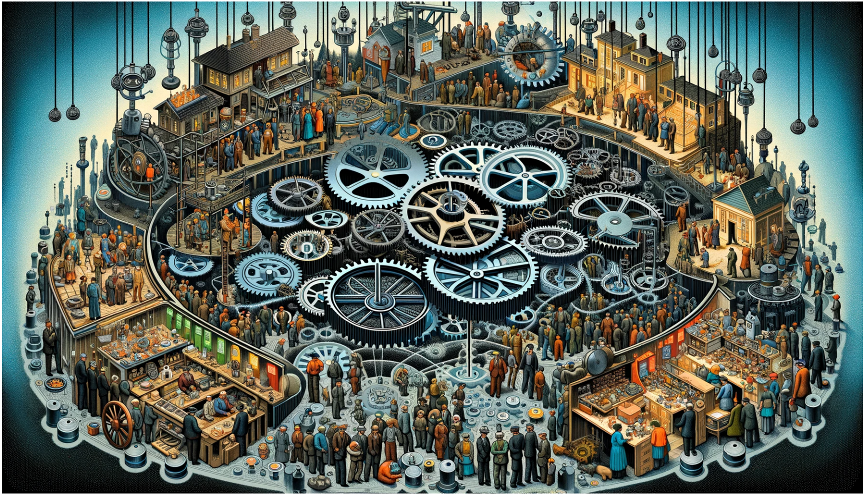
2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies

3. In **Nonlinearities in Production Networks with Distortions**, we obtain a novel second-order approximation for the aggregate TFP in a production network economy with distortions

Thank you!



Output from DALL-E after introducing title and abstract

Upper Decile vs The Rest
(Consumer Expenditure Survey 2021)

Higher Expenditure Share in

- Education: 3.4% vs 1.3%
- Entertainment: 6.5% vs 4.9%
- Pensions: 17.4% vs 9.1%
- Lodging: 2.6% vs 1.1%

Lower Expenditure Share in

- Shelter: 17.6% vs 20.5%
- Home Food: 5.9% vs 8.5%
- Utilities: 4.1% vs 7.0%
- Healthcare: 6.2% vs 8.3%

From 2004 to 2019

Income share for top quintile ↑ from 48% to 53%

Back

Literature Review

- **Disaggregated National Accounts**

  Cantillon (1756), Quesnay (1758), Leontief (1928), Meade & Stone (1941), Kuznetz (1946), Stone (1961), Andersen et al. (2022)

- **Production Networks**

  Hulten (1978), Long & Plosser (1983), Gabaix (2011), Jones (2011, 2013), Acemoglu et al. (2012), Baqaee (2018), Baqaee & Farhi (2019, 2020, 2023), Bigio & La'O (2020)

- **Growth Accounting**

  Solow (1957), Domar (1961), Jorgenson et al. (1987), Basu & Fernanld (2022), Petrin & Levinsohn (2012), Baqaee & Farhi (2020)

Back

Dixit-Stiglitz Aggregation

- Sector i has a sectoral aggregator for $z_i \in [0, 1]$

$$y_i = \left(\int y_{z_i}^{\mu_i} \; d\, z_i \right)^{\frac{1}{\mu_i}}$$

- Demand for variaties

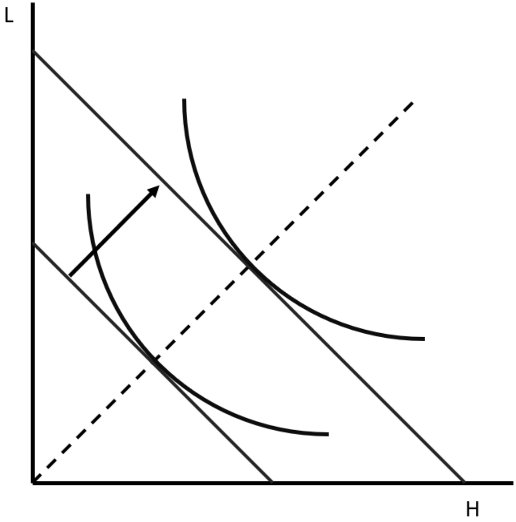
$$y_{z_i} = \left(\frac{p_i}{p_{z_i}} \right)^{\frac{1}{1-\mu_i}} \; y_i$$

- Intermediate's problem

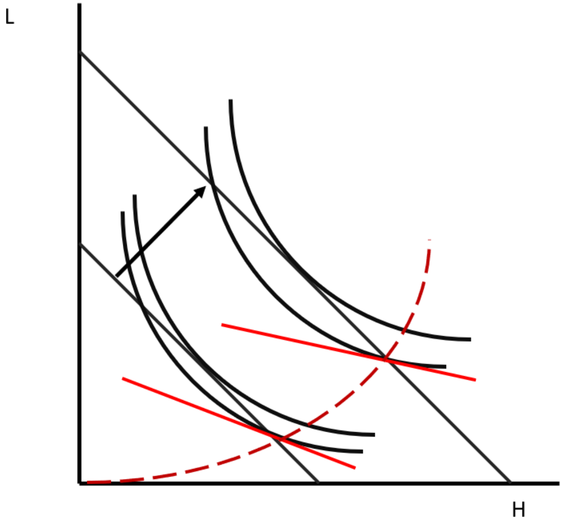
$$\underset{p_{z_i}, y_{z_i}, \ell_{z_i \mathbf{h}}, \ell_{z_i \mathbf{l}}}{Max} \quad \pi_{z_i} = p_{z_i} y_{z_i} - w_{\mathbf{h}} \, \ell_{z_i \, \mathbf{h}} - w_{\mathbf{l}} \, \ell_{z_i \, \mathbf{l}}$$

$$y_{z_i} = A_i \; \ell_{z_i \, \mathbf{h}}{}^{\alpha_i} \; \ell_{z_i \, \mathbf{l}}^{1-\alpha_i}$$

Aggregate Non-Homotheticity



Aggregate Non-Homotheticity



Equilibrium Definition

$$\boldsymbol{e = (A, \mu, \beta, \alpha)} \in \mathscr{E} \text{ into}$$

$$\vartheta \equiv \left\{ \left\{ y_i, \{\ell_{ir}, C_{ri}\}_{r\in\{h,l\}} \right\}_{i\in\{H,L\}}, \{C_r, L_r\}_{r\in\{h,l\}} \right\}$$

$$\rho \equiv \{p_H, p_L, w_h, w_l, p_h^c, p_l^c\}$$

Back

Necessary & sufficient equilibrium conditions

(ϑ, ρ) are an equilibrium iff

$$-\frac{w_b}{w_r}\frac{U_{L_r}}{U_{C_{ri}}} = \mu_i \frac{\partial y_i}{\partial \ell_{ib}} \quad i \in \{H, L\}, r, b \in \{h, l\},$$

such that $C_{ri} > 0$, and $\ell_{ib} > 0$,

and resource constraints

$$y_i(e) = C_{hi}(e) + C_{li}(e) \quad i \in \{H, L\}$$
$$L_r(e) = \ell_{Hr}(e) + \ell_{Lr}(e) \quad r \in \{h, l\}.$$

Back

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_l \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Back

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_l \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

Back

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_l \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2}\Big((1-\mu_H)\lambda_H + (1-\mu_L)\lambda_L\Big)$$

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_l \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \, \mu_H \, \lambda_H + \alpha_L \, \mu_L \, \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2}\Big((1-\mu_H)\, \lambda_H + (1-\mu_L)\, \lambda_L\Big)$$

Value Added Share

$$\tilde{\Lambda}_h = \alpha_H \, \lambda_H + \alpha_L \, \lambda_L \qquad \tilde{\Lambda}_h + \tilde{\Lambda}_l = 1$$

Back

Sales Distribution

$$\lambda_H = \overbrace{\left(\frac{1}{2 - \underbrace{(\beta_h - \beta_l)}_{\geq\ 0}\underbrace{(\alpha_H - \alpha_L)}_{\geq\ 0}}\right)}^{\theta\ \geq\ 1/2}\left(1 - \mu_L\underbrace{(\beta_h - \beta_l)}_{\geq\ 0}\underbrace{(\alpha_H - \alpha_L)}_{\geq\ 0}\right)$$

Amplification Effect

Contractionary Effect

Consumption Expenditure Distribution

$$\chi_h = \theta\left(1 - \underbrace{(\alpha_H - \alpha_L)}_{\geq\ 0}\underbrace{(\beta_h - \mu_H)}_{?}\right)$$

Back

Labor Income Distribution

$$\Lambda_h = \theta \left[\alpha_L + \mu_H \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 - \mu_L (\beta_h - \beta_l))}_{\geq 0} \right]$$

$$\Lambda_l = \theta \left[\alpha_H - \mu_H \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 + \mu_L (\beta_h - \beta_l))}_{\geq 0} \right]$$

Value-Added Distribution

Back

$$\begin{aligned} \widetilde{\Lambda}_h &= \alpha_H \lambda_H + \alpha_L \lambda_L \\ &= \theta \Big(1 - \underbrace{(\beta_h - \beta_l)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \Big(\alpha_H - \mu_H \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \Big) \Big) \end{aligned}$$

$$\widetilde{\Lambda}_l = \theta \Big(1 - \underbrace{(\beta_h - \beta_l)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \Big(\alpha_L + \mu_H \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \Big) \Big)$$

3 Effects from Distortions on Labor

1. Misallocation comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}$$

Back

3 Effects from Distortions on Labor

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2. Allocative differences ≠ Misallocation

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$$

Intuition
For the undistorted case
$\mu_H = \mu_L = 1/2$
there is a continuum
of property rights on firms

Cases

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Cases

3. Distorted Labor Supply Γ_r

$$-\frac{U_{L_r}}{U_{C_r}} = \underbrace{\frac{\Lambda_r}{\chi_r}}_{=\Gamma_r} \frac{C_r}{L_r}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L / d\,\ell_{Lr}}{d\,y_H / d\,\ell_{Hr}}}_{\textbf{Misallocation}}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1 **Case 2** **Case 3** **Case 4**

$\mu_H = \mu_L$

Symmetric π **All π for h**

Back

$\frac{\ell_{Hh}}{L_H} \neq \alpha_H$ not the same $\underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}}_{\text{Misallocation}}$

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$\mu_H = \mu_L$

Symmetric π **All π for h**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

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$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h / \chi_h)}_{\Gamma_h} \frac{C_h}{L_h} \qquad -\frac{U_{L_l}}{U_{C_l}} = \underbrace{(\Lambda_l / \chi_l)}_{\Gamma_l} \frac{C_l}{L_l}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L / d\,\ell_{Lr}}{d\,y_H / d\,\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1 — **Symmetric π**

Case 2 — $\mu_H = \mu_L$ — **All π for h**

Case 3 — $\alpha_H = \alpha_L$ — **Symmetric π**

Case 4 — $\beta_h = \beta_l$

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

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Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L / d\,\ell_{Lr}}{d\,y_H / d\,\ell_{Hr}}}_{\text{Misallocation}}$$

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$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l) \quad \frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{d\,y_L / d\,\ell_{Lr}}{d\,y_H / d\,\ell_{Hr}}$$

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Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\text{Misallocation}}$$

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Case 2 — $\mu_H = \mu_L$ — **All π for h**

Case 3 — $\alpha_H = \alpha_L$ — **Symmetric π**

Case 4 — $\beta_h = \beta_l$

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \qquad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l) \qquad \frac{\ell_{Hh}}{L_h} > \alpha_H$$

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$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h} \qquad -\frac{U_{L_l}}{U_{C_l}} = \underbrace{(\Lambda_l/\chi_l)}_{\Gamma_l} \frac{C_l}{L_l}$$

Linear Approximation in response to $d\log A_L$

- $$\lambda_H \, d\log S_H = \beta_h \chi_h \, d\log E_h + \beta_l \chi_l \, d\log E_l - (\rho - 1)\beta_h \beta_l \left(d\log A_L + (\alpha_H - \alpha_L)\, d\log \frac{w_h}{w_l} \right)$$
- $$\lambda_L \, d\log S_L = (1 - \beta_h)\chi_h \, d\log E_h + (1 - \beta_l)\chi_l \, d\log E_l + (\rho - 1)\beta_h \beta_l \left(d\log A_L + (\alpha_H - \alpha_L)\, d\log \frac{w_h}{w_l} \right)$$
- $$d\log E_r = \frac{(1 + \zeta^w)\Gamma_r}{1 + \zeta^e \Gamma_r} d\log w_r + \frac{1}{2} \frac{\sum \lambda_i (1 - \mu_i)\, d\log S_i}{(1 + \zeta^e \Gamma_r)\chi_r}$$
- $$d\log w_r = \frac{\zeta^e}{1 + \zeta^w} d\log E_r + \frac{\sum f_{ir}\lambda_i \, d\log S_i}{(1 + \zeta^w)\Lambda_r}$$

Back

Bilateral Centralities

$$\boldsymbol{\Lambda}_h = m_{h\to h}\,\chi_h + m_{l\to h}\,\chi_l$$

$$m_{r\to h} = \beta_r\, f_{H\to h} + (1-\beta_r)\; f_{L\to h}, \qquad f_{i\to h} = \alpha_i\; \mu_i$$

3 definitions for $m_{r\to h}$

1. **Partial equilibrium effect** on h's labor income from one additional **expenditure** unit from r

Back

Bilateral Centralities

$$\boldsymbol{\Lambda}_h = m_{h \to h}\, \chi_h + m_{l \to h}\, \chi_l$$

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2. **Share** of **expenditure** from r that reaches Λ_h

Back

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2. **Share** of **expenditure** from r that reaches Λ_h
3. $\{m_{h\to h}, m_{l\to h}\}$ is a **ranking** for **expenditure** relevance on Λ_h

Back

Bilateral Centralities

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2. **Share** of **expenditure** from r that reaches Λ_h
3. $\{m_{h\to h}, m_{l\to h}\}$ is a **ranking** for **expenditure** relevance on Λ_h

- Similar 3 definitions for $f_{i\to h}$ but for **revenue** of i

Back

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d\, log\, p_r^c C_r}{d\, log\, p_H} = \frac{p_H C_{rH}}{p_r^c C_r}}_{\text{Shephard's Lemma} \quad \frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r$$

In equilibrium as parameter

Back

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d\log p_r^c C_r}{d\log p_H} = \frac{p_H C_{rH}}{p_r^c C_r}}_{\text{Shephard's Lemma} \quad \frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r$$

In equilibrium as parameter

1. New equilibrium with local approximations keep α and β fixed

Substitution Effects

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1. New equilibrium with local approximations keep α and β fixed
2. Estimate β's consistent with the new equilibrium

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H} = \frac{p_H C_{rH}}{p_r^c C_r}}_{\text{Shephard's Lemma} \quad \frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r$$

In equilibrium as parameter

1. New equilibrium with local approximations keep α and β fixed
2. Estimate β's consistent with the new equilibrium

Exact delta hat - Dekle, Eaton & Kortum (2008)

$$\frac{p_H C_{rH}}{E_r} = \beta_r^\rho \left(\frac{p_r^c \overline{C}_r}{p_H \overline{C}_{rH}} \right)^{\rho - 1} \rightarrow \underbrace{d\,\beta_r = (\rho - 1)\beta_r (1 - \beta_r)\, d \log \frac{p_L}{p_H}}_{\text{Increases under substitutability when } p_L/p_H \uparrow}$$

Back

Theorem 1: labor income share variation

$$d\,\boldsymbol{\Lambda}_l = \underbrace{\overbrace{(m_{h\to l} - m_{l\to l})}^{\overbrace{(\beta_h - \beta_l)}^{\geq\ 0}\overbrace{(\boldsymbol{\mu}_H - \boldsymbol{\alpha}_H)}^{?}}\, \boldsymbol{d}\,\chi_h}_{\text{Distributive Income}_l} + \underbrace{\overbrace{(f_{H\to L} - f_{L\to l})}^{\overbrace{(\mu_H - \alpha_H)}^{?}} \sum \chi_r\, \boldsymbol{d}\,\boldsymbol{\beta_r}}_{\text{Income Centrality}_l}$$

Back

Labor Wedge

For factors with endogenous supply...

$$-\frac{U_{L_h}}{U_{C_h}} = \Gamma_h \frac{C_h}{L_h} \quad \text{with} \quad \Gamma_h = \frac{\Lambda_h}{\chi_h}$$

Proof

$d \log \Gamma_h$ - Extension of Bigio & La'O (2020)

(i) Representative Household
(ii) Around Efficient Equilibrium
$\longrightarrow$
(i) Heterogenous Households
(ii) Any Equilibrium

$$\boldsymbol{d \log \Gamma_h = d \log \Lambda_h - d \log \chi_h}$$

Proof of Theorem 1 for $d\,log\,\Gamma_h$

From goods market clearing

$$\begin{pmatrix} y_H \\ y_L \end{pmatrix} = \begin{pmatrix} C_{hH} + C_{lH} \\ C_{hL} + C_{lL} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}}\ y_H \\ \frac{(1-\beta_h)}{C_{hL}}\ y_L \end{pmatrix} = \begin{pmatrix} \frac{\beta_h}{C_{hH}}\left(C_{hH} + C_{lH}\right) \\ \frac{(1-\beta_h)}{C_{hL}}\left(C_{hL} + C_{lL}\right) \end{pmatrix}$$

From FOC and equilibrium $\beta_h \frac{\chi_h}{C_{hH}} = p_H = \beta_l \frac{\chi_l}{C_{lH}}$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}}\ y_H \\ \frac{(1-\beta_h)}{C_{hH}}\ y_L \end{pmatrix} = \begin{pmatrix} \beta_h \frac{\chi_h}{\chi_h} + \beta_l \frac{\chi_l}{\chi_h} \\ (1-\beta_h)\,\frac{\chi_h}{\chi_h} + (1-\beta_l)\,\frac{\chi_l}{\chi_h} \end{pmatrix}$$

Back

Proof of Theorem 1 for $d\,log\,\Gamma_h$

From FOC and equilibrium $-\frac{1}{\beta_h}\frac{U_{L_h}}{U_{C_h}}\frac{C_{hH}}{C_h} = \frac{w_h}{p_H} = \mu_H\,\alpha_H\frac{y_H}{\ell_{Hh}}$

$$\begin{pmatrix} \ell_{Hh} \\ \ell_{Lh} \end{pmatrix} = \begin{pmatrix} -\frac{U_{C_h}}{U_{L_h}}\,\alpha_H\,\mu_H\,y_H\,\beta_h\,\frac{C_h}{C_{hH}} \\ -\frac{U_{C_h}}{U_{L_h}}\,\alpha_L\,\mu_L\,y_L\,(1-\beta_h)\,\frac{C_h}{C_{hL}} \end{pmatrix}$$

From labor market clearing condition

$$L_h = \ell_{Hh} + \ell_{Lh} = -\frac{U_{C_h}}{U_{L_h}}C_h\begin{pmatrix}\alpha_H\,\mu_H & \alpha_L\,\mu_L\end{pmatrix}\begin{pmatrix} \frac{\beta_h}{C_{hH}}\,y_H \\ \frac{(1-\beta_h)}{C_{hH}}\,y_L \end{pmatrix}$$

$$= -\frac{U_{C_h}}{U_{L_h}}C_h\underbrace{\left(\alpha_H\,\mu_H\sum_{r\in\{h,l\}}\beta_r\frac{\chi_r}{\chi_h} + \alpha_L\,\mu_L\sum_{r\in\{h,l\}}(1-\beta_r)\,\frac{\chi_r}{\chi_h}\right)}_{=\Gamma_h}$$

Back

Δ TFP

A.

$$GDP = P_Y\,Y = p_h^c C_h + p_l^c C_l$$

$$d\,log\,GDP = \chi_h\,d\,log\,p_h^c C_h + \chi_l\,d\,log\,p_l^c C_l$$

Δ TFP

A.

$$GDP = P_Y\, Y = p_h^c C_h + p_l^c C_l$$

$$d\,log\,GDP = \chi_h\, d\,log\,p_h^c C_h + \chi_l\, d\,log\,p_l^c C_l$$

B. Divisia Index GDP deflator

$$\begin{aligned} d\,log\,P_Y &\equiv \chi_h\, d\,log\,p_h^c + \chi_l\, d\,log\,p_l^c \\ &= \tilde{\Lambda}_h\, d\,log\,w_h + \tilde{\Lambda}_l\, d\,log\,w_l \\ &\quad - \lambda_H\, d\,log\,(A_H \times \mu_H) - \lambda_L\, d\,log\,(A_L \times \mu_L) \end{aligned}$$

Back Extra Steps

Additional Steps for $d \log P_Y$

Start from

$$p_H = \frac{w_h \, \ell_{Hh} + w_l \, \ell_{Hl}}{\mu_H \, A_H \, \ell_{Hh}^{\alpha_H} \, \ell_{Hl}^{1-\alpha_H}}$$

Take first-order approximation

$$\widehat{p}_H = -\widehat{A}_H - \widehat{\mu}_H + \alpha_H \, \widehat{\alpha}_{Hh} + (1-\alpha_H) \; \widehat{\alpha}_{Hl}$$

Do the same for bundle prices

$$\widehat{p}_h^c = -\beta_h \left(\widehat{A}_H + \widehat{\mu}_H\right) - (1-\beta_h) \left(\widehat{A}_L + \widehat{\mu}_L\right) + \widetilde{\mathscr{C}}_{hh} \, \widehat{w}_h + \widetilde{\mathscr{C}}_{hl} \, \widehat{w}_l$$

Back

Distortion Centrality Heterogeneity

$$d\,\Lambda = d\,\Lambda_h + d\,\Lambda_l$$

$$\textit{Misallocation} = \overbrace{(\delta_l - \delta_h)}^{\geq\ 0}\ d\,\Lambda_l + \delta_h\ d\,\boldsymbol{\Lambda}$$

$$\delta_l - \delta_h = \overbrace{(\mu_H - \mu_L)}^{\geq\ 0}\ \overbrace{(\alpha_H - \alpha_L)}^{\geq\ 0}\ \overbrace{\boldsymbol{a}}^{>0}$$

$$\boldsymbol{a} = \frac{1 + (\beta_h - \beta_l)(\alpha_H - \alpha_L)\left(1 + \overbrace{\mu_H\mu_L(\beta_h - \beta_l)(\alpha_H - \alpha_L)}^{\boldsymbol{b}}\right)}{(\alpha_H\mu_H + \alpha_L\mu_L - \boldsymbol{b})(\alpha_H\mu_L + \alpha_L\mu_H - \boldsymbol{b})}$$

Back

Constant $\boldsymbol{a}$

$$\boldsymbol{a} = \frac{1 + (\beta_h - \beta_l)(\alpha_H - \alpha_L)\left(1 + \overbrace{\mu_H \mu_L (\beta_h - \beta_l)(\alpha_H - \alpha_L)}^{\boldsymbol{b}}\right)}{(\alpha_H \mu_H + \alpha_L \mu_L - \boldsymbol{b})(\alpha_H \mu_L + \alpha_L \mu_H - \boldsymbol{b})}$$

Back

Alternatives: Income distribution → Output

In Auclert & Rognlie (2020)

- Negative Correlation between income and MPC
- $+$ Wage rigidities
- Aggregate Demand $\downarrow$ & Keynesian unemployment

In my model

- Static model, MPC equals 1
- No nominal rigidities
- Supply effect due to Misallocation

Back

Income Centrality

$$\text{Income Centrality}_h = \overbrace{\sum_{i\in\mathcal{N}} \mu_i \lambda_i \, d\,\widetilde{\Omega}^{\ell}_{ih}}^{\text{Labor Demand Recomposition}_h} + \overbrace{\sum_{i\in\mathcal{N}} f_{ih} \lambda_i \, d\log \mu_i}^{\text{Competitive Income}_h}$$

$$+ \overbrace{\sum_{i\in\mathcal{N}} f_{ih} \sum_{b\in\mathcal{H}} \chi_b \, d\,\beta_{bi}}^{\text{Final Demand Recomposition}_h} + \overbrace{\sum_{i\in\mathcal{N}} f_{ih} \sum_{j\in\mathcal{N}} \mu_j \lambda_j \, d\,\widetilde{\Omega}^{x}_{ji}}^{\text{Intermediate Demand Recomposition}_h}$$

Share of sector i's revenue reaching worker h's income (f_{ih})

Sales Share $\lambda_j = \frac{\text{Sales}_j}{GDP}$

Back

Misallocation Decomposition

1. Misallocation ↑ as expenditure rises for households with high M_h
2. Misallocation ↑ as labor demand for workers with high δ rises
3. Misallocation ↑ as profit margins fall in sector with high F_i

$$\overbrace{\sum_{h\in\mathcal{H}} M_h \, d\,\chi_h}^{Distributive} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i \lambda_i \sum_{h\in\mathcal{H}} \delta_h \, d\,\widetilde{\Omega}^{\ell}_{ih}}^{Labor\ Demand} + \overbrace{\sum_{i\in\mathcal{N}} \lambda_i F_i \, d\log\mu_i}^{Competitive}$$

$$+ \overbrace{\sum_{h\in\mathcal{H}} \chi_h \sum_{i\in\mathcal{N}} F_i \, d\,\beta_{hi}}^{Final\ Demand} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i \lambda_i \sum_{j\in\mathcal{N}} F_j \, d\,\widetilde{\Omega}^{x}_{ij}}^{Intermediate\ Demand}$$

4. Misallocation ↑ as demand of goods ↑ from sectors with high F_i

Back

Antisupression Algorithm

1. Significant portion of data supressed to protect confidentiality
2. Since 2007 non-suppressed observations have a random noise infusion multiplier
3. Use information available due to to the industrial and geographical hierarchical nature → manifold of bound and aggregation constraints across hierarchies
4. Two gold standards:
   i. Two-staged algorithm from Isserman & Westervelt (2006)
   ii. Linear programming solution from Eckert et al. (2020)
5. These two methods estimate the number of workers, not their compensation. I develop a three-staged algorithm that starting from the guess Eckert et al. (2020) extends Isserman & Westervelt (2006) to the estimation of labor compensation

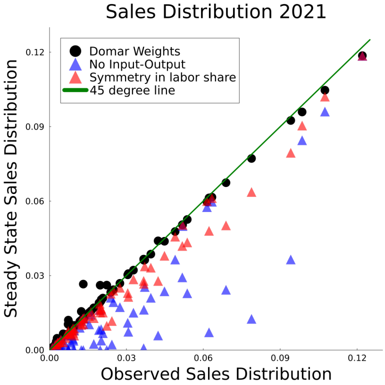
Back

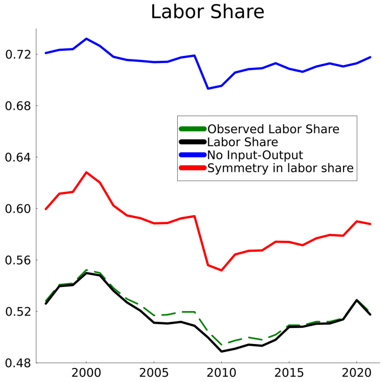
Missing Private Employment

1. The CBP only covers some forms of private employment
2. It does not include workers in
   - Agriculture production
   - Railroads
   - Government
   - Private household
3. To fill this gap, I use the BEA's Regional Economic Information System to obtain state-level employment and income measures for agricultural and production workers
4. Data sources for REIS are the Quarterly Census of Employment and Statistics from the BLS
5. Main limitation from REIS is that it is only provided at the 2-digit NAICS level

Back

Moments under Representative Household





	R^2 on sales distribution	R^2 on labor cost share
Base Model	0.994	0.981
No Input-Output	0.730	0.733
Symmetry in Labor	0.978	0.933

Back

Contribution from each component

Table: Counterfactual TFP Growth Differential in the Absence of Components

A. Between 1997 and 2020

Technology	***Competitiveness***	***Misallocation***
-23.4%	2.5%	2.8%

B. Between 2002 and 2009

Technology	***Competitiveness***	***Misallocation***
-13.0%	**19.3%**	**-8.2%**

C. Between 2010 and 2020

Technology	***Competitiveness***	***Misallocation***
-6.3%	**-9.8%**	**7.6%**

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Covariance Decomposition

Table: Covariance Decomposition

A. Between 1997 and 2020

Technology	***Competitiveness***	***-Misallocation***
44.4%	**34.6%**	**21.0%**

B. Between 2002 and 2009

Technology	***Competitiveness***	***-Misallocation***
28.3%	**61.2%**	10.5%

C. Between 2010 and 2020

Technology	***Competitiveness***	***-Misallocation***
58.1%	4.9%	**37.0%**

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Model without Intermediate Inputs

	Rep. Household		*Occupation*		*County*		*State & Ocupation*	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
$d\log TFP$	0.523		0.503		0.388		-0.265	
	(0.366)		(0.350)		(0.316)		(0.264)	
Technology		1.341***		0.789***		0.796***		0.847***
		(0.308)		(0.267)		(0.266)		(0.289)
Competitiveness		0.212		0.320		0.454		0.986
		(0.423)		(0.489)		(0.373)		(0.695)
Misallocation		0.573*		0.450		0.335		-0.105
		(0.329)		(0.437)		(0.315)		(0.360)
Intercept	0.012***	0.011***	0.012***	0.012***	0.013***	0.012***	0.015***	0.012***
	(3.2e-3)	(2.0e-3)	(3.2e-3)	(2.2e-3)	(3.2e-3)	(2.1e-3)	(3.0e-3)	(2.2e-3)
Observations	22							
N	66							
H	1		750		3,136		38,190	
R^2	9.2%	71.4%	9.35%	62.4%	7.00%	62.5%	4.8%	60.4%
Adj. R^2	9.2%	68.4%	9.35%	58.4%	7.00%	58.6%	4.8%	56.2%

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Model with Intermediate Inputs

	Rep. Household (1)	(2)	*Occupation* (3)	(4)	*County* (5)	(6)	*State & Ocupation* (7)	(8)
dlogTFP	0.370***		0.311***		0.316***		0.311***	
	(0.072)		(0.069)		(0.065)		(0.069)	
Technology		0.478***		0.414***		0.416***		0.413***
		(0.097)		(0.081)		(0.083)		(0.082)
Competitiveness		0.398***		0.341***		0.350***		0.342***
		(0.062)		(0.054)		(0.053)		(0.054)
Misallocation		0.074		0.172		0.164		0.168
		(0.138)		(0.125)		(0.135)		(0.125)
Intercept	0.010***	0.009	0.011***	0.010***	0.011***	0.010***	0.011***	0.010***
	(2.1e-3)	(2.0e-3)	(2.2e-3)	(1.8e-3)	(2.1e-3)	(1.9e-3)	(2.3e-3)	(1.9e-3)
Observations	22							
N	66							
H	1		750		3,136		38,190	
R^2	56.9%	75.2%	49.9%	75.8%	54.0%	75.4%	49.9%	75.5%
Adj. R^2	56.9%	72.6%	49.9%	73.3%	54.0%	72.8%	49.9%	73.2%

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Technological Sources

A. Between 1998 and 2020

1	Oil & gas extraction	-11.11%
2	Computer & electronics	-6.64%
3	Telecommunications	-2.85%
4	Computer systems design	-2.30%
5	Administrative services	-1.74%
6	Insurance carriers	-1.45%
7	Farms	-1.34%
8	Primary metals	-1.28%
	⋮	
63	Rental & leasing	1.41%
64	Credit intermediation	1.77%
65	Chemical Products	2.84%
66	Construction	2.87%

C. Between 2010 and 2020

1	Oil & gas extraction	-5.41%
2	Computer systems design	-1.29%
3	Management of companies	-1.26%
4	Housing	-1.14%
5	Other real estate	-1.01%
	⋮	
64	Air transportation	1.03%
65	Chemical products	1.90%
66	Credit intermediation	2.73%

B. Between 2002 and 2009

1	Oil & gas extraction	-5.35%
2	Computer & electronics	-2.84%
3	Telecommunications	-2.27%
4	Utilities	-1.92%
5	Administrative services	-1.06%
	⋮	
66	Construction	1.76%

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Competitiveness Sources

A. Between 1998 and 2020

1	Housing	-1.65%
2	Insurance carriers	-1.53%
3	Misc. professional services	-1.10%
4	Other services	-0.89%
	⋮	
63	Publishing industries	0.80%
64	Computer and electronics	1.34%
65	Chemical products	2.57%
66	Credit intermediation	4.10%

C. Between 2010 and 2020

1	Oil & gas extraction	-6.34%
2	Housing	-3.09%
3	Insurance carriers	-0.98%
4	Misc. professional services	-0.87%
5	Administrative services	-0.82%
	⋮	
64	Primary metals	0.80%
65	Chemical products	0.84%
66	Credit intermediation	3.86%

B. Between 2002 and 2009

1	Securities & investment	-0.86%
	⋮	
58	Wholesale trade	0.92%
59	Publishing industries	0.93%
60	Internet, & inf. services	0.99%
61	Chemical products	1.35%
62	Telecommunications	1.43%
63	Computer and electronics	1.48%
64	Housing	1.57%
65	Utilities	1.87%
66	Oil & gas extraction	6.59%

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Sources of Misallocation

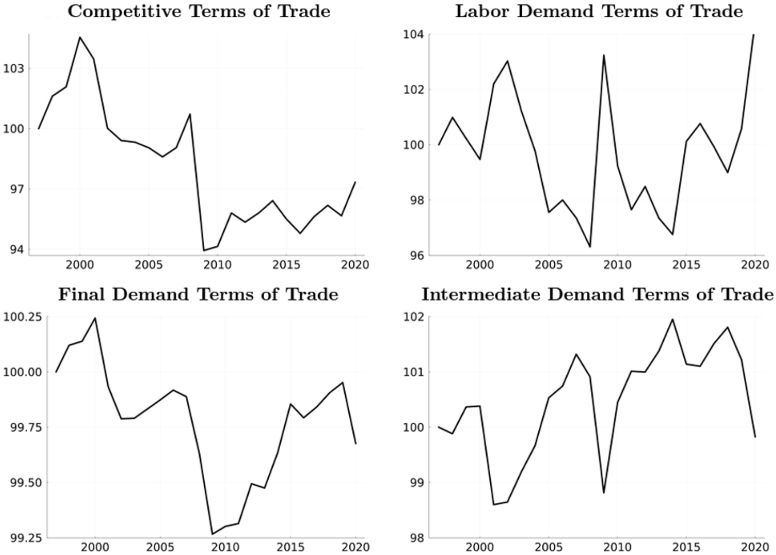

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Table 11: Counterfactual TFP Growth Differential in the Absence of Misallocation Components

A. Between 1998 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	-3.4%	6.3%	0.4%	-1.3%
Occupation	0%	-5.9%	15.1%	-2.0%	-4.2%
County	0.1%	-5.2%	14.2%	-0.9%	-4.4%
State & Occupation	0.1%	-5.9%	15.6%	-2.6%	-4.5%

B. Between 2002 and 2009

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	-9.3%	1.1%	-0.9%	-0.2%
Occupation	0%	-11.0%	3.4%	-1.9%	-0.8%
County	0.1%	-10.4%	3.4%	-0.7%	-1.0%
State & Occupation	0.1%	-11.1%	3.4%	-2.0%	-0.9%

C. Between 2010 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	3.9%	1.2%	1.7%	0.9%
Occupation	0%	2.9%	7.2%	0.2%	-1.8%
County	0.1%	3.0%	3.5%	2.1%	-1.5%
State & Occupation	0.1%	2.8%	7.4%	-0.1%	-1.7%

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Table 12: Counterfactual TFP Growth Without Sectoral Competitive TT

A. Between 1998 and 2020

1	Credit intermediation	-2.16%
2	Chemical products	-1.06%
3	Computer & electronics	-0.98%
4	Publishing industries	-0.80%
5	Internet & inf. services	-0.69%
	⋮	
64	Insurance carriers	0.77%
65	Other services	0.81%
66	Misc. professional services	0.87%

B. Between 2002 and 2009

1	Oil & gas extraction	-1.46%
2	Computer & electronics	-1.11%
3	Internet & inf. services	-1.01%
4	Wholesale trade	-0.92%
5	Telecommunications	-0.86%
6	Utilities	-0.84%
7	Publishing industries	-0.82%

C. Between 2010 and 2020

1	Credit intermediation	-2.0%
2	Securities & investment	-0.52%
	⋮	
64	Administrative services	0.62%
65	Misc. professional services	0.70%
66	Oil & gas extraction	1.91%

Table 13: Counterfactual TFP Growth Without Sectoral Labor Demand TT

A. Between 1998 and 2020

1	Wholesale trade	-1.62%
2	Insurance carriers	-1.61%
3	Other retail	-1.07%
	⋮	
61	Utilities	0.69%
62	Computer systems design	0.82%
63	Publishing industries	1.34%
64	Oil & gas extraction	1.79%
65	Computer & electronics	2.28%
66	Credit intermediation	2.40%

B. Between 2002 and 2009

1	Securities & investment	-0.96%
	⋮	
64	Computer & electronicss	0.85%
65	Utilities	1.02%
66	Oil & gas extraction	2.20%

C. Between 2010 and 2020

1	Wholesale trade	-1.70%
2	Insurance carriers	-1.03%
3	Administrative services	-0.93%
4	Other retail	-0.83%
	⋮	
64	Publishing industries	0.89%
65	Computer & electronics	0.98%
66	Credit intermediation	2.44%

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Table 14: Counterfactual TFP Growth Without Sectoral Final Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.50%
2	Motor vehicles	-0.91%
3	Machinery	-0.88%
4	Apparel & leather	-0.51%
	⋮	
62	Securities & investment	0.87%
63	Misc. professional services	0.94%
64	Hospitals	0.95%
65	Internet & inf. services	1.01%
66	Wholesale trade	1.18%

B. Between 2002 and 2009

1	Construction	-1.22%
2	Motor vehicles	-0.82%
	⋮	
66	Hospitals	0.58%

C. Between 2010 and 2020

1	Computer & electronis	-0.52%
	⋮	
63	Other retail	0.59%
64	Internet & inf. services	0.60%
65	Construction	0.89%
66	Wholesale trade	1.08%

Table 15: Counterfactual TFP Growth Without Sectoral Intermediate Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.24%
2	Credit intermediation	-0.90%
3	Publishing industries	-0.76%
4	Computer systems design	-0.45%
5	Ambulatory health	-0.42%
	⋮	
61	Telecommunications	0.52%
62	Administrative services	0.54%
63	Hospitals	0.56%
64	Insurance carriers	0.74%
65	Other retail	0.90%
66	Wholesale trade	1.21%

B. Between 2002 and 2009

1	Computer & electronics	-0.48%
	⋮	
66	Securities & investment	0.49%

C. Between 2010 and 2020

1	Credit intermediation	-0.97%
2	Publishing industries	-0.51%
3	Computer & electronics	-0.49%
	⋮	
63	Insurance carriers	0.52%
64	Administrative services	0.63%
65	Other retail	0.66%
66	Wholesale trade	1.12%

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Normalized nested CES environment - Firms

Firms

$$\frac{y_i}{\overline{y}_i} = A_i \left(\sum_{h \in \mathscr{H}} \widetilde{\Omega}^{\ell}_{ih} \left(\frac{\ell_{ih}}{\overline{\ell}_{ih}} \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{j \in \mathscr{N}} \widetilde{\Omega}^{x}_{ij} \left(\frac{x_{ij}}{\overline{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}$$

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Normalized nested CES environment - Households

Households

$$U_h\left(c_h, \widetilde{L}_h\right) = \frac{\left(c_h\left(1 - E_h^{-\gamma_h}\widetilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1-\sigma} \quad s.t. \quad \frac{C_h}{\overline{C}_h} = \left(\sum_{i\in\mathcal{N}} \beta_{hi}\left(\frac{C_{hi}}{\overline{C}_{hi}}\right)^{\frac{\rho_h - 1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h - 1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \widetilde{L}_h$

Normalized nested CES environment - Households

Households

$$U_h\left(c_h, \widetilde{L}_h\right) = \frac{\left(c_h\left(1 - E_h^{-\gamma_h}\widetilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1-\sigma} \quad s.t. \quad \frac{C_h}{\overline{C}_h} = \left(\sum_{i\in\mathcal{N}} \beta_{hi}\left(\frac{C_{hi}}{\overline{C}_{hi}}\right)^{\frac{\rho_h - 1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h - 1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \widetilde{L}_h$

The change in labor supply from type h workers is, to a first-order

$$d\log L_h = \zeta_h^n \, d\log n_h + \zeta_h^w \, d\log w_h - \zeta_h^e \, d\log E_h$$

Where the corresponding elasticities are given by

$$\zeta_h^n = \frac{E_h^{\gamma_h}}{1-\varphi_h\gamma_h}\frac{n_h}{L_h}, \qquad \zeta_h^w = \frac{1}{1-\varphi_h\gamma_h}\frac{\varphi_h}{\Gamma_h}, \qquad \zeta_h^e = \zeta_h^w - \gamma_h\zeta_h^n.$$

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Solution - Expenditure & Wages

$$d\log E_h = \overbrace{\frac{\zeta_h^n \Gamma_h}{1+\zeta_h^e\Gamma_h}\, d\log n_h}^{\substack{\textit{Demographic Effect}\\ \textit{on Expenditure (PE)}}} + \overbrace{\frac{\left(1+\zeta_h^w\right)\Gamma_h}{1+\zeta_h^e\Gamma_h} d\log w_h}^{\substack{\textit{Wage Effect}\\ \textit{on Expenditure (GE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\kappa_{ih}\lambda_i}{\left(1+\zeta_h^e\Gamma_h\right)\chi_h}\left(\left(1-\mu_i\right) d\log S_i - \mu_i\; d\log \mu_i\right)}^{\substack{\textit{Corporate Income Effect}\\ \textit{on Expenditure (PE + GE)}}};$$

$$d\log w_h = \overbrace{\frac{\zeta_h^e}{1+\zeta_h^w}\, d\log E_h}^{\substack{\textit{Expenditure Effect}\\ \textit{on Wages (GE)}}} - \overbrace{\frac{\zeta_h^n}{1+\zeta_h^w} d\log n_h}^{\substack{\textit{Demographic Effect}\\ \textit{on Wages (PE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{\left(1+\zeta_h^w\right)\Lambda_h}\left(\left(\theta_i-1\right) d\log A_i + \theta_i\; d\log \mu_i\right)}^{\substack{\textit{Direct Effect}\\ \textit{on Wages (PE)}}}$$

$$\overbrace{-\sum_{j\in\mathcal{N}}\left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{\left(1+\zeta_h^w\right)\Lambda_h}\left(\theta_i-1\right)\widetilde{\psi}_{ij}^{x}\right)\left(d\log A_j + d\log \mu_j\right)}^{\substack{\textit{Supplier Effect}\\ \textit{on Wages (PE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{\left(1+\zeta_h^w\right)\Lambda_h}\; d\log S_i}^{\substack{\textit{Sales Effect}\\ \textit{on Wages (GE)}}}$$

$$\overbrace{-\left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{\left(1+\zeta_h^w\right)\Lambda_h}\left(\theta_i-1\right)\right) d\log w_h}^{\substack{\textit{Direct Substitution Effect}\\ \textit{on Wages (GE)}}} + \overbrace{\sum_{b\in\mathcal{H}}\left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{\left(1+\zeta_h^w\right)\Lambda_h}\left(\theta_i-1\right)\widetilde{\psi}_{ib}^{\ell}\right) d\log w_b}^{\substack{\textit{Supplier Substitution Effect}\\ \textit{on Wages (GE)}}};$$

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Solution - Sales

$$d \log S_i = \overbrace{\sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} \, d \log E_h}^{\substack{\textit{Expenditure Effect} \\ \textit{on Sales (GE)}}} + \overbrace{\sum_{j \in \mathcal{N}} \frac{\Omega^x_{ji} \lambda_j}{\lambda_i} \, d \log S_j}^{\substack{\textit{Sales Effect} \\ \textit{on Sales (GE)}}} + \overbrace{\sum_{j \in \mathcal{N}} \frac{\Omega^x_{ji} \lambda_j}{\lambda_i} \left(\left(\theta_j - 1 \right) d \log A_j + \theta_j \, d \log \mu_j \right)}^{\substack{\textit{Direct Effect} \\ \textit{on Sales (PE)}}}$$

$$\overbrace{+ \sum_{j \in \mathcal{N}} \left(\sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} \left(\rho_h - 1 \right) \left(\widetilde{\psi}^x_{ij} - \widetilde{\mathscr{B}}_{hj} \right) + \sum_{q \in \mathcal{N}} \frac{\Omega^x_{qi} \lambda_q}{\lambda_i} \left(\theta_q - 1 \right) \left(\widetilde{\psi}^x_{ij} - \widetilde{\psi}^x_{qj} \right) \right) \left(d \log A_j + d \log \mu_j \right)}^{\substack{\textit{Supplier Effect} \\ \textit{on Sales (PE)}}}$$

$$\overbrace{+ \sum_{h \in \mathcal{H}} \left(\sum_{b \in \mathcal{H}} \frac{\beta_{bi} \chi_b}{\lambda_i} \left(\rho_b - 1 \right) \left(\widetilde{\mathscr{C}}_{bh} - \widetilde{\psi}^\ell_{ih} \right) + \sum_{j \in \mathcal{N}} \frac{\Omega^x_{ji} \lambda_j}{\lambda_i} \left(\theta_j - 1 \right) \left(\widetilde{\psi}^\ell_{jh} - \widetilde{\psi}^\ell_{ih} \right) \right)}^{\substack{\textit{Supplier Substitution Effect} \\ \textit{on Sales (GE)}}} d \log w_h \, .$$

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