Inequality and Misallocation under Production Networks

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Research Question & Motivation

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Upper Decile vs The Rest

Higher Expenditure Share in
Education and Entertainment

Lower Expenditure Share in
Shelter, Utilities, Healthcare

Data from Consumer Expenditure Survey

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Data from Consumer Expenditure Survey

Income share for the top has increased

In this Presentation

In economies with distortions, variations in distributions (labor income & expenditure) can influence misallocation

Novel TFP decomposition and sufficient statistics that measures aggregate misallocations effects

Implementation of the model with US data: between 2010-2019, distributional sources of variation reduced TFP growth by 7.5%

Approach   Contribution

$\triangle$**Consumption**   & $\triangle$**Demand**
Distribution   **Structure**

$\Downarrow$

$\triangle$ **Income Distribution**

Approach   ## Contribution

Δ **Consumption** & Δ **Demand**
  Distribution   **Structure**

                              Δ Labor Income Shares

⇓

Δ **Income Distribution**

Approach

Contribution

Bigio & La'O (2020)

(i) Rep Household

(ii) Efficient Equilibrium

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$\downarrow$ **Δ Labor Income Shares**

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My Model

(i) Het Households
(ii) Any Equilibrium

Literature Review

Approach

$\triangle$**Consumption Distribution** & $\triangle$**Demand Structure**

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$\triangle$ **Income Distribution**

$\Downarrow$

$\triangle$ **Misallocation**

$\Downarrow$

$\triangle$ **TFP**

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Δ **Misallocation**

Baqaee & Farhi (2020)
Rep Household
↓
My Model
Het Households

⇓

Δ **TFP**

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } \boldsymbol{H} \\ \text{Less Competitive: } \boldsymbol{L} \end{cases}$ **+** **Two Workers** : $\begin{cases} \text{High-Skill: } \boldsymbol{h} \\ \text{Low-Skill: } \boldsymbol{l} \end{cases}$

Caveat: Paper is more general than this case

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1. Good markets face exogenous distortions

$$Cost \quad = \boldsymbol{\mu} \quad \times \quad Revenue$$

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1. Good markets face exogenous distortions

$$Cost = \mu \times Revenue$$

2. Labor markets are competitive

3. Correlations *"HhH"*:
- **_H_** has high μ
- **_H_** requires more **_h_**
- **_h_** have a higher expenditure in **_H_**

Mechanism's Intuition

1. **μ heterogeneity** $\longrightarrow$ allocates more workers to **H**
   - **H** operates with low marginal productivity
   - **L** operates with high marginal productivity

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2. **Skill-bias heterogeneity** $\longrightarrow$ asymmetries in the income exposure in response to local perturbations

3. **Preference heterogeneity** expenditure flows
   - As **h** income increase, expenditure in **H** rises
   - Workers relocate from **L** to **H**
   - Misallocation is accentuated

Firm Heterogeneity $\begin{cases}$ **a. Skill Bias** \
$i \in \{H, L\}$ $\quad$ **b. Distortions**

$$\underset{y_i, \ell_{i\mathbf{h}}, \ell_{i\mathbf{l}}}{Max} \quad \pi_i = p_i y_i - w_{\mathbf{h}} \ell_{i\mathbf{h}} - w_{\mathbf{l}} \ell_{i\mathbf{l}}$$

$$y_i = A_i \, \ell_{i\mathbf{h}}^{\alpha_i} \, \ell_{i\mathbf{l}}^{1-\alpha_i}$$

Skill Bias

$$\alpha_L \leq \alpha_H$$

$$\text{Firm Heterogeneity} \begin{cases} \text{a. Skill Bias} \\ \text{b. Distortions} \end{cases}$$
$$i \in \{H, L\}$$

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Skill Bias
$$\boldsymbol{\alpha_L} \leq \boldsymbol{\alpha_H}$$

Markdown
$$0 < \boldsymbol{\mu_L} \leq \boldsymbol{\mu_H} \leq 1$$

$$\text{Cost}_i = \boldsymbol{\mu_i} \times \text{Revenue}_i$$

Alternative Narrative: Sectoral Dixit-Stiglitz Aggregation

Household Heterogeneity
$r \in \{h, l\}$

$\begin{cases} & \\ & \\ & \\ & \end{cases}$

Preferences $\rightarrow$ Aggregate Non-Homotheticity

Graphic Argument

Unique Skill $\rightarrow$ Structural Income Heterogeneity

Household Heterogeneity
$r \in \{h, l\}$

$\left\{ \begin{array}{l} \text{\color{red}\textbf{Preferences}} \quad \rightarrow \quad \begin{array}{c} \text{Aggregate} \\ \text{Non-Homotheticity} \end{array} \\[1em] \text{\color{blue}\textbf{Unique Skill}} \quad \rightarrow \quad \begin{array}{c} \text{Structural Income} \\ \text{Heterogeneity} \end{array} \end{array} \right.$

Graphic Argument

$$\underset{C_r, C_{rH}, C_{rL}}{Max} U_r\left(C_r\right) \text{ s.t. } \frac{C_r}{\overline{C}_r} = \left(\boldsymbol{\color{red}\beta_r} \left(\frac{C_{rH}}{\overline{C}_{rH}} \right)^{\frac{\rho-1}{\rho}} + \left(1 - \boldsymbol{\color{red}\beta_r}\right) \left(\frac{C_{rL}}{\overline{C}_{rL}} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$E_r = p_r^c C_r = p_H C_{rH} + p_L C_{rL} \leq \boldsymbol{\color{blue}w_r L_r} + 0.5 \text{ profits}$$

Consumption Bias
$$\boldsymbol{\color{red}\beta_l \leq \beta_h}$$

Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H \, C_{rH} = \beta_r \, p_r^c \, C_r \qquad\qquad w_h \, \ell_{ih} = \alpha_i \, \mu_i \, p_i \, y_i$$

Equilibrium   Equilibrium Definition   Necessary and sufficient conditions for equilibrium

Solve for Equilibrium Distributions

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In **market clearing conditions**

$$y_i = C_{hi} + C_{li} \qquad\qquad L_r = \ell_{Hr} + \ell_{Lr}$$

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Equilibrium in terms of

$$\boldsymbol{\lambda_i} = \frac{p_i\, y_i}{GDP} \qquad\qquad \boldsymbol{\chi_r} = \frac{p_r^c\, C_r}{GDP}$$

Sales (Domar weights) $\qquad\qquad$ Expenditure

$$\boldsymbol{\Lambda_h} = \frac{w_h\, L_h}{GDP} = \sum \alpha_i \mu_i \lambda_i \qquad \boldsymbol{\tilde{\Lambda}_h} = \sum \alpha_i \lambda_i$$

Labor income $\qquad\qquad$ Value added

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

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$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

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$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

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Additional Restriction

$$\mu_H + \mu_L = 1$$

Solution   Allocative $\neq$

What I Don't Do

- Misallocation literature **distorted vs. efficient** equilibrium

- Parametric assumptions (usually CD) → analytic TFP

- Evaluate how getting rid of distortions has an effect on TFP

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What I Do

- Local TFP Δ around distorted equilibrium to any shock

- Distributional $\Delta \rightarrow$ Misallocation $\Delta \rightarrow \Delta$ TFP

- To ilustrate: $d \log A$

Local Variation to $d\,log\,A$

$$d\,\chi_h = \frac{\color{red}(\alpha_H - \alpha_L)}{2}\,d\,\lambda_H$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

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Expenditure elasticity requires $\alpha_H \neq \alpha_L$

$$d \lambda_H = (\rho - 1) \underbrace{(\chi_h - \chi_l)}_{> 0} \beta_h \, \beta_l \, \Phi \, d \log \frac{A_H}{A_L}$$

Sales elasticity requires $\rho \neq 1$

First-Order Approximation   Constant $\Phi > 0$

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Sales elasticity requires $\rho \neq 1$

$\left\{ \begin{array}{l} \textbf{Under } \rho > 1\text{: } \text{consumers increase expenditure on } H \text{ \& } \lambda_H \uparrow \\ \textbf{Under } \rho < 1\text{: } \underbrace{\text{consumers increase expenditure on } L \text{ \& } \lambda_H \downarrow}_{\textbf{Baumol's Cost Disease}} \end{array} \right.$

First-Order Approximation   Constant $\Phi > 0$

Income Distribution & Bilateral Centralities

$$
\Lambda_h = \overbrace{\left(\underbrace{\alpha_H\, \mu_H}_{f_{H \to h}}\, \beta_h + \underbrace{\alpha_L\, \mu_L}_{f_{L \to h}}\, (1 - \beta_h) \right)}^{m_{h \to h}} \chi_h
$$

$$
+ \underbrace{\left(\overbrace{\alpha_H\, \mu_H}\, \beta_l + \overbrace{\alpha_L\, \mu_L}\, (1 - \beta_l) \right)}_{m_{l \to h}} \chi_l
$$

$$
\begin{array}{c}
m_{l \to h} \\
f_{L \to h}
\end{array}
\quad \textbf{\%} \ \text{of} \quad
\begin{array}{c}
\textbf{expenditure} \ \text{from} \ l \\
\textbf{revenue} \ \text{from} \ L
\end{array}
\quad \text{reaching} \ \Lambda_h
$$

Comparative Statics

$$\Lambda_h = m_{h \to h} \, \chi_h + m_{l \to h} \, \chi_l$$

$$m_{r \to h} = \beta_r \, f_{H \to h} + (1 - \beta_r) \, f_{L \to h} \qquad f_{i \to h} = \alpha_i \, \mu_i$$

Substitution Effects $d \, \beta_r$ Low skill variation $d \, \Lambda_l$ Labor Wedge $d \, log \, \Gamma_r$ General Income Centrality

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Take total derivative

$$d\, \Lambda_h = \underbrace{\chi_h\, d\, m_{h \to h} + \chi_l\, d\, m_{l \to h}}_{\textbf{Income Centrality}_h} + \underbrace{m_{h \to h}\, d\, \chi_h + m_{l \to h}\, d\, \chi_l}_{\textbf{Distributive Income}_h}$$

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We know: $d\,\chi_h + d\,\chi_l = 0$ & $d\, m_{r \to h} = (f_{H \to h} - f_{L \to h})\, d\,\beta_r$

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$$d\,\Lambda_h = \underbrace{\overbrace{(f_{H\to h} - f_{L\to h})}^{\overset{\geq}{(\alpha_H-\mu_L)}\;0}\sum\chi_r\,d\,\beta_r}_{\textbf{Income Centrality}_h} + \underbrace{\overbrace{(m_{h\to h} - m_{l\to h})}^{\overset{\geq}{(\beta_h-\beta_l)}\;0\;\overset{\geq}{(\alpha_H-\mu_L)}\;0}\,d\,\chi_h}_{\textbf{Distributive Income}_h}$$

Substitution Effects $d\,\beta_r$ Low skill variation $d\,\Lambda_l$ Labor Wedge $d\,log\,\Gamma_r$ General Income Centrality

Baqaee & Fahri (2020)

$$d\ log\ TFP = \underbrace{\boldsymbol{\lambda}_H\ d\ log\ A_H + \boldsymbol{\lambda}_L\ d\ log\ A_L}_{\text{Technology}}$$

$$+ \underbrace{\boldsymbol{\lambda}_H\ d\ log\ \mu_H + \boldsymbol{\lambda}_L\ d\ log\ \mu_L}_{\text{Competitiveness}}$$

$$- \underbrace{\left(\widetilde{\boldsymbol{\Lambda}}_h\ d\ log\ \Lambda_h + \widetilde{\boldsymbol{\Lambda}}_l\ d\ log\ \Lambda_l\right)}_{\text{Misallocation}}$$

Without distortions $\rightarrow$ Hulten (1978)

Intermediate Steps

Distortion Centralities δ

$$\textbf{\textit{Misallocation}} = \widetilde{\Lambda}_h \, d \log \Lambda_h + \widetilde{\Lambda}_l \, d \log \Lambda_l$$

$$= \frac{\widetilde{\boldsymbol{\Lambda}}_h}{\boldsymbol{\Lambda}_h} \, d \, \Lambda_h + \frac{\widetilde{\boldsymbol{\Lambda}}_l}{\boldsymbol{\Lambda}_l} \, d \, \Lambda_l$$

$$= \boldsymbol{\delta}_h \, d \, \Lambda_h + \boldsymbol{\delta}_l \, d \, \Lambda_l$$

Distortion Centralities δ

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δ measures how **undervalue** a worker is

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$$= \boldsymbol{\delta}_h \, d \, \Lambda_h + \boldsymbol{\delta}_l \, d \, \Lambda_l$$

δ measures how **undervalue** a worker is

$$\boldsymbol{\delta}_l - \boldsymbol{\delta}_h = \overbrace{(\mu_H - \mu_L)}^{\geq \ 0} \, \overbrace{(\alpha_H - \alpha_L)}^{\geq \ 0} \, \overbrace{\boldsymbol{a}}^{> \, 0}$$

$\boxed{\delta_l - \delta_h > 0}$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textit{Misallocation} = \underbrace{(F_H - F_L)\sum \chi_r\,d\,\beta_r}_{\textbf{1. } \textit{Final Demand}} + \underbrace{(M_h - M_l)\,d\,\chi_h}_{\textbf{2. } \textit{Expenditure}}$$

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Sufficient Statistics

Revenue Centrality F

$$F_i = f_{i \to h}\, \delta_h + f_{i \to l}\, \delta_l$$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textbf{\textit{Misallocation}} = \underbrace{(F_H - F_L)\sum \chi_r\, \boldsymbol{d}\,\beta_r}_{\textbf{1.}\ \textit{Final Demand}} + \underbrace{(M_h - M_l)\, \boldsymbol{d}\,\chi_h}_{\textbf{2.}\ \textit{Expenditure}}$$

Sufficient Statistics

Revenue Centrality F Expenditure Centrality M

$$F_i = f_{i\to h}\,\delta_h + f_{i\to l}\,\delta_l \qquad M_r = m_{r\to h}\,\delta_h + m_{r\to l}\,\delta_l$$

Introduce $d\,\Lambda_h$ and $d\,\Lambda_l$

$$\textbf{\textit{Misallocation}} = \underbrace{(F_H - F_L)\sum \chi_r\, d\,\beta_r}_{\textbf{1.}\ \textit{Final Demand}} + \underbrace{(M_h - M_l)\,d\,\chi_h}_{\textbf{2.}\ \textit{Expenditure}}$$

Sufficient Statistics

Revenue Centrality F Expenditure Centrality M

$$F_i = f_{i\to h}\,\delta_h + f_{i\to l}\,\delta_l \qquad M_r = m_{r\to h}\,\delta_h + m_{r\to l}\,\delta_l$$

1. F_i is high for firms that operate in relatively competitive supply chains and directly or indirectly demand high δ workers

2. M_r is high for households that consume from relatively competitive supply chains that demand workers with high δ

Final Demand ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(F_H - F_L) \sum \chi_r \, d \, \beta_r}_{\text{Final Demand}}$$

Final Demand ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(F_H - F_L) \sum \chi_r \, d \, \beta_r}_{\text{Final Demand}}$$

$$F_H - F_L = \overbrace{\underbrace{(\mu_H - \mu_L)}}^{\geq \quad 0}$$
$$\times \underbrace{[\delta_I - (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L)\, a]}_{> \quad 0}$$

Final Demand $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP $\downarrow$

$$\underbrace{(F_H - F_L) \sum \chi_r \, \boldsymbol{d} \, \beta_r}_{\text{Final Demand}}$$

$$F_H - F_L = \overbrace{(\mu_H - \mu_L)}^{\geq \quad 0}$$

$$\times \underbrace{[\delta_I - (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \, \boldsymbol{a}]}_{> \quad 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_I$
Final Demand Recomposition	❌	✅	✅

Expenditure ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(M_h - M_l)\ d\,\chi_h}_{\text{Distributive}} \qquad d\,\chi_h = \frac{(\alpha_H - \alpha_L)}{2}\,d\,\lambda_H$$

Constant a

Expenditure $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP $\downarrow$

$$\underbrace{(M_h - M_l)\ d\,\chi_h}_{\text{Distributive}} \qquad d\,\chi_h = \frac{(\alpha_H - \alpha_L)}{2}\ d\,\lambda_H$$

$$\overbrace{M_h - M_l}^{\geq \quad 0} = \overbrace{(\mu_H - \mu_L)}^{\geq \quad 0}\ \overbrace{(\beta_h - \beta_l)}^{\geq \quad 0}$$

$$\times \underbrace{\left[\delta_l - (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L)\,\boldsymbol{a}\right]}_{> \quad 0}$$

Constant $\boldsymbol{a}$

Expenditure $\uparrow \rightarrow$ **Misallocation** $\uparrow \rightarrow$ **TFP** $\downarrow$

$$\underbrace{(M_h - M_l)\ d\,\chi_h}_{\text{\textit{Distributive}}} \qquad d\,\chi_h = \frac{(\alpha_H - \alpha_L)}{2}\,d\,\lambda_H$$

$$\overbrace{M_h - M_l}^{\geq\ 0} = \overbrace{(\mu_H - \mu_L)}^{\geq\ 0}\ \overbrace{(\beta_h - \beta_l)}^{\geq\ 0}$$

$$\times \underbrace{[\delta_l - (\alpha_H - \alpha_L)(\alpha_H\mu_H - \alpha_L\mu_L)\,\boldsymbol{a}]}_{>\ 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Expenditure Redistribution	❌	❌	❌

Constant *a*

d log A_L = 1% under $\rho > 1$: $\lambda_L \uparrow$ & $\lambda_H \downarrow$
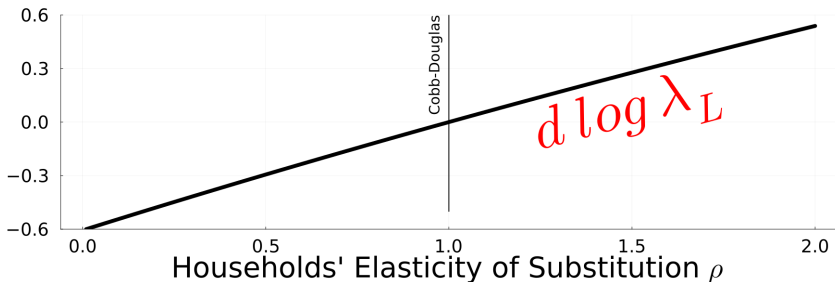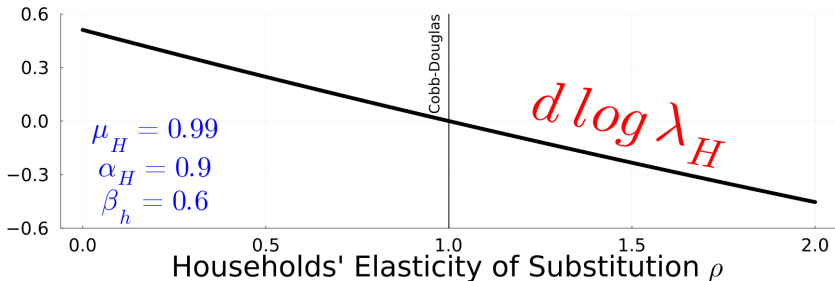
Under $\rho > 1$: **Distributive ↓ & Final Demand ↓**
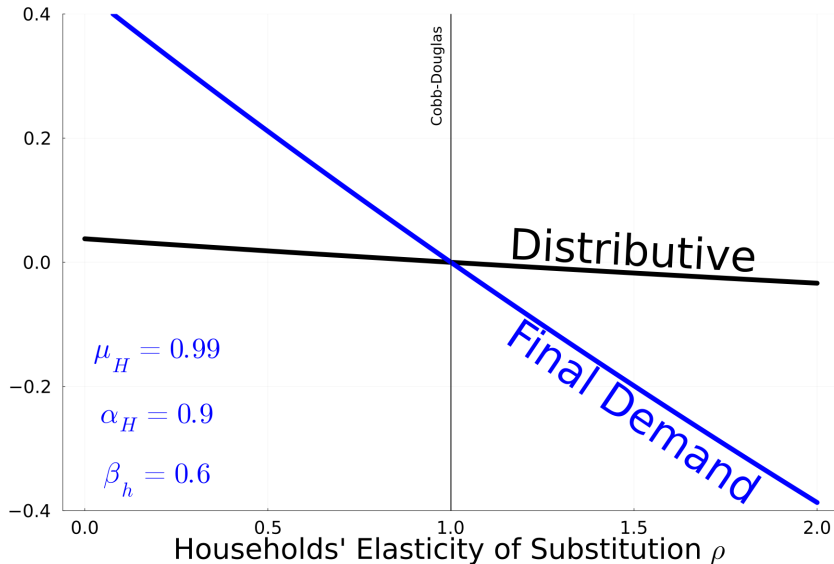
Under $\rho > 1$: Misallocation ↓ & $d \log TFP > \lambda_L$
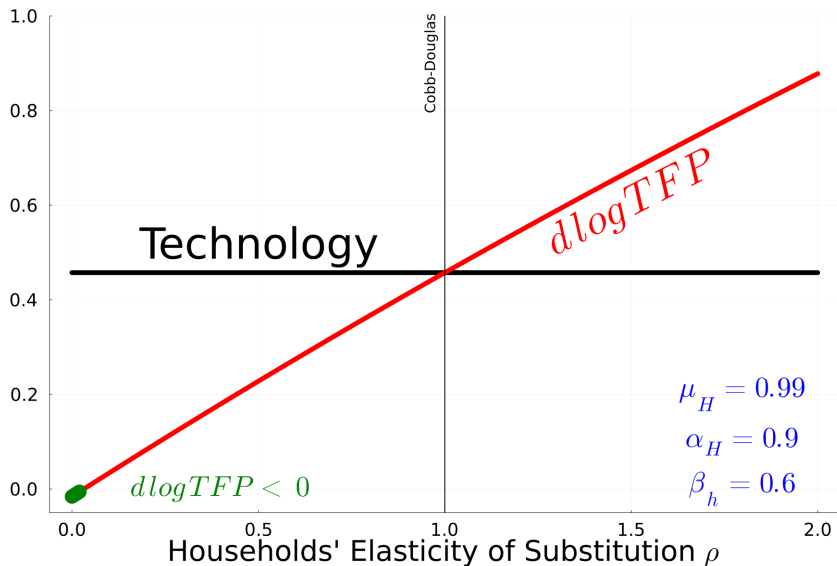
In the Paper...

- General Non-Parametric CRS model for production & consumption

- General Input-Output Networks

- General Equity Distribution

General Income Centrality   General Decomposition for Misallocation

In the Paper...

- General Non-Parametric CRS model for production & consumption

- General Input-Output Networks

- General Equity Distribution

Additional Channels

1. $\mu \uparrow$ & stronger for sectors with high $\lambda_i F_i$

2. $\alpha \uparrow$ for high δ workers & stronger if $\mu_i \lambda_i$ high

3. **Intermediate demand** $\uparrow$ on sectors with high F_i

General Income Centrality | General Decomposition for Misallocation

Data for money flows from 1997 to 2021
Household to Firm

1. State level **P**ersonal **C**onsumption **E**xpenditure (BEA)

$\beta_{state, industry}$: $\begin{cases} \text{PCE provides expenditure on types of goods} \\ \text{IO Make matrix: type of good} \rightarrow \text{industry} \end{cases}$

Data for money flows from 1997 to 2021

Household to Firm

1. State level **P**ersonal **C**onsumption **E**xpenditure (BEA)

$\beta_{state,\,industry}$: $\begin{cases} \text{PCE provides expenditure on types of goods} \\ \text{IO Make matrix: type of good}\rightarrow\text{industry} \end{cases}$

Firm to Firm

2. **I**nput-**O**utput tables (BEA) for 66 NAICS industries

$\mu_i = \dfrac{\text{Total Cost}_i}{\text{Sales}_i}$

$\text{Intermediate Intensity}_{ij} = \dfrac{p_j\, x_{ij}}{\text{Total Cost}_i}$

$\text{Total Cost}_i = \text{Labor Costs}_i + \text{Intermediate Cost}_i$

3. Industry Level Production Accounts (BEA)

$d \log A_i$

3. **I**ndustry **L**evel **P**roduction **A**ccounts (BEA)

d log A_i

4. **C**ounty **B**usiness **P**atterns (Census)
Industry specific **geographic (state)** bias in labor

Antisupression Algorithm   Missing Private Employment

3. Industry Level Production Accounts (BEA)

$$d\ log\ A_i$$

4. County Business Patterns (Census)

Industry specific **geographic (state)** bias in labor

Antisupression Algorithm   Missing Private Employment

5. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

From **4** & **5** → industry specific heterogeneity by worker type. Worker type comes from State & Occupational interactions $H = 38,189$ ($\approx$ 1.5 bill m's)

Financial Sector in Economists
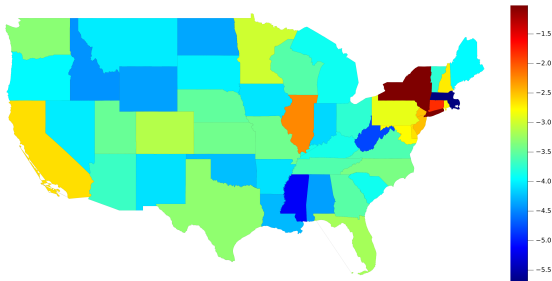
Ambulatory Health in Dentists
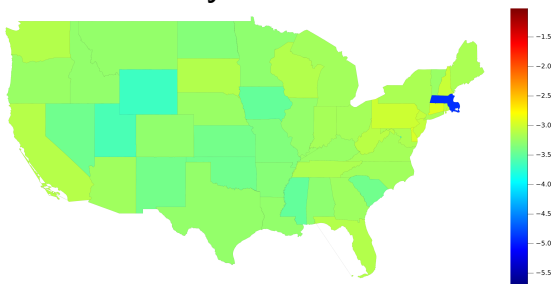
R^2 **rises from** 5% **to** 50% **with IO Networks**
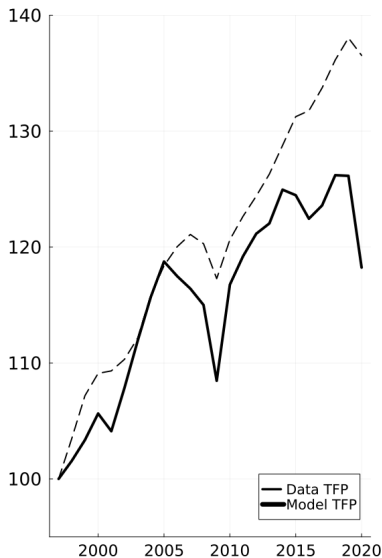
	Observed d log TFP on	
	Without Networks	**With Networks**
	(1)	**(2)**
d log $\widehat{TFP}$	-0.265	0.311***
	(0.264)	(0.069)
Adj R^2	0.048	0.499
	(3)	**(4)**
Technology	0.847***	0.413***
	(0.289)	(0.082)
Competitive	0.986	0.342***
	(0.695)	(0.054)
Misallocation	-0.105	0.0168
	(0.360)	(0.125)
Adj R^2	0.562	0.732

Moments   Implementation   Growth Dec   Covariance Dec   Without Int.   With Int.

$d\ log\ TFP =$ Technology **$+$** Competitiveness **$-$** Misallocation

Technology ↑

$$\sum_{i \in \mathscr{N}} \widetilde{\lambda}_i\ d\ log\ A_i$$
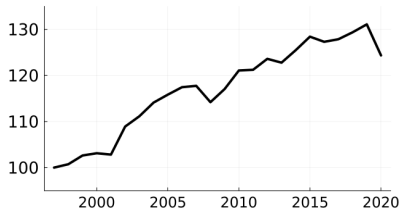
Between 1997 and 2020	
Oil & gas extraction	**-11.1%**
Computer & electronic	**-6.6%**

Competitiveness ↓

$$\sum_{i \in \mathscr{N}} \widetilde{\lambda}_i\ d\ log\ \mu_i$$
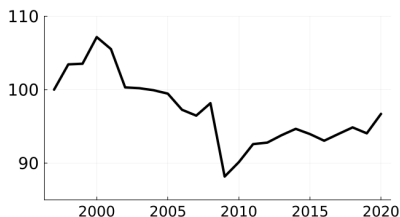
Between 1997 and 2020	
Credit intermediation	**4.1%**
Between 2002 and 2009	
Oil & gas extraction	**6.6%**

Technological Sources

Competitiveness Sources

Without Misallocation↑ after 2009, TFP↑ 7.5%

- Misallocation↓ between 2001 and 2010 by **-8.2%**
- Misallocation↑ between 2010 and 2020 by **7.5%**
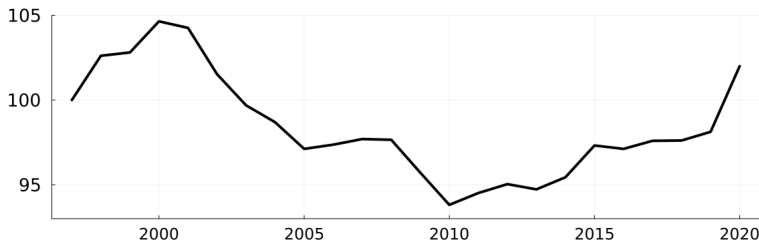
Increasing profit margins

- Oil & gas extraction: **-1.5%**
- Computer & electronics: **-1.1%**

Increasing labor demand

- Credit intermediation: **2.4%**

Final and Intermediate Demand

- Wholesale Trade: **2.2%**

Distribution of δ | Sources of Misallocation (Graph) | Sources of Misallocation (Counterfactual)

Industry variation from μ and Labor Demand | Industry variation from Final and Intermediate Demand

Normalized Nested CES

Introduced by **de La Grandville (1989)** and **Klump & de La Grandville (1989)** and as in **Baqaee & Farhi (2019,a,b, 2020, 2022)**

Normalized Nested CES

Parameters - **Atalay (2017)**, **Boehm et al. (2014)**

1. Elasticity of substitution **between worker types**: 1.0

2. Elasticity of substitution **between sectoral intermediate inputs**: 0.2

3. Elasticity of substitution **between labor and intermediate inputs**: 0.5

4. Elasticity of substitution in **final consumption**: 0.9

5. **Substitution effect** in labor supply: 2

6. **Income effect** in labor supply: 2

$$d \log TFP = \underbrace{\textbf{\textit{Technology}}}_{= 1\%} - \textit{Misallocation}$$

Best Sectors	**d log TFP**
1. Nursing & Residential Care	1.041%
2. Social Assistance	1.039%
3. General Merchandise Store	1.029%
4. Ambulatory health care	1.027%
5. Hospitals	1.026%

Worst Sectors	**d log TFP**
1. Oil & Gas extraction	0.587%
2. Primary Metals	0.610%
3. Chemical Products	0.618%
4. Mining, except Oil & Gas	0.630%
5. Utilities	0.647%

	d log TFP on			
μ_i	0.359*** (0.09)			0.207 (0.13)
λ_i		0.170 (0.56)		0.854* (0.50)
F_i			0.212*** (0.05)	0.148** (0.07)
R^2	0.20	$1e^{-3}$	0.21	0.27
N			66	

We want productivity shocks in sectors with high F_i!

Positional Terms of Trade

$$C_r = \textbf{PTT}_r \times f_r\left(L_h, L_l\right)$$

Positional **T**erms of **T**rade

$$C_r = \textbf{PTT}_{\textbf{r}} \times f_r\left(L_h, L_l\right)$$

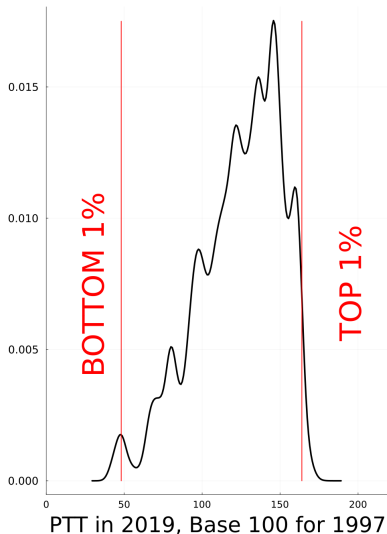$$d\,log\,TFP = \sum \chi_r\,d\,log\,\textbf{PTT}_{\textbf{r}}$$

Positional Terms of Trade

$$C_r = \mathbf{PTT_r} \times f_r \left(L_h, L_l \right)$$

$$d \log TFP = \sum \chi_r \; d \log \mathbf{PTT_r}$$

$$d \log \mathbf{PTT_h} = \underbrace{\beta_h \; d \log A_H + (1 - \beta_h) \; d \log A_L}_{\text{Technology}_h}$$

$$+ \underbrace{\beta_h \; d \log \mu_H + (1 - \beta_h) \; d \log \mu_L}_{\text{Competitiveness}_h}$$

$$- \underbrace{\left(\frac{\widetilde{m}_{h \leftarrow h}}{\mathbf{\Lambda_h}} \; d \, \Lambda_h + \frac{\widetilde{m}_{h \leftarrow l}}{\mathbf{\Lambda_l}} \; d \, \Lambda_l - d \log \chi_h \right)}_{\text{Misallocation}_h}$$

$C_r =$ **Positional Terms of Trade**$_r \times f_r(L_h, L_l)$

PTT in 2019, Base 100 for 1997

Top 1%	
Occupation	
Logging Workers	37%
Computer Occupations	13%
Mathematical Sciences Occupations	10%
Compensation Managers	7%
Bottom 1%	
Occupation	
Printing Workers	40%
Shoe & Leather Operator	26%
Textile Machine Operator	15%
Miscellaneous Textile	12%

COUNTERFACTUAL SHOOCK: Antitrust Policy in Housing

Conclusion

- First comprehensive study for join heterogeneity in multisector economies with distortions and input-output networks

- **Theoretical Contribution** in production network + distortions + heterogeneous households:
  - Variation of the income distribution
  - Variations for TFP
  - Variations for PTT

- **Empirical Contribution**: First implementation of a production network model with household heterogeneity for the US
  - In the absence of distributional sources of misallocation, TFP would have grown 7.5% more after Great Recession

Upper Decile vs The Rest
(Consumer Expenditure Survey 2021)

Higher Expenditure Share in

- Education: 3.4% vs 1.3%
- Entertainment: 6.5% vs 4.9%
- Pensions: 17.4% vs 9.1%
- Lodging: 2.6% vs 1.1%

Lower Expenditure Share in

- Shelter: 17.6% vs 20.5%
- Home Food: 5.9% vs 8.5%
- Utilities: 4.1% vs 7.0%
- Healthcare: 6.2% vs 8.3%

From 2004 to 2019

Income share for top quintile ↑ from 48% to 53%

Back

Literature Review

- **Diggregated National Accounts**
  - Cantillon (1756), Quesnay (1758), Leontief (1928), Meade & Stone (1941), Kuznetz (1946), Stone (1961), Andersen et al. (2022)

- **Production Networks**
  - Hulten (1978), Long & Plosset (1983), Gabaix (2011), Jones (2011, 2013), Acemoglu et al. (2012), Baqaee (2018), Baqaee & Farhi (2019, 2020, 2023), Bigio & La'O (2020)

- **Growth Accounting**
  - Solow (1957), Domar (1961), Jorgenson et al. (1987), Basu & Fernanld (2022), Petrin & Levinsohn (2012), Baqaee & Farhi (2020)

Back

Dixit-Stiglitz Aggregation

- Sector i has a sectoral aggregator for $z_i \in [0,1]$

$$y_i = \left(\int y_{z_i}^{\mu_i} \, d z_i \right)^{\frac{1}{\mu_i}}$$
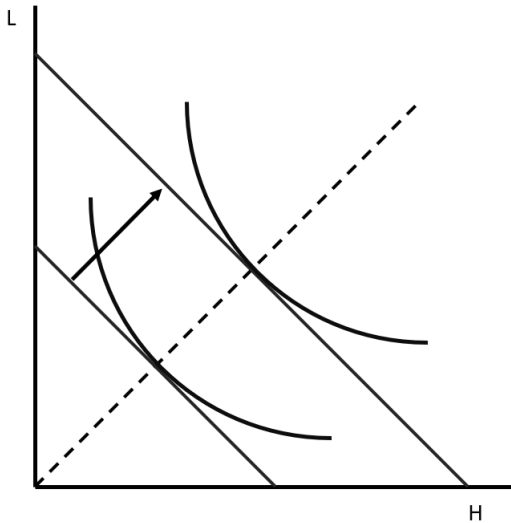
- Demand for variaties

$$y_{z_i} = \left(\frac{p_i}{p_{z_i}} \right)^{\frac{1}{1-\mu_i}} y_i$$

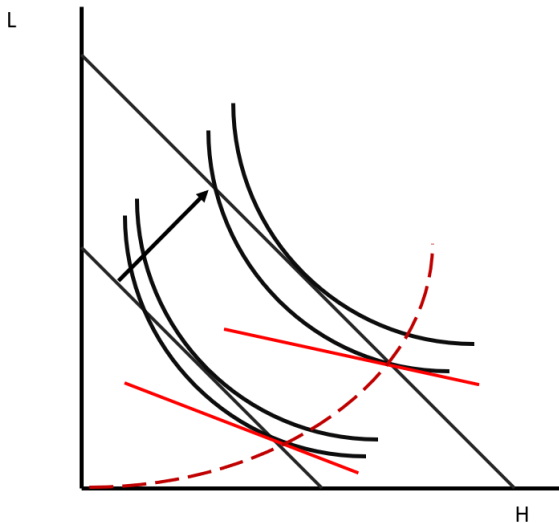- Intermediate's problem

$$\underset{p_{z_i}, y_{z_i}, \ell_{z_i h}, \ell_{z_i l}}{Max} \quad \pi_{z_i} = p_{z_i} y_{z_i} - w_h \, \ell_{z_i h} - w_l \, \ell_{z_i l}$$

$$y_{z_i} = A_i \, \ell_{z_i h}^{\alpha_i} \, \ell_{z_i l}^{1-\alpha_i}$$

Back

Motivation
○○○
Model
○○○○
Solution
○○○○
Income
○○
TFP
○○○○○
Example
○○○○
Data
○○○
Empirics
○○○
Policy
○○
Distributive
○○
Conclusion
○
Appendix

Aggregate Non-Homotheticity

Aggregate Non-Homotheticity

Equilibrium Definition

For $e = (A, \mu, \beta, \alpha) \in \mathscr{E}$, prices and allocations:

(i) **Firms'** labor demand and output decisions maximize profits;

(ii) **Households'** consumption and labor supply maximize utility satisfying budget constraints;

(iii) Goods and labor markets clear.

Back

Equilibrium Definition

$$\boldsymbol{e = (A, \mu, \beta, \alpha)} \in \mathscr{E} \text{ into}$$

$$\vartheta \equiv \left\{ \left\{ y_i, \{\ell_{ir}, C_{ri}\}_{r \in \{h,l\}} \right\}_{i \in \{H,L\}}, \{C_r, L_r\}_{r \in \{h,l\}} \right\}$$

$$\rho \equiv \left\{ p_H, p_L, w_h, w_l, p_h^c, p_l^c \right\}$$

Back

Necessary & sufficient equilibrium conditions

(ϑ, ρ) are an equilibrium iff

$$\frac{U_{C_{rj}}}{U_{C_{ri}}} = \frac{\mu_i \, \partial y_i / \partial \ell_{ib}}{\mu_j \, \partial y_j / \partial \ell_{jb}} \quad i, j \in \{H, L\}, r, b \in \{h, l\},$$

such that $C_{ri} > 0$, and $\ell_{ib} > 0$,

and resource constraints

$$y_i(e) = C_{hi}(e) + C_{li}(e) \quad i \in \{H, L\}$$
$$L_r(e) = \ell_{Hr}(e) + \ell_{Lr}(e) \quad r \in \{h, l\}.$$

Back

Sales Share

$$\lambda_H = \beta_h \, \chi_h + \beta_l \, \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Sales Share

$$\lambda_H = \beta_h \, \chi_h + \beta_l \, \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \, \mu_H \, \lambda_H + \alpha_L \, \mu_L \, \lambda_L$$

Sales Share

$$\lambda_H = \beta_h \, \chi_h + \beta_l \, \chi_l \qquad\qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \, \mu_H \, \lambda_H + \alpha_L \, \mu_L \, \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2}\left((1 - \mu_H) \, \lambda_H + (1 - \mu_L) \, \lambda_L\right)$$

Sales Share

$$\lambda_H = \beta_h \, \chi_h + \beta_l \, \chi_l \qquad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \, \mu_H \, \lambda_H + \alpha_L \, \mu_L \, \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2}\left((1 - \mu_H) \, \lambda_H + (1 - \mu_L) \, \lambda_L\right)$$

Value Added Share

$$\widetilde{\Lambda}_h = \alpha_H \, \lambda_H + \alpha_L \, \lambda_L \qquad \widetilde{\Lambda}_h + \widetilde{\Lambda}_l = 1$$

Sales Distribution

$$\lambda_H = \overbrace{\left(\cfrac{1}{2 - \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0}} \right)}^{\theta \ \geq \ 1/2} \left(1 - \mu_L \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \right)$$

$$\qquad\qquad\qquad\text{Amplification Effect} \qquad\qquad\qquad \text{Contractionary Effect}$$

Consumption Expenditure Distribution

$$\chi_h = \theta \left(1 - \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \underbrace{(\beta_h - \mu_H)}_{?} \right)$$

Labor Income Distribution

$$\Lambda_h = \theta \left[\alpha_H \mu_H + \alpha_L \mu_L - \mu_H \mu_L \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \right]$$

$$\Lambda_l = \theta \left[\alpha_L \mu_H + \alpha_H \mu_L - \mu_H \mu_L \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \right]$$

Value-Added Distribution [Back]

$$\begin{aligned}
\widetilde{\Lambda}_h &= \alpha_H \lambda_H + \alpha_L \lambda_L \\
&= \theta \Big(1 - \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \big(\alpha_H \mu_L + \alpha_L \mu_H \big) \Big) \\
\widetilde{\Lambda}_l &= \theta \Big(1 - \underbrace{(\beta_h - \beta_l)}_{\geq \ 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq \ 0} \big(\alpha_H \mu_H + \alpha_L \mu_L \big) \Big)
\end{aligned}$$

3 Effects from Distortions on Labor

1. Misallocation comes from MRS wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

Back

3 Effects from Distortions on Labor

1. **Misallocation** comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\boldsymbol{\mu_L}}{\boldsymbol{\mu_H}} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

2. Allocative differences $\neq$ **Misallocation**

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$$

Intuition
For the undistorted case
$\mu_H = \mu_L = 1/2$
there is a continuum
of property rights on firms

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \qquad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\boldsymbol{\mu_L}}{\boldsymbol{\mu_H}} \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}}_{\textbf{Misallocation}}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\textbf{Misallocation}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$$\mu_H = \mu_L$$

Symmetric π **All** π **for h**

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\textcolor{red}{\mu_L}}{\textcolor{red}{\mu_H}} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\textcolor{red}{\textbf{Misallocation}}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$$\mu_H = \mu_L$$

Symmetric π **All** π **for h**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L \left(\beta_h - \beta_l\right)$$

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\textbf{Misallocation}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

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Symmetric π   **All π for h**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L \left(\beta_h - \beta_l\right)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\textbf{\textcolor{red}{Misallocation}}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$$\mu_H = \mu_L$$

Symmetric π **All** π **for h**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L \left(\beta_h - \beta_l \right)$$

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Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\text{\textcolor{red}{Misallocation}}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$\mu_H = \mu_L$ $\alpha_H = \alpha_L$ $\beta_h = \beta_l$

Symmetric π   **All π for h**   **Symmetric π**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L \left(\beta_h - \beta_l \right)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}}_{\textbf{Misallocation}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$$\mu_H = \mu_L \qquad\qquad \alpha_H = \alpha_L \quad \beta_h = \beta_l$$

Symmetric π   **All π for h**   **Symmetric π**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L \left(\beta_h - \beta_l \right) \qquad \frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{d\, y_L / d\, \ell_{Lr}}{d\, y_H / d\, \ell_{Hr}}$$

Back

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same} \quad \underbrace{\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H}\frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}}_{\textbf{Misallocation}}$$

Case 1   **Case 2**   **Case 3**   **Case 4**

$$\mu_H = \mu_L \qquad\qquad \alpha_H = \alpha_L \quad \beta_h = \beta_l$$

Symmetric π   **All π for h**   **Symmetric π**

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H\alpha_L\left(\beta_h - \beta_l\right) \qquad \frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}} \qquad\qquad \frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H}\frac{d\,y_L/d\,\ell_{Lr}}{d\,y_H/d\,\ell_{Hr}}$$

Back

Linear Approximation in response to $d \log A_H/A_L$

■ $d\Lambda_h = (\alpha_H - \mu_L)\, d\lambda_H,$ ■ $d\Lambda_l = (\mu_H - \alpha_H)\, d\lambda_H,$

■ $d\chi_h = d\Lambda_h + \dfrac{1}{2}(\mu_L - \mu_H)\, d\lambda_H,$

■ $d\lambda_H = (\rho - 1)(\chi_h - \chi_l)\,\beta_h\beta_l\, d\log\dfrac{A_H}{A_L} + (\beta_h - \beta_l)\, d\chi_h$

$\qquad\qquad + (\rho - 1)(\alpha_H - \alpha_L)\,\beta_h\beta_l\, d\log\dfrac{\Lambda_l}{\Lambda_h}$

Back

Constant Φ in $d\,\lambda_H = (\rho - 1)\,(\chi_h - \chi_I)\,\beta_h\,\beta_I\,\Phi\,d\log\frac{A_H}{A_L}$

$$\Phi = \frac{1 + \Upsilon\,(\phi_h\phi_I + (\rho-1)\beta_h\beta_I(\alpha_H - \alpha_L)(\alpha_H - \mu_H)(\alpha_H - \mu_L)((\beta_h - \beta_I)\Lambda_h - (\rho-1)\beta_h\beta_I(\alpha_H - \alpha_L)))^{-1}}{1 + \frac{1}{2}\,(\beta_h - \beta_I)\,(\mu_H - \mu_L)},$$

with

$$\Upsilon = ((\beta_h - \beta_I)\Lambda_h - (\rho-1)\beta_h\beta_I(\alpha_H - \alpha_L))(\alpha_H - \mu_L)\Lambda_I\left(1 + \frac{1}{2}(\beta_h - \beta_I)(\mu_H - \mu_L)\right)$$

$$+ (\rho-1)\beta_h\beta_I(\alpha_H - \alpha_L)(\mu_H - \alpha_H)\Lambda_h\left(1 - \frac{1}{2}(\beta_h - \beta_I)(\alpha_H - \alpha_L)\right),$$

$$\phi_h = \Lambda_h + (\beta_h - \beta_I)\,\Lambda_h\left(\mu_L - \alpha_H + \frac{1}{2}\,(\mu_H - \mu_L)\right) + (\varrho - 1)\,\beta_h\beta_I\,(\alpha_H - \alpha_L)\,(\alpha_H - \mu_L),$$

$$\phi_I = \Lambda_I + (\beta_h - \beta_I)\,\frac{\Lambda_I}{2}\,(\mu_H - \mu_L) + (\varrho - 1)\,\beta_h\beta_I\,(\alpha_H - \alpha_L)\,(\alpha_H - \mu_H).$$

Substitution Effects

$$\beta_r \quad \equiv \quad \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\substack{\text{Shephard's} \\ \text{Lemma}}}_{\frac{d \, p_r^c C_r}{d \, p_H} = C_{rH}} = \quad \frac{p_H C_{rH}}{p_r^c C_r} = \beta_r \; \substack{\text{In equilibrium} \\ \text{as parameter}}$$

Substitution Effects

$$\beta_r \quad \equiv \quad \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\substack{\text{Shephard's} \\ \text{Lemma}}} \underbrace{\quad = \quad \frac{p_H C_{rH}}{p_r^c C_r}}_{\frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r \; \substack{\text{In equilibrium} \\ \text{as parameter}}$$

1. New equilibrium with local approximations keep α and β fixed

Substitution Effects

$$\beta_r \quad \equiv \quad \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\substack{\text{Shephard's} \\ \text{Lemma}}} \quad = \quad \underbrace{\frac{p_H C_{rH}}{p_r^c C_r}}_{\frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r \; \substack{\text{In equilibrium} \\ \text{as parameter}}$$

1. New equilibrium with local approximations keep α and β fixed

2. Estimate β's consistent with the new equilibrium

Substitution Effects

$$\beta_r \quad \equiv \quad \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\substack{\text{Shephard's} \\ \text{Lemma}}} \underbrace{= \frac{p_H C_{rH}}{p_r^c C_r}}_{\frac{d\, p_r^c C_r}{d\, p_H} = C_{rH}} = \beta_r \;\; \substack{\text{In equilibrium} \\ \text{as parameter}}$$

1. New equilibrium with local approximations keep α and β fixed

2. Estimate β's consistent with the new equilibrium

Exact delta hat - Dekle, Eaton & Kortum (2008)

$$\frac{p_H C_{rH}}{E_r} = \beta_r^\rho \left(\frac{p_r^c \overline{C}_r}{p_H \overline{C}_{rH}} \right)^{\rho-1} \rightarrow \underbrace{d\,\beta_r = \boldsymbol{(\rho-1)}\beta_r\,(1-\beta_r)\,\boldsymbol{d \log \frac{p_L}{p_H}}}_{\text{Increases under substitutability when } p_L/p_H \uparrow}$$

Back

Theorem 1: labor income share variation

$$d\,\Lambda_l = \underbrace{\overbrace{(\beta_h - \beta_l)}^{\geq\ 0}\overbrace{(\mu_H - \alpha_H)}^{?}(m_{h\to l} - m_{l\to l})\,d\,\chi_h}_{\textbf{Distributive Income}_l} + \underbrace{\overbrace{(\mu_H - \alpha_H)}^{?}(f_{H\to L} - f_{L\to l})\sum\chi_r\,d\,\beta_r}_{\textbf{Income Centrality}_l}$$

Back

Labor Wedge

For factors with endogenous supply...

$$-\frac{U_{L_h}}{U_{C_h}} = \Gamma_h \frac{C_h}{L_h}$$

with

$$\Gamma_h = \frac{\Lambda_h}{\chi_h}$$

Proof

$d \log \Gamma_h$ - Extension of Bigio & La'O (2020)

(i) Representative Household   (i) Heterogenous Households
(ii) Around Efficient Equilibrium $\longrightarrow$ (ii) Any Equilibrium

$$d \log \Gamma_h = d \log \Lambda_h - d \log \chi_h$$

Back

Proof of Theorem 1 for $d\,log\,\Gamma_h$

From goods market clearing

$$\begin{pmatrix} y_H \\ y_L \end{pmatrix} = \begin{pmatrix} C_{hH} + C_{lH} \\ C_{hL} + C_{lL} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hL}} y_L \end{pmatrix} = \begin{pmatrix} \frac{\beta_h}{C_{hH}} (C_{hH} + C_{lH}) \\ \frac{(1-\beta_h)}{C_{hL}} (C_{hL} + C_{lL}) \end{pmatrix}$$

From FOC and equilibrium $\quad \beta_h \dfrac{\chi_h}{C_{hH}} = p_H = \beta_l \dfrac{\chi_l}{C_{lH}}$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hH}} y_L \end{pmatrix} = \begin{pmatrix} \beta_h \frac{\chi_h}{\chi_h} + \beta_l \frac{\chi_l}{\chi_h} \\ (1-\beta_h) \frac{\chi_h}{\chi_h} + (1-\beta_l) \frac{\chi_l}{\chi_h} \end{pmatrix}$$

Back

Proof of Theorem 1 for $d \log \Gamma_h$

From FOC and equilibrium $\quad -\dfrac{1}{\beta_h} \dfrac{U_{L_h}}{U_{C_h}} \dfrac{C_{hH}}{C_h} = \dfrac{w_h}{p_H} = \mu_H \, \alpha_H \dfrac{y_H}{\ell_{Hh}}$

$$\begin{pmatrix} \ell_{Hh} \\ \ell_{Lh} \end{pmatrix} = \begin{pmatrix} -\dfrac{U_{C_h}}{U_{L_h}} \, \alpha_H \, \mu_H \, y_H \, \beta_h \, \dfrac{C_h}{C_{hH}} \\ -\dfrac{U_{C_h}}{U_{L_h}} \, \alpha_L \, \mu_L \, y_L \, (1 - \beta_h) \, \dfrac{C_h}{C_{hL}} \end{pmatrix}$$

From labor market clearing condition

$$L_h = \ell_{Hh} + \ell_{Lh} = -\dfrac{U_{C_h}}{U_{L_h}} \, C_h \begin{pmatrix} \alpha_H \, \mu_H & \alpha_L \, \mu_L \end{pmatrix} \begin{pmatrix} \dfrac{\beta_h}{C_{hH}} \, y_H \\ \dfrac{(1-\beta_h)}{C_{hH}} \, y_L \end{pmatrix}$$

$$= -\dfrac{U_{C_h}}{U_{L_h}} C_h \underbrace{\left(\alpha_H \, \mu_H \sum_{r \in \{h, l\}} \beta_r \dfrac{\chi_r}{\chi_h} + \alpha_L \, \mu_L \sum_{r \in \{h, l\}} (1 - \beta_r) \dfrac{\chi_r}{\chi_h} \right)}_{= \Gamma_h}$$

Income Centrality

$$\underset{\text{Centrality}_h}{\text{Income}} = \overbrace{\sum_{i \in \mathcal{N}} \mu_i \; \lambda_i \; d\, \widetilde{\Omega}^{\ell}_{ih}}^{\substack{\text{Labor Demand} \\ \text{Recomposition}_h}} + \overbrace{\sum_{i \in \mathcal{N}} f_{i \to h} \; \lambda_i \; d\, log \; \mu_i}^{\substack{\text{Competitive} \\ \text{Income}_h}}$$

$$+ \overbrace{\sum_{i \in \mathcal{N}} f_{i \to h} \sum_{b \in \mathcal{H}} \chi_b \, d\, \beta_{bi}}^{\substack{\text{Final Demand} \\ \text{Recomposition}_h}} + \overbrace{\sum_{i \in \mathcal{N}} f_{i \to h} \sum_{j \in \mathcal{N}} \mu_j \; \lambda_j \; d\, \widetilde{\Omega}^{x}_{ji}}^{\substack{\text{Intermediate Demand} \\ \text{Recomposition}_h}}$$

Share of sector
i's revenue reaching
worker h's income

Sales Share $\lambda_j = \frac{\text{Sales}_j}{GDP}$

Back

Δ TFP

A.

$$GDP = P_Y\, Y = p_h^c\, C_h + p_l^c\, C_l$$

$$d \log GDP = \chi_h\, d \log p_h^c C_h + \chi_l\, d \log p_l^c C_l$$

Δ TFP

A.

$$GDP = P_Y\, Y = p_h^c\, C_h + p_l^c\, C_l$$

$$d\, log\, GDP = \chi_h\, d\, log\, p_h^c C_h + \chi_l\, d\, log\, p_l^c C_l$$

B. Divisia Index GDP deflator

$$d\, log\, P_Y \equiv \chi_h\, d\, log\, p_h^c + \chi_l\, d\, log\, p_l^c$$

$$= \tilde{\boldsymbol{\Lambda}}_h\, d\, log\, w_h + \tilde{\boldsymbol{\Lambda}}_l\, d\, log\, w_l$$

$$- \boldsymbol{\lambda}_H\, d\, log\,(A_H \times \mu_H) - \boldsymbol{\lambda}_L\, d\, log\,(A_L \times \mu_L)$$

Additional Steps for *d log P_Y*

Start from

$$p_H = \frac{w_h\,\ell_{Hh} + w_l\,\ell_{Hl}}{\mu_H\,A_H\,\ell_{Hh}^{\alpha_H}\,\ell_{Hl}^{1-\alpha_H}}$$

Take first-order approximation

$$\widehat{p}_H = -\widehat{A}_H - \widehat{\mu}_H + \alpha_H\,\widehat{\alpha}_{Hh} + (1-\alpha_H)\,\widehat{\alpha}_{Hl}$$

Do the same for bundle prices

$$\widehat{p}_h^c = -\beta_h\left(\widehat{A}_H + \widehat{\mu}_H\right) - (1-\beta_h)\left(\widehat{A}_L + \widehat{\mu}_L\right) + \widetilde{}_{hh}\,\widehat{w}_h + \widetilde{}_{hl}\,\widehat{w}_l$$

Back

Distortion Centrality Heterogeneity

$$d\,\Lambda = d\,\Lambda_h + d\,\Lambda_l$$

$$\textbf{\textit{Misallocation}} = \overbrace{(\boldsymbol{\delta_l} - \boldsymbol{\delta_h})}^{\geq \quad 0}\; d\,\Lambda_l + \delta_h\; d\,\boldsymbol{\Lambda}$$

$$\boldsymbol{\delta_l} - \boldsymbol{\delta_h} = \overbrace{(\mu_H - \mu_L)}^{\geq \quad 0}\,\overbrace{(\alpha_H - \alpha_L)}^{\geq \quad 0}\,\overbrace{\boldsymbol{a}}^{>\,0}$$

$$\boldsymbol{a} = \frac{1 - (\alpha_H - \alpha_L)\left(1 - (\beta_h - \beta_l)\overbrace{\mu_H \mu_L\,(\alpha_H - \alpha_L)\,(\beta_h - \beta_l)}^{=\,\boldsymbol{b}}\right)}{(\alpha_H \mu_H + \alpha_L \mu_L - \boldsymbol{b})\,(\alpha_L \mu_H + \alpha_H \mu_L - \boldsymbol{b})}$$

Misallocation Decomposition

1. Misallocation $\uparrow$ as expenditure rises for households with high $\boxed{M_h}$

2. Misallocation $\uparrow$ as labor demand for workers with high δ rises

3. Misallocation $\uparrow$ as profit margins fall in sector with high $\boxed{F_i}$

$$
\overbrace{\sum_{h\in\mathscr{H}} \boxed{M_h}\, d\,\chi_h}^{\text{Distributive}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\, \lambda_i \sum_{h\in\mathscr{H}} \delta_h\, d\,\widetilde{\Omega}_{ih}^{\ell}}^{\text{Labor Demand}} + \overbrace{\sum_{i\in\mathscr{N}} \lambda_i\, \boxed{F_i}\, d\,log\,\mu_i}^{\text{Competitive}}
$$

$$
+ \overbrace{\sum_{h\in\mathscr{H}} \chi_h \sum_{i\in\mathscr{N}} \boxed{F_i}\, d\,\beta_{hi}}^{\text{Final Demand}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\, \lambda_i \sum_{j\in\mathscr{N}} \boxed{F_j}\, d\,\widetilde{\Omega}_{ij}^{x}}^{\text{Intermediate Demand}}
$$

4. Misallocation $\uparrow$ as demand of goods $\uparrow$ from sectors with high $\boxed{F_i}$
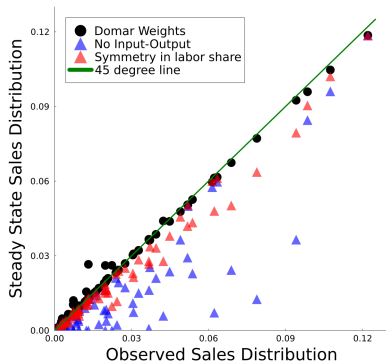
Antisupression Algorithm

1. Significant portion of data supressed to protect confidentiality

2. Since 2007 non-suppressed observations have a random noise infusion multiplier

3. Use information available due to to the industrial and geographical hierarchical nature → manifold of bound and aggregation constraints across hierarchies

4. Two gold standards:
   i. Two-staged algorithm from Isserman & Westervelt (2006)
   ii. Linear programming solution from Eckert et al. (2020)

5. These two methods estimate the number of workers, not their compensation. I develop a three-staged algorithm that starting from the guess Eckert et al. (2020) extends Isserman & Westervelt (2006) to the estimation of labor compensation

Back

Missing Private Employment

1. The CBP only covers some forms of private employment

2. It does not include workers in

- Agriculture production
- Railroads
- Government
- Private household

3. To fill this gap, I use the BEA's Regional Economic Information System to obtain state-level employment and income measures for agricultural and production workers

4. Data sources for REIS are the Quarterly Census of Employment and Statistics from the BLS

5. Main limitation from REIS is that it is only provided at the 2-digit NAICS level

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Moments under Representative Household

Sales Distribution 2021

- Domar Weights
- No Input-Output
- Symmetry in labor share
- 45 degree line

Labor Share

- Observed Labor Share
- Labor Share
- No Input-Output
- Symmetry in labor share
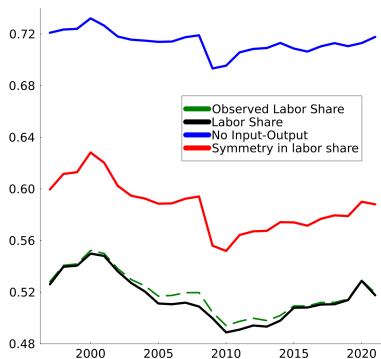
	R^2 *on sales distribution*	R^2 *on labor cost share*
Base Model	0.994	0.981
No Input-Output	0.730	0.733
Symmetry in Labor	0.978	0.933

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Contribution from each component

Table: Counterfactual TFP Growth Differential in the Absence of Components

A. Between 1997 and 2020

Technology	Competitiveness	Misallocation
-23.4%	2.5%	2.8%

B. Between 2002 and 2009

Technology	Competitiveness	Misallocation
-13.0%	**19.3%**	**-8.2%**

C. Between 2010 and 2020

Technology	Competitiveness	Misallocation
-6.3%	**-9.8%**	**7.6%**

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Covariance Decomposition

Table: Covariance Decomposition

A. Between 1997 and 2020

Technology	*Competitiveness*	*-Misallocation*
44.4%	**34.6%**	**21.0%**

B. Between 2002 and 2009

Technology	*Competitiveness*	*-Misallocation*
28.3%	**61.2%**	10.5%

C. Between 2010 and 2020

Technology	*Competitiveness*	*-Misallocation*
58.1%	4.9%	**37.0%**

Back

Moments with Heterogeneous Households

Sales Distribution 2021

Income Distribution 2021
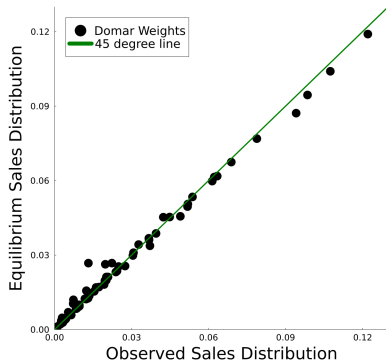
	Observed λ on
Model λ	1.021^{***}
	(0.003)
R^2	0.981
N	$1,650$
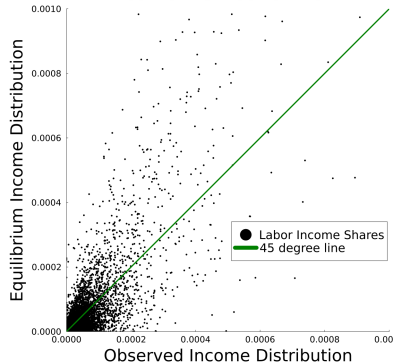
	Observed Λ on
Model Λ	0.435^{***}
	(0.0015)
R^2	0.682
N	$38,189$

Implementation

$$d \log TFP_t = \sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1} \, d \log A_{i,t}$$

$$\underbrace{\phantom{\sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1} \, d \log A_{i,t}}}_{\text{Technology}_t}$$

$$+ \sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1} \, d \log \mu_{i,t}$$

$$\underbrace{\phantom{\sum_i \widetilde{\boldsymbol{\lambda}}_{i,t\text{-}1} \, d \log \mu_{i,t}}}_{\text{Competitiveness}_t}$$

$$- \sum_r \widetilde{\boldsymbol{\Lambda}}_{r,t\text{-}1} \, d \log \Lambda_{r,t}$$

$$\underbrace{\phantom{\sum_r \widetilde{\boldsymbol{\Lambda}}_{r,t\text{-}1} \, d \log \Lambda_{r,t}}}_{\text{Misallocation}_t}$$

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Model without Intermediate Inputs

	Rep. Household		Occupation		County		State & Occupation	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$d \log TFP$	0.523		0.503		0.388		-0.265	
	(0.366)		(0.350)		(0.316)		(0.264)	
Technology		1.341***		0.789***		0.796***		0.847***
		(0.308)		(0.267)		(0.266)		(0.289)
Competitiveness		0.212		0.320		0.454		0.986
		(0.423)		(0.489)		(0.373)		(0.695)
Misallocation		0.573*		0.450		0.335		-0.105
		(0.329)		(0.437)		(0.315)		(0.360)
Intercept	0.012***	0.011***	0.012***	0.012***	0.013***	0.012***	0.015***	0.012***
	(3.2e-3)	(2.0e-3)	(3.2e-3)	(2.2e-3)	(3.2e-3)	(2.1e-3)	(3.0e-3)	(2.2e-3)
Observations				22				
N				66				
H		1		750		3,136		38,190	
R^2	9.2%	71.4%	9.35%	62.4%	7.00%	62.5%	4.8%	60.4%
Adj. R^2	9.2%	68.4%	9.35%	58.4%	7.00%	58.6%	4.8%	56.2%

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Model with Intermediate Inputs

	Rep. Household		Occupation		County		State & Occupation	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$d \log TFP$	0.370***		0.311***		0.316***		0.311***	
	(0.072)		(0.069)		(0.065)		(0.069)	
Technology		0.478***		0.414***		0.416***		0.413***
		(0.097)		(0.081)		(0.083)		(0.082)
Competitiveness		0.398***		0.341***		0.350***		0.342***
		(0.062)		(0.054)		(0.053)		(0.054)
Misallocation		0.074		0.172		0.164		0.168
		(0.138)		(0.125)		(0.135)		(0.125)
Intercept	0.010***	0.009	0.011***	0.010***	0.011***	0.010***	0.011***	0.010***
	(2.1e-3)	(2.0e-3)	(2.2e-3)	(1.8e-3)	(2.1e-3)	(1.9e-3)	(2.3e-3)	(1.9e-3)
Observations				22				
N				66				
H		1		750		3,136		38,190
R^2	56.9%	75.2%	49.9%	75.8%	54.0%	75.4%	49.9%	75.5%
$Adj.\ R^2$	56.9%	72.6%	49.9%	73.3%	54.0%	72.8%	49.9%	73.2%

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Technological Sources

A. Between 1998 and 2020

1	Oil & gas extraction	-11.11%
2	Computer & electronics	-6.64%
3	Telecommunications	-2.85%
4	Computer systems design	-2.30%
5	Administrative services	-1.74%
6	Insurance carriers	-1.45%
7	Farms	-1.34%
8	Primary metals	-1.28%
	⋮	
63	Rental & leasing	1.41%
64	Credit intermediation	1.77%
65	Chemical Products	2.84%
66	Construction	2.87%

C. Between 2010 and 2020

1	Oil & gas extraction	-5.41%
2	Computer systems design	-1.29%
3	Management of companies	-1.26%
4	Housing	-1.14%
5	Other real estate	-1.01%
	⋮	
64	Air transportation	1.03%
65	Chemical products	1.90%
66	Credit intermediation	2.73%

B. Between 2002 and 2009

1	Oil & gas extraction	-5.35%
2	Computer & electronics	-2.84%
3	Telecommunications	-2.27%
4	Utilities	-1.92%
5	Administrative services	-1.06%
	⋮	
66	Construction	1.76%

Competitiveness Sources

A. Between 1998 and 2020

1	Housing	-1.65%
2	Insurance carriers	-1.53%
3	Misc. professional services	-1.10%
4	Other services	-0.89%
	⋮	
63	Publishing industries	0.80%
64	Computer and electronics	1.34%
65	Chemical products	2.57%
66	Credit intermediation	4.10%

B. Between 2002 and 2009

1	Securities & investment	-0.86%
	⋮	
58	Wholesale trade	0.92%
59	Publishing industries	0.93%
60	Internet, & inf. services	0.99%
61	Chemical products	1.35%
62	Telecommunications	1.43%
63	Computer and electronics	1.48%
64	Housing	1.57%
65	Utilities	1.87%
66	Oil & gas extraction	6.59%

C. Between 2010 and 2020

1	Oil & gas extraction	-6.34%
2	Housing	-3.09%
3	Insurance carriers	-0.98%
4	Misc. professional services	-0.87%
5	Administrative services	-0.82%
	⋮	
64	Primary metals	0.80%
65	Chemical products	0.84%
66	Credit intermediation	3.86%

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Distortion Centralities δ

Distortion Centrality Density in 2019
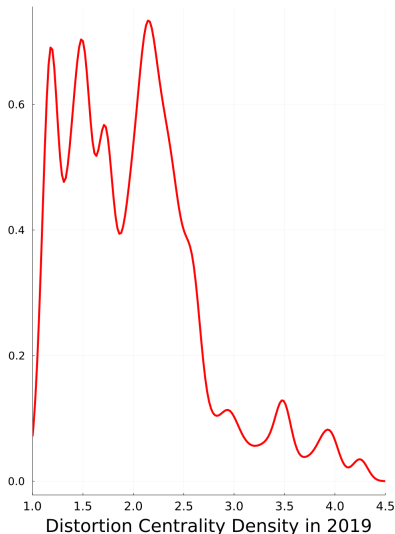
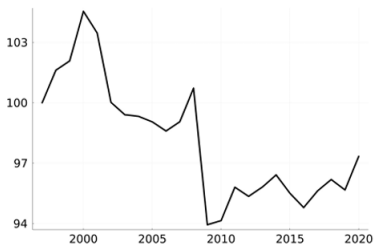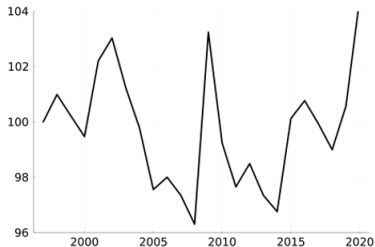| *Lowest* δ	
Nursing Assistant	1.05 - 1.08
Residential Advisor	1.06 - 1.22
Rehabilitation Counselor	1.07 - 1.08
Recreational Therapist	1.07 - 1.09
Food Server	1.07 - 1.45

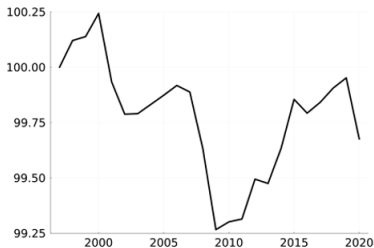| *Highest* δ	
Teller	4.27 - 4.28
New Accounts Clerk	4.24 - 4.27
Loan Interviewer	4.21 - 4.26
Loan Officer	4.23 - 4.26
Credit Analyst	3.89 - 4.22

Sources of Misallocation

Competitive Terms of Trade

Labor Demand Terms of Trade

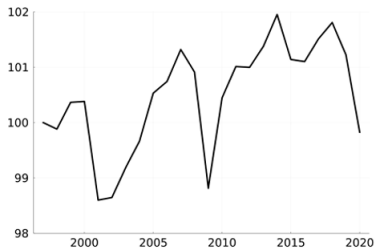Final Demand Terms of Trade

Intermediate Demand Terms of Trade

Table 11: Counterfactual TFP Growth Differential in the Absence of Misallocation Components

A. Between 1998 and 2020

Heterogeneity	*Distributive TT*	*Competitive TT*	*Labor DTT*	*Final DTT*	*Intermediate DTT*
Rep. Household	0%	-3.4%	6.3%	0.4%	-1.3%
Occupation	0%	-5.9%	15.1%	-2.0%	-4.2%
County	0.1%	-5.2%	14.2%	-0.9%	-4.4%
State & Occupation	0.1%	-5.9%	15.6%	-2.6%	-4.5%

B. Between 2002 and 2009

Heterogeneity	*Distributive TT*	*Competitive TT*	*Labor DTT*	*Final DTT*	*Intermediate DTT*
Rep. Household	0%	-9.3%	1.1%	-0.9%	-0.2%
Occupation	0%	-11.0%	3.4%	-1.9%	-0.8%
County	0.1%	-10.4%	3.4%	-0.7%	-1.0%
State & Occupation	0.1%	-11.1%	3.4%	-2.0%	-0.9%

C. Between 2010 and 2020

Heterogeneity	*Distributive TT*	*Competitive TT*	*Labor DTT*	*Final DTT*	*Intermediate DTT*
Rep. Household	0%	3.9%	1.2%	1.7%	0.9%
Occupation	0%	2.9%	7.2%	0.2%	-1.8%
County	0.1%	3.0%	3.5%	2.1%	-1.5%
State & Occupation	0.1%	2.8%	7.4%	-0.1%	-1.7%

Table 12: Counterfactual TFP Growth Without Sectoral Competitive TT

A. Between 1998 and 2020

1	Credit intermediation	-2.16%
2	Chemical products	-1.06%
3	Computer & electronics	-0.98%
4	Publishing industries	-0.80%
5	Internet & inf. services	-0.69%
	⋮	
64	Insurance carriers	0.77%
65	Other services	0.81%
66	Misc. professional services	0.87%

B. Between 2002 and 2009

1	Oil & gas extraction	-1.46%
2	Computer & electronics	-1.11%
3	Internet & inf. services	-1.01%
4	Wholesale trade	-0.92%
5	Telecommunications	-0.86%
6	Utilities	-0.84%
7	Publishing industries	-0.82%

C. Between 2010 and 2020

1	Credit intermediation	-2.0%
2	Securities & investment	-0.52%
	⋮	
64	Administrative services	0.62%
65	Misc. professional services	0.70%
66	Oil & gas extraction	1.91%

Table 13: Counterfactual TFP Growth Without Sectoral Labor Demand TT

A. Between 1998 and 2020

1	Wholesale trade	-1.62%
2	Insurance carriers	-1.61%
3	Other retail	-1.07%
	⋮	
61	Utilities	0.69%
62	Computer systems design	0.82%
63	Publishing industries	1.34%
64	Oil & gas extraction	1.79%
65	Computer & electronics	2.28%
66	Credit intermediation	2.40%

B. Between 2002 and 2009

1	Securities & investment	-0.96%
	⋮	
64	Computer & electronicss	0.85%
65	Utilities	1.02%
66	Oil & gas extraction	2.20%

C. Between 2010 and 2020

1	Wholesale trade	-1.70%
2	Insurance carriers	-1.03%
3	Administrative services	-0.93%
4	Other retail	-0.83%
	⋮	
64	Publishing industries	0.89%
65	Computer & electronics	0.98%
66	Credit intermediation	2.44%

Table 14: Counterfactual TFP Growth Without Sectoral Final Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.50%
2	Motor vehicles	-0.91%
3	Machinery	-0.88%
4	Apparel & leather	-0.51%
	⋮	
62	Securities & investment	0.87%
63	Misc. professional services	0.94%
64	Hospitals	0.95%
65	Internet & inf. services	1.01%
66	Wholesale trade	1.18%

B. Between 2002 and 2009

1	Construction	-1.22%
2	Motor vehicles	-0.82%
	⋮	
66	Hospitals	0.58%

C. Between 2010 and 2020

1	Computer & electronis	-0.52%
	⋮	
63	Other retail	0.59%
64	Internet & inf. services	0.60%
65	Construction	0.89%
66	Wholesale trade	1.08%

Table 15: Counterfactual TFP Growth Without Sectoral Intermediate Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.24%
2	Credit intermediation	-0.90%
3	Publishing industries	-0.76%
4	Computer systems design	-0.45%
5	Ambulatory health	-0.42%
	⋮	
61	Telecommunications	0.52%
62	Administrative services	0.54%
63	Hospitals	0.56%
64	Insurance carriers	0.74%
65	Other retail	0.90%
66	Wholesale trade	1.21%

B. Between 2002 and 2009

1	Computer & electronics	-0.48%
	⋮	
66	Securities & investment	0.49%

C. Between 2010 and 2020

1	Credit intermediation	-0.97%
2	Publishing industries	-0.51%
3	Computer & electronics	-0.49%
	⋮	
63	Insurance carriers	0.52%
64	Administrative services	0.63%
65	Other retail	0.66%
66	Wholesale trade	1.12%

Normalized nested CES environment - Firms

Firms

$$\frac{y_i}{\overline{y}_i} = A_i \left(\sum_{h \in \mathcal{H}} \widetilde{\Omega}_{ih}^{\ell} \left(\frac{\ell_{ih}}{\overline{\ell}_{ih}} \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{j \in \mathcal{N}} \widetilde{\Omega}_{ij}^{x} \left(\frac{x_{ij}}{\overline{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}$$

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Normalized nested CES environment - Households

Households

$$U_h\left(c_h, \widetilde{L}_h\right) = \frac{\left(c_h\left(1 - E_h^{-\gamma_h}\widetilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1-\sigma} \quad s.t. \quad \frac{C_h}{\overline{C}_h} = \left(\sum_{i \in \mathcal{N}} \beta_{hi}\left(\frac{C_{hi}}{\overline{C}_{hi}}\right)^{\frac{\rho_h-1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h-1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \widetilde{L}_h$

Normalized nested CES environment - Households

Households

$$U_h\left(c_h, \widetilde{L}_h\right) = \frac{\left(c_h \left(1 - E_h^{-\gamma_h}\widetilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1 - \sigma} \quad s.t. \quad \frac{C_h}{\overline{C}_h} = \left(\sum_{i \in \mathcal{N}} \beta_{hi} \left(\frac{C_{hi}}{\overline{C}_{hi}}\right)^{\frac{\rho_h - 1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h - 1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \widetilde{L}_h$

The change in labor supply from type h workers is, to a first-order

$$d \log L_h = \zeta_h^n \, d \log n_h + \zeta_h^w \, d \log w_h - \zeta_h^e \, d \log E_h$$

Where the corresponding elasticities are given by

$$\zeta_h^n = \frac{E_h^{\gamma_h}}{1 - \varphi_h\gamma_h} \frac{n_h}{L_h}, \qquad \zeta_h^w = \frac{1}{1 - \varphi_h\gamma_h} \frac{\varphi_h}{\Gamma_h}, \qquad \zeta_h^e = \zeta_h^w - \gamma_h\zeta_h^n.$$

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Solution - Expenditure & Wages

$$d\ log\ E_h = \overbrace{\frac{\zeta_h^n \Gamma_h}{1 + \zeta_h^e \Gamma_h}}^{\substack{\text{Demographic Effect} \\ \text{on Expenditure (PE)}}}\ d\ log\ n_h + \overbrace{\frac{\left(1 + \zeta_h^w\right)\Gamma_h}{1 + \zeta_h^e \Gamma_h}}^{\substack{\text{Wage Effect} \\ \text{on Expenditure (GE)}}} d\ log\ w_h + \overbrace{\sum_{i \in \mathcal{N}} \frac{\kappa_{ih} \lambda_i}{\left(1 + \zeta_h^e \Gamma_h\right)\chi_h}\left((1 - \mu_i)\ d\ log\ S_i - \mu_i\ d\ log\ \mu_i\right)}^{\substack{\text{Corporate Income Effect} \\ \text{on Expenditure (PE + GE)}}};$$

$$d\ log\ w_h = \overbrace{\frac{\zeta_h^e}{1 + \zeta_h^w}}^{\substack{\text{Expenditure Effect} \\ \text{on Wages (GE)}}}\ d\ log\ E_h - \overbrace{\frac{\zeta_h^n}{1 + \zeta_h^w}}^{\substack{\text{Demographic Effect} \\ \text{on Wages (PE)}}}d\ log\ n_h + \overbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{\left(1 + \zeta_h^w\right)\Lambda_h}\left((\theta_i - 1)\ d\ log\ A_i + \theta_i\ d\ log\ \mu_i\right)}^{\substack{\text{Direct Effect} \\ \text{on Wages (PE)}}}$$

$$-\overbrace{\sum_{j \in \mathcal{N}}\left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{\left(1 + \zeta_h^w\right)\Lambda_h}\left(\theta_i - 1\right)\widetilde{\psi}_{ij}^\times\right)\left(d\ log\ A_j + d\ log\ \mu_j\right)}^{\substack{\text{Supplier Effect} \\ \text{on Wages (PE)}}} + \overbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{\left(1 + \zeta_h^w\right)\Lambda_h}\ d\ log\ S_i}^{\substack{\text{Sales Effect} \\ \text{on Wages (GE)}}}$$

$$-\overbrace{\left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{\left(1 + \zeta_h^w\right)\Lambda_h}\left(\theta_i - 1\right)\right)d\ log\ w_h}^{\substack{\text{Direct Substitution Effect} \\ \text{on Wages (GE)}}} + \overbrace{\sum_{b \in \mathcal{H}}\left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{\left(1 + \zeta_h^w\right)\Lambda_h}\left(\theta_i - 1\right)\widetilde{\psi}_{ib}^\ell\right)d\ log\ w_b}^{\substack{\text{Supplier Substitution Effect} \\ \text{on Wages (GE)}}};$$
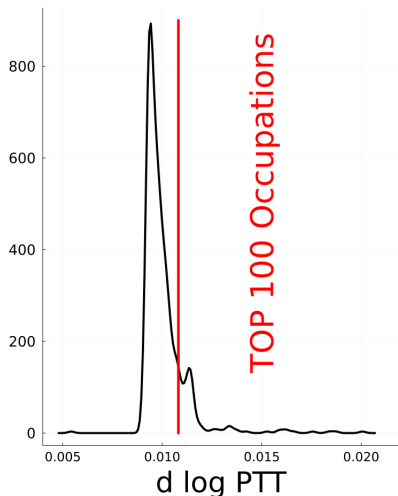
Solution - Sales

$$d \log S_i = \overbrace{\sum_{h \in \mathcal{H}} \frac{\beta_{hi}\chi_h}{\lambda_i} \, d \log E_h}^{\substack{\text{Expenditure Effect} \\ \text{on Sales (GE)}}} + \overbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} \, d \log S_j}^{\substack{\text{Sales Effect} \\ \text{on Sales (GE)}}} + \overbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} \left(\left(\theta_j - 1\right) d \log A_j + \theta_j \, d \log \mu_j \right)}^{\substack{\text{Direct Effect} \\ \text{on Sales (PE)}}}$$

$$+ \overbrace{\sum_{j \in \mathcal{N}} \left(\sum_{h \in \mathcal{H}} \frac{\beta_{hi}\chi_h}{\lambda_i} (\rho_h - 1) \left(\widetilde{\psi}_{ij}^x - \widetilde{\mathscr{B}}_{hj} \right) + \sum_{q \in \mathcal{N}} \frac{\Omega_{qi}^x \lambda_q}{\lambda_i} \left(\theta_q - 1 \right) \left(\widetilde{\psi}_{ij}^x - \widetilde{\psi}_{qj}^x \right) \right) \left(d \log A_j + d \log \mu_j \right)}^{\substack{\text{Supplier Effect} \\ \text{on Sales (PE)}}}$$

$$+ \overbrace{\sum_{h \in \mathcal{H}} \left(\sum_{b \in \mathcal{H}} \frac{\beta_{bi}\chi_b}{\lambda_i} (\rho_b - 1) \left(\widetilde{\mathscr{C}}_{bh} - \widetilde{\psi}_{ih}^\ell \right) + \sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} \left(\theta_j - 1 \right) \left(\widetilde{\psi}_{jh}^\ell - \widetilde{\psi}_{ih}^\ell \right) \right) d \log w_h}^{\substack{\text{Supplier Substitution Effect} \\ \text{on Sales (GE)}}} .$$

Back

$$d \log TFP = \underbrace{\textbf{\textit{Competitiveness}}}_{= 1\%} - \textit{Misallocation}$$

Best Sectors	**d log TFP**
1. Housing	0.766%
2. Credit Intermediation	0.414%
3. Oil & Gas extraction	0.384%
4. Furniture	0.370%
5. Mining, except Oil & Gas	0.364%

Worst Sectors	**d log TFP**
1. Nursing & Residential Care	-0.329%
2. Social Assistance	-0.303%
3. General Merchandise Store	-0.274%
4. Hospitals	-0.219%
5. Ambulatory health care	-0.201%

	d log TFP on		
μ_i	-0.974*** (0.12)		-0.919*** (0.18)
λ_i		1.351 (0.95)	-0.132 (0.73)
F_i		-0.427*** (0.08)	-0.046 (0.11)
R^2	0.48	0.03	0.29	0.48
N		66		

We want competition shocks in sectors with low μ_i!

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Effects from more competition in Housing

Top 100 occupations

Construction workers 48

Painters, Carpet Installer, Tile Setter,

Stonemason, Plasterer, Drywall Installer,

Septic Servicer, Construction Supervisor

Financial specialist 7

Property appraiser, Loan Officer

Credit Analyst, Financial Examiner

Extraction Workers 7

Rock Splitter, Roof Bolter

Woodworkers 6

Cabinetmaker, Furnite Finisher

Installation & Maintenance 5

Heating & AC, Mobile Home Installer